CHARGE SYMMETRY PROPERTIES AND REPRESENTATIONS OF THE EXTENDED LORENTZ GROUP IN THE THEORY OF ELEMENTARY PARTICLES

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The extended Lorentz group, including the complete Lorentz group and charge conjugation, is considered. It is shown that the use of irreducible projective representations of this extended group requires the existence of charge multiplets. Charge symmetry and associated production of strange particles follow from the invariance under reflections and charge conjugation and from the conservation laws for the electric and baryonic charges. The Pauli-Gürsey transformation holds for free nucleons. The extension of the condition of invariance under this transformation to the case of interactions leads to isobaric invariance for strong interactions of all particles.

1. INTRODUCTION

It is known that the strongly interacting particles form charge multiplets (p, n, π^+, π^0, π^-, K^+, K^0, etc.). Particles belonging to the same multiplet have almost identical masses and identical spins, but differ in their electric charges. In agreement with experiment one adopts the hypothesis of charge symmetry and the stronger hypothesis of charge independence. In the conventional theory this is expressed by the invariance under rotations in a certain formal isobaric space. The particles of a given multiplet are considered as states of the same particle with different projections of the isobaric spin. For example, the proton and the neutron form the nucleon. The description of the nucleon makes use of a reducible eight-component representation of the full Lorentz group. We have a similar situation (reducibility of the representation of the full Lorentz group) for the other strongly interacting particles.

Now the question arises: if it is required that the elementary particles be described only by irreducible representations, would it then be possible to extend the Lorentz group and to find irreducible representations of this extended group which automatically lead to the existence of charge multiplets and charge symmetries? The solution of this question is the subject of the present paper.

We extend the Lorentz group in the following fashion.

The wave functions of quantum theory are complex functions. The operation of charge conjugation C, which takes a particle into its antiparticle, is always represented by the product of a linear operator (matrix) and the antilinear operator of complex conjugation:

$$C: \qquad \psi_c = C_0\psi^*, \qquad (1)$$

where C_0 is determined such that ψ_C transforms according to the same irreducible representation of the proper Lorentz group as ψ.

Besides the proper Lorentz group L, the spatial reflections I, and the time reversal T, we also include the charge conjugation C in the extended group. Together with the conventional irreducible representations of the extended group, we also consider its irreducible projective representations.*

The importance of using the projective representations of the full Lorentz group was pointed out by Gel'fand and Tsetlin[1] in connection with the theory of parity doublets of Lee and Yang. The possibility of using projective representations is connected with the indeterminacy of the phase factor of the quantum theoretical wave function. Subsequent to Gel'fand and Tsetlin, the projective representations of the full Lorentz group were discussed

*We are given a projective representation of the group G, if an operator R(g) is given for each element g of the group G such that the operator $R(g_1g_2) = \alpha(g_1, g_2)R(g_1)R(g_2)$ corresponds to the product of group elements g_1g_2. If $\alpha(g_1, g_2) \equiv 1$, the projective representation reduces to the conventional one. In general, $\alpha(g_1, g_2)$ can also be equal to -1. Anticommuting operators of the projective representation may thus correspond to commuting elements of the group. In particular, the conventional spinor representation is a projective representation (the operations $I = \gamma_4$ and $T = \gamma_4\gamma_5$ anticommute, whereas the spatial and time reflections commute).

by Taylor and McLennan.[2] Taylor notes the connection between these representations and the isobaric invariance. As only the full Lorentz group is considered, protons and neutrons, and $\pi^{\pm}$ and π^0 mesons, differ with respect to their spatial parities. Salam and Pais, at the Seventh Rochester Conference, also discussed the need for a new definition of the operations of space-time reflections from which the charge symmetries would follow.

In this paper we do not consider all irreducible projective representations of the extended group. We restrict ourselves to those necessary for the description of the strongly interacting particles. We shall show that multiplets, charge symmetries, and associated production of strange particles are immediate consequences of the standard conservation laws for the number of baryons and electric charge, and of the invariance with respect to the full Lorentz group and charge conjugation, if the nucleons, Ξ particles, and K mesons are described by the new projective representations of the extended group, while the remaining particles are described in the usual fashion.

In our theory the Pauli-Gürsey transformation holds for free nucleons. This transformation is connected with the isobaric invariance in a natural way. If the requirement of invariance under this transformation is extended to the interaction Lagrangian for the nucleons, the isobaric invariance for strong interactions follows for all particles.

