

FIG. 2. ●)  $\beta$  spectrum; ○)  $e^- - \gamma$  coincidence spectrum.

the intensity of the  $\gamma$  ray in cascade with the 134-keV line in  $\text{Ce}^{144}$  is less than  $4 \times 10^{-4}$  of that for the 134-keV transition. This value is defined by the statistical accuracy of the measurements. However, in a previous study,<sup>7</sup> we found an indication of a soft  $\beta$  spectrum with  $E_\beta \approx 130$  keV and we suggested that there may be a 41 to 134 keV cascade.

The absence of coincidences among the  $\gamma$  rays has prompted us to investigate coincidences of the 134-keV  $\gamma$  line with the 35-keV conversion line that had been observed in the primary  $\beta$  spectrum and that was identified as an L conversion transition of the 41-keV level. These coincidences were measured with a device described previously.<sup>6,7</sup> Figure 2 shows a portion of the coincidence spectrum which was obtained from one of a series of measurements. Also shown in Fig. 2 is the corresponding portion of the singles  $\beta$  spectrum. These data indicate that the 41L conversion line is in coincidence with the 134-keV  $\gamma$  line.

All these data are in agreement with a decay scheme which includes a 175-keV level.<sup>2,4,7</sup> In this case one must assume that the 41-keV transition undergoes practically total conversion. The values computed for the conversion coefficient in the L shell<sup>9</sup> show that for a multipolarity which is not less than E2, or even for E2 with an admixture of M1, the conversion coefficient is close to unity, and this is in complete agreement with the data from our measurements of  $\gamma - \gamma$  and  $e^- - \gamma$  coincidences.

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<sup>8</sup> Hickok, McKinley, and Fultz, Phys. Rev. **109**, 113 (1958).

<sup>9</sup> G. F. Dranitsina, Коэффициенты внутренней конверсии на  $L_I, L_{II}, L_{III}$  подоболочках (The Coefficients of Internal Conversion in the  $L_I, L_{II},$  and  $L_{III}$  Subshells) Acad. Sci. Press (1957).

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### THE ELASTIC SCATTERING OF POLARIZED DIRAC PARTICLES BY A SHORT RANGE CENTER OF FORCE IN THE DAMPING THEORY

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THE elastic scattering of Dirac particles by a fixed short range center of force including the effect of damping was investigated in references 1 and 2 (in the following we shall adhere to the notations of these papers). Reference 2 also took into account the polarization effects for the special case of scattering from a  $\delta$ -function potential.

In the present note we consider the elastic scattering of Dirac particles by an arbitrary center of force with the inclusion of the polarization effects, using the damping theory. The solution of the fundamental integral equation of the damping theory is [cf. formula (28) of reference 2]

$$B'_{m_s} = \frac{8\pi^2 c \hbar}{kK} \sum_{n,l,m} \frac{c_l^{(n)}}{1 + ic_l^{(n)}} \Omega_{lm}^{(n)}(\mathbf{k}', m_s) \Omega_{lm}^{*(n)}(\mathbf{k}, m_s) B_{m_s}. \quad (1)$$

$B_{m_s} \equiv B_{m_s}(\mathbf{k})$  and  $B'_{m_s} \equiv B_{m_s}(\mathbf{k}')$  are the amplitudes corresponding to the projection of the spin of the particle on the  $z$  axis ( $m_s = \pm \frac{1}{2}$ ) in the initial and final states;  $\Omega_{lm}^{(n)}(\mathbf{k}, m_s)$  are the components of the spherical spinors; the quantities

$$c_l^{(n)} = -\tan \gamma_{l'}^{(n)} = \frac{1}{2} \left[ \left( 1 + \frac{k_0}{K} \right) c_l + \left( 1 - \frac{k_0}{K} \right) c_{l \pm 1} \right] \quad (n = 1, 2), \quad (2)$$

$$c_l = -\tan \gamma_l = \frac{\pi K}{c \hbar} \int_0^\infty r V(r) J_{l \pm 1/2}^2(kr) dr \quad (3)$$

determine the scattering phase shifts for Dirac and spinless relativistic particles, respectively;  $n = 1, 2$  denotes the value of the quantum number  $j$  for given  $l$ :  $j = l + \frac{1}{2}$  for  $n = 1$ , and  $j = l - \frac{1}{2}$  for  $n = 2$ .

