

As an illustration we shall consider the case of a constant field ( $A_\alpha = 0$ ,  $\alpha = 1, 2, 3$ ) in the approximation  $\Gamma_i(q', q'') = 1$ . We shall assume (compare reference 8; the equation has a meaning also for the determination of the Green function as an ensemble average)

$$G_0(p) = -\frac{1}{(2\pi)^4} \left\{ \frac{n_p}{p_0 - \omega_p - i\gamma'_p} - \frac{1 - n_p}{p_0 - \omega_p + i\gamma''_p} \right\}$$

( $\omega_p, \gamma'_p, \gamma''_p$  — real;  $\gamma', \gamma'' > 0$ ). (5)

We must note that Eq. (5) is by no means general (in particular, this assumption means that we restrict ourselves to one — perhaps spin-degenerate-band). A number of interesting cases, however, are included here. One can easily evaluate  $\Delta G(x, y)$  under the given conditions. Taking (1) into account we get for the change in the occupation numbers in momentum space:

(a) if  $\gamma' = \gamma'' = \eta \rightarrow 0$ :

$$\Delta n_p = -\frac{eE_\alpha}{s + ik_0} \frac{\partial n_p}{\partial p_\alpha}; \quad (6)$$

(b) if  $\gamma', \gamma''$  are finite and  $k_0 = 0$  (the calculation is also simple for  $k_0 \neq 0$ , but leads to a more unwieldy result):

$$\Delta n_p = -eE_\alpha \frac{\partial}{\partial p_\alpha} \frac{n_p(1 - n_p)}{\gamma_p}. \quad (7)$$

Here  $\mathbf{E}$  is the field strength (with potential  $\varphi$ ),  $\gamma_p = \gamma'_p + \gamma''_p$ . The right hand side of (6) agrees, as was to be expected, exactly with the "accelerating" term of the transport equation, if we understand by  $s^{-1}$  the time of action of the field. The static electrical conductivity tensor has from (7) the form (in the usual units,  $m$  is the true electron mass)

$$\sigma_{\alpha\beta} = -\frac{e^2}{m} \frac{2}{(2\pi\hbar)^3} \int d\mathbf{p} p_\alpha \frac{\partial}{\partial p_\beta} \frac{n_p(1 - n_p)}{\gamma_p}. \quad (8)$$

In the particular case of Boltzmann statistics and isotropic model ( $n_p \sim \exp(-\hbar\omega_p/kT)$ ,  $\omega_p = \omega(|\mathbf{p}|)$ ,  $\gamma_p = \gamma(|\mathbf{p}|)$ ) Eq. (7) agrees formally with the result of applying the transport equation, if we take for the "relaxation time" the quantity

$$\tau_p = \frac{1}{\gamma_p} \left[ 1 + kT \frac{\partial \ln \gamma_p}{\partial \hbar\omega_p} \right]. \quad (9)$$

We emphasize, however, that Eq. (8) is valid over a far wider range than the transport equation.

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168

### THERMODYNAMIC THEORY OF FERROMAGNETIC ABSORPTION

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IF a ferromagnet is in an alternating magnetic field, its macroscopic state changes; this means that the specimen is absorbing energy from the field (ferromagnetic absorption). The basic causes of ferromagnetic absorption are, as is known, the following: the nonvanishing time for establishment of equilibrium within the spin system (intraspin relaxation), and the nonvanishing time for establishment of equilibrium between the spin system and the lattice (spin-lattice relaxation). In this article it is shown that for an isotropic ferromagnetic medium near the Curie temperature, it is possible to determine the law of ferromagnetic resonance if the specific form of the thermodynamic potential is known.

A nonequilibrium state of the system can be described by a thermodynamic potential that depends on nonequilibrium thermodynamic parameters. We suppose that a ferromagnet without hysteresis is in an external magnetic field, and that the amplitudes of the constant and alternating fields are pointed in mutually perpendicular planes (the case of perpendicular fields). In the second case  $H_0$  and  $h$  are pointed along the  $z$  axis

(the case of parallel fields). The nonequilibrium thermodynamic potential in the stated problem has the form

$$\Phi = \Phi_0 + \alpha M^2 + 1/2 \beta M^4 - MH', \quad (1)$$

where  $\Phi_0$ ,  $\alpha$ , and  $\beta$  are functions of temperature and pressure,  $M$  is the magnetization, and  $H'$  is the effective magnetic field.<sup>1</sup> In the case of weak fields the magnetization  $M$  coincides with the spontaneous magnetization  $M_S$ .

A nonequilibrium state of the system may be described<sup>1-3</sup> by nonequilibrium thermodynamic parameters, which are given by the relations

$$\mathbf{h} = \mathbf{H}' - \mathbf{H}_0 = \mathbf{h}_0 e^{i\omega t},$$

$$\mathbf{m} = \mathbf{M} - \mathbf{M}_0 = \mathbf{m}_0 e^{i\omega t}, \quad \vartheta = T - T_0. \quad (2)$$