In this theory the Lagrangian for the interaction with electromagnetic fields can be easily written down with the help of the charge operator. It appears that the electromagnetic interactions are invariant only under Wigner time reversal, but not under Schwinger time reversal.

The case of weak interactions, which do not conserve spatial parity, is more complicated and will not be discussed in the present paper.

To be definite, we shall assume that the relative parities of all baryons are identical and that the reflection of the conventional spinors is performed with the help of the operator γ_4. All bosons are considered as pseudoscalars. We start with the discussion of the nucleons.

2. THE FREE NUCLEON FIELD

It is easily shown that the requirement

$$I^2 = T^2 = C^2 = 1, \qquad (2)$$

leads, for the case of four-component spinors, to the following expressions for the operators I, T, and C:

$$\begin{aligned} &\text{a) } I: && \psi' = \gamma_4\psi, \\ &\text{b) } T: && \psi' = i\gamma_4\gamma_5\psi, \\ &\text{c) } C: && \psi_c = i\gamma_2\psi^*, \end{aligned} \qquad (3)$$

where T is the Schwinger time reversal for spinors,[3] and the matrices γ_i are expressed in the Pauli representation.

The following commutation relations hold between the operators I, T, and C:

$$\text{a) } IT = -TI, \quad \text{b) } IC = -CI, \quad \text{c) } TC = -CT. \qquad (4)$$

We retain relations (2), but we require that, in contrast to the conventional theory, the sign in relation (4a) is changed for nucleons, i.e., we demand that

$$\text{a) } IT = TI, \quad \text{b) } IC = -CI, \quad \text{c) } TC = -CT. \qquad (4')$$

The commutation relations (2) and (4′) can only be satisfied by 8×8 matrices:

$$\begin{aligned} &\text{a) } I: && \psi' = \tau_3 \times \gamma_4\psi, \\ &\text{b) } T: && \psi' = 1 \times \gamma_4\psi, \\ &\text{c) } C: && \psi_c = i\tau_3 \times \gamma_2\psi^*, \end{aligned} \qquad (5)$$

where τ are the Pauli matrices. These operators, together with the operators of the proper Lorentz group (in which γ_μ should everywhere be replaced by $1 \times \gamma_\mu$), form the irreducible projective representation of the extended Lorentz group. The spinors $\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$ have eight components.

In this representation the Lagrangian for the free field is uniquely determined:*

$$L = \bar{\psi}(1 \times \gamma_\mu\partial_\mu + i\tau_2 \times \gamma_5 m)\psi, \qquad (6)$$

where $\bar{\psi} = \psi^{*T}(1 \times \gamma_4)$. The field equations have the form

$$1 \times \gamma_\mu\partial_\mu\psi = -i\tau_2 \times \gamma_5 m\psi. \qquad (7)$$

The Lagrangian (6) as well as Eq. (7) are invariant with respect to the two one-parameter groups of transformations

$$\psi' = \exp(i\lambda)\psi, \qquad (8)$$

$$\psi' = \exp[i\tau_1 \times \gamma_5\lambda]\psi, \qquad (9)$$

and with respect to the three-parameter group

$$\psi' = a\psi + b\tau_3 \times \gamma_5\psi_c, \qquad (10)$$

where $|a|^2 + |b|^2 = 1$.

The transformations (9) and (10) are the analogs of the Pauli transformations[4] for the neutrino. They are different only in that γ_5 is replaced by $\tau_1 \times \gamma_5$ and $\tau_3 \times \gamma_5$, respectively.

*According to Schwinger $L \to L^T$, where the superscript T signifies transposition of the operators of the Hilbert space.[3]

We introduce the new four-component spinors

$$\psi_p = (\psi_1 + \gamma_5\psi_2)/\sqrt{2}, \quad \psi_{pc} = (\psi_{c1} + \gamma_5\psi_{c2})/\sqrt{2},$$
$$\psi_{nc} = (-\gamma_5\psi_1 + \psi_2)/\sqrt{2}, \quad \psi_n = (\gamma_5\psi_{c1} - \psi_{c2})/\sqrt{2}, \tag{11}$$

they satisfy the ordinary Dirac equation

$$\gamma_\mu \partial_\mu \psi = -m\psi, \tag{12}$$

Under the transformation (9) we obtain

$$\psi_p' = \exp(i\lambda)\psi_p, \quad \psi_n' = \exp(i\lambda)\psi_n,$$
$$\psi_{pc}' = \exp(-i\lambda)\psi_{pc}, \quad \psi_{nc}' = \exp(-i\lambda)\psi_{nc}. \tag{13}$$

Transformation (9) may thus be viewed as a gauge transformation connected with the conservation law for the number of baryons. ψp, ψn, ψpc, and ψ_{nc} refer to the proton, neutron, antiproton, and antineutron fields respectively.