The differential cross section for elastic scattering, taking into account the direction of the spin of the incoming and scattered particles, is given by

$$d\sigma_{m_s, m_s} = (K^2 / 4\pi^2 c^2 \hbar^2) \left( B'_{m_s} B_{m_s} / \sum_{m_s} B'_{m_s} B_{m_s} \right) d\Omega', \quad (4)$$

where  $d\Omega' = \sin \theta' d\theta' d\varphi'$  is the element of solid angle.

With the  $z$  axis taken along the momentum of the incoming particle ( $\mathbf{k} \parallel z$ ), we obtain for the amplitudes  $B'_{m_s}$ :

$$\begin{aligned} B'_{1/2} &= \frac{2\pi c \hbar}{K} [f(\theta') B_{1/2} - g(\theta') e^{-i\varphi'} B_{-1/2}], \\ B'_{-1/2} &= \frac{2\pi c \hbar}{K} [g(\theta') e^{i\varphi'} B_{1/2} + f(\theta') B_{-1/2}], \end{aligned} \quad (5)$$

where

$$f(\theta') = \frac{1}{2ik} \sum_l \left[ (l+1)(e^{2i\eta_l^{(1)}} - 1) + l(e^{2i\eta_l^{(2)}} - 1) \right] \times P_l(\cos \theta'), \quad (6)$$

$$g(\theta') = \frac{1}{2ik} \sum_l \left[ -e^{2i\eta_l^{(1)}} + e^{2i\eta_l^{(2)}} \right] P_l^1(\cos \theta'). \quad (7)$$

From (4) we find the following expression for the differential cross section, for an arbitrary initial direction of the spins,

$$d\sigma = \sum_{m_s} d\sigma_{m_s, m_s} = |f(\theta')|^2 + |g(\theta')|^2 + (g^*(\theta')f(\theta') - g(\theta')f^*(\theta')) \times \{ (B'_{1/2} B_{-1/2} e^{-i\varphi'} - B'_{-1/2} B_{1/2} e^{i\varphi'}) / (B'_{1/2} B_{1/2} + B'_{-1/2} B_{-1/2}) \}. \quad (8)$$

It follows from these formulas that the scattering of an initially unpolarized beam ( $B'_{1/2} B_{1/2} + B'_{-1/2} B_{-1/2} = 1$ ;  $B'_{1/2} B_{1/2} + B'_{-1/2} B_{-1/2} = B'_{1/2} B_{-1/2} = B'_{-1/2} B_{1/2} = 0$ ) is accompanied by a partial polarization according to the relations

$$\begin{aligned} B'_{1/2} B'_{1/2} - B'_{-1/2} B'_{-1/2} &= 0, \\ B'_{1/2} B'_{-1/2} &= -e^{2i\varphi'} B'_{-1/2} B'_{1/2} \\ &= 2\pi^2 c^2 \hbar^2 K^{-2} e^{i\varphi'} (f^*(\theta')g(\theta') - g^*(\theta')f(\theta')), \\ B'_{1/2} B'_{1/2} + B'_{-1/2} B'_{-1/2} &= 4\pi^2 c^2 \hbar^2 K^{-2} (|f(\theta')|^2 + |g(\theta')|^2). \end{aligned} \quad (9)$$