Here the quantities with index 0 correspond to an equilibrium state of the system;  $T$  is the temperature of the spin system,  $T_0$  is the temperature of the lattice,  $\omega$  is the frequency of the alternating field. The equation of motion for the alternating part of the magnetization is expressed by the formula

$$\dot{\mathbf{m}} = -\varphi(H', T) \nabla \Phi + g[\mathbf{H}' \cdot \mathbf{M}], \quad (3)$$

where  $\varphi(H', T)$  is some function, which remains undetermined within the framework of thermodynamic theory;  $g$  is the gyromagnetic ratio. On limiting ourselves to terms linear in  $\mathbf{h}$  and  $\mathbf{m}$ , we shall have for the perpendicular field

$$\begin{aligned} \tau \dot{m}_x + A_x m_x - \tau \omega_{0x} m_y &= \chi_f h_x, \\ \tau \dot{m}_y + A_y m_y + \tau \omega_{0y} m_x &= \chi_f \omega_{0x} h_x, \\ \tau \dot{m}_z + A_z m_z + (\chi_f H'/T) \vartheta &= 0, \\ \omega_0 &\equiv (\omega_{0x} \omega_{0y})^{1/2} \\ &= g \{H_0 + (N_x - N_y) M_0\}^{1/2} \{H_0 + (N_y - N_z) M_0\}^{1/2}, \\ A_{(x,y,z)} &\equiv 1 + \alpha^{-1} [\beta M_S^2 + N_{(x,y,z)}], \end{aligned} \quad (4)$$

where  $\tau \equiv \chi_f / \varphi$  is the spin-spin relaxation time for  $\mathbf{h}$  perpendicular to  $\mathbf{H}_0$ ,  $\chi_f$  is the magnetic susceptibility, which depends on  $\alpha$  (reference 4),  $\omega_0$  is the resonance frequency, and  $N$  is a demagnetizing factor.

For complete solution of the problem it is necessary to introduce another equilibrium equation in  $\mathbf{m}$  and  $\theta$ ; it will correspond to spin-lattice interaction. According to the first law of thermodynamics, with (1) and (2) taken into account, we find:

$$(A'T/\chi_f H') \delta \tau' \dot{m} - (1 - \delta) \tau' \dot{\vartheta} - \vartheta = 0, \quad (5)$$

where  $\tau' = \alpha_1 T^2 / (\alpha + CH')$  is the spin-lattice relaxation time,  $A' = 1 + 3\beta M_S^2 / \alpha$ ,  $\delta = CH'^2 / (\alpha + CH'^2)$ , and  $\alpha_1$ ,  $\alpha$ , and  $C$  are constant coefficients.

For parallel fields the last equation in (4) gives

$$\tau \dot{m} + A'm + A'' (\chi_f H'/T) \vartheta = \chi_f h, \quad (6)$$

where  $A'' = 1 + KM_S^2$ , and where  $K$  is a constant. On eliminating  $\theta$  from (5) and (6), we get an equation relating to the magnetization  $\mathbf{m}$ , namely

$$\begin{aligned} (1 - \delta) \ddot{m} + (\gamma/\tau + 1/\tau') \dot{m} + A'm / (\tau'\tau) \\ = [(1 - \delta) \dot{h} + h/\tau'] \chi_f / \tau, \end{aligned} \quad (7)$$

where  $\tau$  represents the "transverse" relaxation time and  $\tau'$  the "longitudinal" relaxation time according to Bloch, and  $\gamma = A'[1 - \delta(A'' - 1)]$ .

If we bear in mind that  $\mathbf{m}$  and  $\mathbf{h}$  are the periodic functions (2), solution of equation (4) in the case of perpendicular fields gives

$$\begin{aligned} \frac{\chi'}{\chi_f} &= \frac{[A_x A_y + \tau^2 (\omega_0^2 - \omega^2)] (A_y + \tau^2 \omega^2) + \tau^2 \omega^2 (A_x + A_y)}{[A_x A_y + (\omega_0^2 - \omega^2) \tau^2]^2 + (A_x + A_y)^2 \tau^2 \omega^2}, \\ \frac{\chi''}{\chi_f} &= \frac{\{[A_y + \tau^2 \omega_0^2 (A_x + A_y)] - [A_x A_y + \tau^2 (\omega_0^2 - \omega^2)]\} \tau \omega}{[A_x A_y + (\omega_0^2 - \omega^2) \tau^2]^2 + (A_x + A_y)^2 \tau^2 \omega^2}. \end{aligned} \quad (8)$$

In the case of parallel fields, we find from (7):

$$\begin{aligned} \frac{\chi'}{\chi_f} &= \frac{\{(\tau + \gamma\tau') - (1 - \delta) [A' - \tau\tau' (1 - \delta) \omega^2] \tau'\} \omega}{[A' - (1 - \delta) \tau\tau' \omega^2]^2 + (\tau + \gamma\tau')^2 \omega^2}, \\ \frac{\chi''}{\chi_f} &= - \frac{\{\tau + [\gamma - (1 - \delta) A'] \tau'\} \omega + (1 - \delta)^2 \tau\tau'^2 \omega^3}{[A' - (1 - \delta) \tau\tau' \omega^2]^2 + (\tau + \gamma\tau')^2 \omega^2}. \end{aligned} \quad (9)$$

The mean absorption energy of the spin system per unit time, by virtue of (2) and (8), has the form:

$$\varepsilon = \frac{1}{2} \chi_f h_0^2 \frac{\{(A_y + \tau^2 \omega_0^2) (A_x + A_y) - [A_x A_y + \tau^2 (\omega_0^2 - \omega^2)]\} \tau \omega^2}{[A_x A_y + (\omega_0^2 - \omega^2) \tau^2]^2 + (A_x + A_y)^2 \tau^2 \omega^2}.$$

Above the Curie point,  $M_S = 0$ ,  $A = A' = A'' = \gamma = 1$ , and  $\omega_0 = gH_0$ ; i.e., results are obtained that agree with the results of references 2 and 5.

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