

NON-UNIQUENESS OF THE PHASE SHIFT ANALYSIS OF PROTON-PROTON COLLISIONS

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A method based on the optical model is proposed for the phase shift analysis of p-p collisions at energies $E \lesssim 10$ Bev. The refractive and absorption coefficients for energies $E \sim 1$ Bev are found to be of the same order of magnitude. The calculated differential cross sections for elastic scattering, the total cross section, and the cross section for inelastic processes agree with experiment.

THE analysis of nucleon-nucleon scattering experiments at high energies on the basis of the phenomenological optical model has been the subject of many papers.¹⁻⁴ In references 1,* 2, and 4 it is assumed that the real part of the complex phase η_l can be neglected in comparison to the imaginary part

$$\text{Re } \eta_l \ll \text{Im } \eta_l. \tag{1}$$

Without assumption (1) the phase shift analysis of the experimental angular distributions becomes exceedingly complicated. No results of such calculations have so far been published.

Since the validity of assumption (1) becomes better as the energy increases owing to the fast increase in the number of possible channels for inelastic processes,⁵ we can, for sufficiently high energy $E = E^*$, compute $\eta_l(E^*) \approx i \text{Im } \eta_l(E^*)$ and the corresponding spatial distribution of the absorption coefficient $K = K(E^*; r)$ by the method described in reference 6. The refractive coefficient is then

$$N(E^*; r) = 1 + N_0(E^*; r) \approx 1.$$

Applying the optical model to the interaction of nucleons with nuclei, we can write the coefficients $K(E; r)$ and $N_0(E; r)$ in the form

$$K(E; r) = k(E)\rho(r), \quad N_0(E; r) = n(E)\rho(r), \tag{2}$$

where $\rho(r)$ is the nuclear matter density. For nucleon-nucleon interactions relation (2) is not satisfied in general, since the refraction and absorption coefficients in different regions of the nucleon describe an interaction between different kinds of particles. For example, the energy dependence of $K(E; r)$ may be notably different for central collisions and for grazing collisions,

*We thank the authors of reference 1 for sending us the manuscript before publication.

where only the pion clouds interact with each other. However, it can be assumed that for energies < 10 Bev, where the wave length λ is still large, it is admissible within the present-day experimental errors to write the experimental coefficients $K(E; r)$ and $N_0(E; r)$, averaged over the internal structure of the nucleon, in the form (2).

Let us now consider the relative coefficients of absorption, k , and refraction, n :

$$\begin{aligned} K(E; r) &= k(E)K(E^*; r); \\ N(E; r) &= n(E)K(E^*; r), \end{aligned} \tag{3}$$

where $K(E^*; r) \equiv K^*(r)$ is the average coefficient computed by Grishin;⁴ $E^* = 6.15$ Bev. The imaginary and real parts of the phase shift $\eta_l(E)$ can then be determined from the equations

$$\begin{aligned} \sigma_{in} &= \pi\lambda^2 \sum_{l=0}^{\infty} (2l+1) [1 - \exp(-4k\eta_l^*)], \\ \sigma_t &= 2\pi\lambda^2 \sum_{l=0}^{\infty} (2l+1) [1 - \exp(-2k\eta_l^*) \cos(2n\eta_l^*)], \end{aligned} \tag{4}$$

where

$$\eta_l^* = \int_0^{\infty} K^* (\sqrt{\lambda^2 l(l+1) + s^2}) ds, \tag{5}$$

and σ_t and σ_{in} are the experimental values of the total cross section and the cross section for all inelastic processes in p-p collisions at the energy E .

The results of the calculation are listed in Table I. In accordance with the results obtained at Dubna,⁷ we set $\sigma_t = 30 \times 10^{-27}$ cm² and $\sigma_{in} =$

TABLE I

$E, \text{ Bev}$	1.5	2.24	2.75	4.40	6.15	10.0
$k \cdot 10^{13} \text{ cm}$	1.28	1.16	1.16	1.02	1.00	0.94
$n \cdot 10^{13} \text{ cm}$	1.48	1.36	1.10	0.52	0.0	0.050

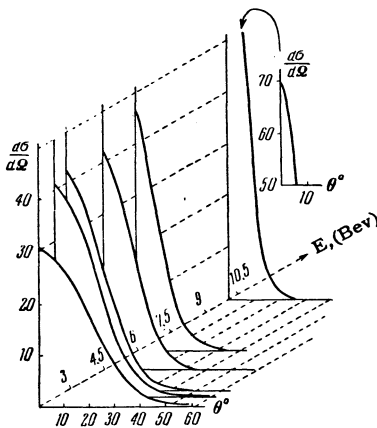
$23.0 \times 10^{-27} \text{ cm}^2$ at the energy $E = 10 \text{ Bev}$; the cross sections for the other energies are the same as in the paper by Grishin, Saitov, and Chuvilo.³

It follows from Table I that condition (1) is not valid for our kind of phase shift analysis in the region of energies of the order of a few Bev. However, we do have the relation $\text{Re } F < \text{Im } F$ in this energy region. This can be seen from Table II, where we list the computed values of the ratio $\nu(\theta) = \text{Re } F(\theta)/\text{Im } F(\theta)$ for $\theta = 0^\circ$ and $\theta = 10^\circ$. The values of σ_t [see formula (4)] are therefore less sensitive to the choice of n than to the choice of k . At $E = 10 \text{ Bev}$ the computed value of n is ≈ 0 . This confirms our assumption that $n(E^*) = 0$.

TABLE II

E, Bev	1.5	2.24	2.75	4.40	6.15	10.0
$\nu(0)$	0.47	0.49	0.44	0.31	0.00	0.036
$\nu(10)$	0.47	0.46	0.42	0.29	0.00	0.031

Using the values of the coefficients k and n of Table I and the values of $K^*(r)$ from the paper by Grishin,⁴ we calculated the angular distribution $d\sigma/d\Omega$ of the elastically scattered protons. The results of this calculation are shown in the figure.



Differential cross sections for the elastic scattering of protons in the center of mass system in mbn/sterad.

The theoretical and experimental values are in satisfactory agreement within the experimental limits of error (for the bibliography see references 3 and 4).*

*There is a small discrepancy between the computed and experimental distributions in the small angle region at $E = 2.24 \text{ Bev}$. This comes from the fact that we used the average experimental value of σ_t for the calculation of $n(E)$. The angular distribution agrees better with experiment, if one chooses for σ_t a value which is closer to the lowest experimental value $\sigma_t = (44.1 \pm 4) \times 10^{-27} \text{ cm}^2$.

All the presently available experimental results can in this way be adequately described on the basis of quite different assumptions about the magnitude of the phase shifts. However, in our opinion the phase shifts calculated by formulas (3) to (5) are preferable to those calculated on the basis of assumption (1). Our assumptions include condition (1) as a special case and define the limits of its applicability. Even in this case the results depend, of course, on the values of σ_t and σ_{in} used (within the experimental errors).

For a definite conclusion one needs more exact measurements of the angular distributions at energies of ~ 1 to 3 Bev .

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