The polarization of the scattered beam can be observed by a second scattering. Indeed, denoting the quantities corresponding to the second scattering by a double prime ( $\mathbf{k}'' \parallel z$ ), we find for the differential cross section [formulas (8) and (9)]:

$$d\sigma(\theta'', \varphi'') = d\sigma_0(\theta'') (1 + \varepsilon(\theta', \theta'') \cos(\varphi'' - \varphi')), \quad (10)$$

where

$$\begin{aligned} \varepsilon(\theta', \theta'') &= p(\theta') p(\theta''), \quad p(\theta) \\ &= i(f(\theta)g^*(\theta) - g(\theta)f^*(\theta)) / (|f(\theta)|^2 + |g(\theta)|^2), \\ d\sigma_0(\theta'') &= (|f(\theta'')|^2 + |g(\theta'')|^2) d\Omega''; \end{aligned}$$

$d\sigma_0$  is the differential cross section for the scattering of an unpolarized beam in the damping theory; the quantities  $p(\theta')$  and  $p(\theta'')$  determine the polarization acquired by the initially unpolarized beam in the first and second scattering, respectively.

Formulas (8) and (10) have the same appearance as the formulas of Mott.<sup>3</sup> However, in the case considered by Mott, the quantities  $f(\theta)$  and  $g(\theta)$  must be determined from differential equations and can only in special cases be obtained in exact form. In the damping theory, on the other hand, these quantities have a completely definite form for an arbitrary short range scattering potential with spherical symmetry.

It is seen from formulas (10) that the asymmetry effect disappears in the general case for  $\varepsilon(\theta', \theta'') \rightarrow 0$ , and that the unpolarized incoming beam remains unpolarized after the second scattering. In particular, for small energies ( $k \ll k_0$ ) there will be no polarization at all, since in this case the scattering amplitude  $g(\theta)$ , and hence  $\varepsilon(\theta', \theta'')$  go to zero. Reversely, at high energies, the polarization plays an essential role.

It can be easily shown that formula (10) goes over into formula (36) of reference 2 in the case of scattering from a  $\delta$ -function potential. The latter formula describes the double scattering of Dirac particles by a  $\delta$ -function potential.

We remark that the polarization effects disappear if the damping effects are neglected [ $f^*(\theta) = f(\theta)$  and  $g^*(\theta) = g(\theta)$ ]. This corresponds to the first approximation in perturbation theory.

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<sup>2</sup>Sokolov, Kerimov, and Guseinov, Nucl. Phys. 5, 390 (1958).

<sup>3</sup>N. Mott and G. Massey, Theory of Atomic Collisions, Clarendon Press, Oxford, 1933 (Russ. trans. IIL, 1951).

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### DECAY OF A BERYLLIUM HYPERFRAGMENT

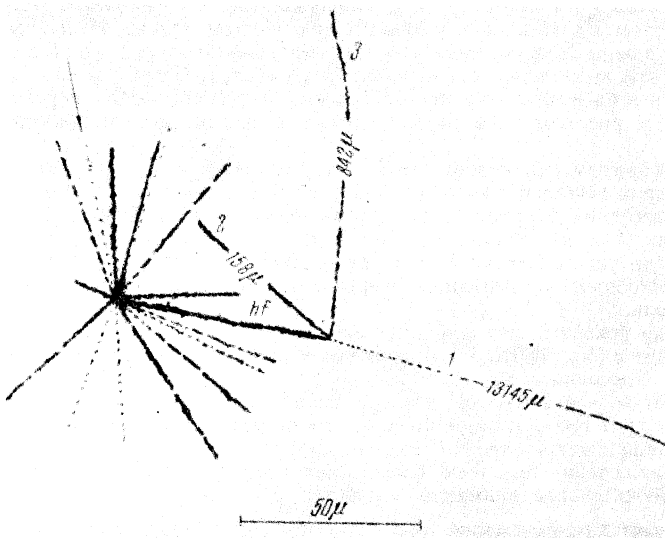
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A systematic scanning of Ilford G-5 emulsions<sup>1</sup> irradiated at an approximate altitude of 25 km we observed a non-mesonic decay of a beryllium hyperfragments, which permits a relatively accurate measurement of the binding energy of the  $\Lambda^0$  particle.

A primary star of the 12 + 4p type (see micro-photograph) emits a slow particle hf. It is stopped in the same layer and forms a secondary three-prong star. The hf range is 60 microns. An estimate of the charge, made by comparing the thickness of hf track with the thicknesses of the tracks of the  $\text{Be}^8$  fragments and the alpha particles from the  $\text{Be}^8$  decay yields  $Z \approx 4$ . An analogous estimate was made for tracks 1, 2, and 3. The measurement details are listed in the table.



#### Track of Hyperfragment

Primary star	12 + 4p
Connected phenomena	None observed
Length, $\mu$	$60 \pm 2$
Angle with undeveloped emulsion	$28^\circ + 30'$
Proof of stopping	Thinning
Charge hf	4
Energy of nucleon, Mev	$\approx 3.9$ , if $\text{Be}^8$

#### Secondary Star

Track	1	2	3
Nature of particle	p	$\text{He}^4$	d
Range	13145	158	842
Experimental error	$\pm 120$	$\pm 2$	$\pm 10$
Measured mass, $m_e(\alpha, R)$	$1820 \pm 250$	-	$3400 \pm 1300$
Energy, Mev	$61.4 \pm 0.8$	$19.1 \pm 0.2$	$17.6 \pm 0.3$
Angle of inclination, $\theta$	$-1^\circ 10'$	$41^\circ 50'$	$-15^\circ 50'$
Error $\Delta\theta$	$\pm 10'$	$\pm 30'$	$\pm 30'$
Polar angle $\varphi$	$182^\circ 40'$	$17^\circ 4p'$	$64^\circ$
Error $\Delta\varphi$	$\pm 0.50$	$\pm 1^\circ$	$\pm 1^\circ$

The total momentum of particles 1, 2, and 3 is  $345.7 \pm 2$  Mev. Assuming that an equal and opposite momentum has been carried away by the neutron, we arrive at the following decay scheme:

$${}^*_{\Lambda}\text{Be}^8 \rightarrow \text{He}^4 + d + p + n + Q, \quad Q = (160.0 \pm 1.3) \text{ Mev.}$$

We obtain for the binding energy of  $\Lambda^0$  in the  $\text{Be}^8$  nucleus  $B_{\Lambda^0} = 9.2 \pm 1.6$  Mev.

The measured values of  $B_{\Lambda^0}$  for the known decays of  ${}_{\Lambda^0}\text{Be}^8$  are  $3.7 \pm 3$  (reference 2),  $0 \pm 5$  (reference 3), 9.3 or 6.6 (depending on the decay scheme, reference 4), and  $5.9 \pm 0.5$  (mesonic decay, reference 5).

The three decay schemes

$${}^*_{\Lambda}\text{Be}^9 \rightarrow \text{He}^5 + d + p + n + Q;$$

$${}^*_{\Lambda}\text{Be}^8 \rightarrow \text{He}^3 + \text{H}^3 + p + n + Q;$$

$${}^*_{\Lambda}\text{Be}^9 \rightarrow \text{He}^4 + \text{H}^3 + p + n + Q$$

can be eliminated, for they lead to large negative values of  $B_{\Lambda^0}$ , equal respectively to  $13.6 \pm 1.8$ ,  $-18.3 \pm 1.8$ , and  $-27.5 \pm 2.2$  Mev.

Decay schemes with several neutral particles cannot be excluded, but are less probable. In the processing of the data we used the values of the constants from Shapiro's review<sup>6</sup> and the range-energy relations from the paper by Fry, Gottstein, and Hain.<sup>7</sup>

<sup>1</sup>A. O. Vaïsenberg and V. A. Smirnitskiĭ, J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 736 (1957), Soviet Phys. JETP 5, 607 (1957).

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<sup>3</sup>M. Blau, Phys. Rev. 102, 495 (1956).