The following transformation should be related to the conservation law for the electric charge:

$$E: \psi' = \exp[(i/2)(1 \times 1 + \tau_1 \times \gamma_5)\lambda]\psi. \tag{14}$$

Indeed, under this transformation:

$$E: \psi_p' = \exp(i\lambda)\psi_p, \quad \psi_{pc}' = \exp(-i\lambda)\psi_{pc},$$
$$\psi_n' = \psi_n, \quad \psi_{nc}' = \psi_{nc}. \tag{15}$$

The three-parameter trnnsformation (10) is isomorphic to a rotation in the isobaric space. Indeed, if we form the conventional eight-component nucleon field $\psi_N = \begin{pmatrix}\psi_p \\ \psi_n\end{pmatrix}$, we have, under the transformation (10),

$$\psi_N' = \exp[i(\boldsymbol{\tau}\cdot\boldsymbol{\lambda})]\psi_N, \tag{16}$$

where λ is a vector with real components λ_1, λ_2, λ_3; $\boldsymbol{\tau}$ are the usual 2×2 Pauli matrices, and

$$a = \cos|\lambda| + \frac{i\lambda_3}{|\lambda|}\sin|\lambda|; \quad b = \frac{\sin|\lambda|}{|\lambda|}(\lambda_2 - i\lambda_1). \tag{17}$$

An analogous isomorphism arising from a purely formal doubling of the number of components was pointed out by Gürsey.[5]

3. INTERACTION OF NUCLEONS WITH ORDINARY BOSONS

We first consider the interaction of nucleons with a neutral pseudoscalar field φ_0 with positive "time-parity:"

$$\begin{aligned} I: &\quad \varphi_0' = -\varphi_0, \\ T: &\quad \varphi_0' = \varphi_0, \\ C: &\quad \varphi_{0c} = \varphi_0. \end{aligned} \tag{18}$$

The requirement of invariance with respect to I, T, C, transformation (9), and E lead to a unique interaction Lagrangian (in the following we only consider interaction Lagrangians without derivatives):

$$L_0 = ig_0\bar\psi\tau_3 \times \gamma_5\psi\varphi_0 = ig_0(\bar p\gamma_5 p - \bar n\gamma_5 n)\varphi_0 = ig_0\bar\psi_N\tau_3\gamma_5\psi_N\varphi_0, \tag{19}$$

the meson field φ_0 couples to the protons and neutrons with a different sign, and φ_0 may be identified with the neutral π^0 meson.

If the neutral meson φ_0' were a spatial pseudoscalar, but had negative time-parity, we would uniquely obtain the interaction Lagrangian

$$L_0' = g_0'\bar\psi\tau_2 \times 1\psi\varphi_0' = ig_0'(\bar p\gamma_5 p + \bar n\gamma_5 n)\varphi_0', \tag{20}$$

φ_0' may be related to the hypothetical ρ_0 meson.

For the Lagrangian describing the interaction of nucleons with a charged pseudoscalar boson field φ:

$$\begin{aligned} I: &\quad \varphi' = -\varphi, \\ T: &\quad \varphi' = \varphi, \\ C: &\quad \varphi_c = \varphi^* \end{aligned}$$

we similarly obtain the unique expression

$$L = ig(\bar\psi\psi_c\varphi^* - \bar\psi_c\psi\varphi) = 2ig(\bar p\gamma_5 n\varphi^* + \bar n\gamma_5 p\varphi). \tag{21}$$

We may thus assume that φ (φ^*) describes π^- (π^+) mesons.

The charge symmetry (i.e., the possibility of the simultaneous interchange p $\rightleftarrows$ n, $\pi^+ \rightleftarrows \pi^-$, $\pi^0 \rightleftarrows -\pi^0$) of the interactions (19) and (20) is obvious. The general form of the Lagrangian for interactions with π mesons is

$$L_\pi = ig_0(\bar p\gamma_5 p - \bar n\gamma_5 n)\pi^0 + 2ig(\bar p\gamma_5 n\pi^+ + \bar n\gamma_5 p\pi^-). \tag{22}$$

If we now require that not only the free nucleon Lagrangian, but also the interaction Lagrangian be invariant under transformations of the three-parameter group, then $g = g_0/\sqrt{2} = g_\pi/\sqrt{2}$, and

$$L_\pi = ig_\pi\bar\psi_N(\boldsymbol{\tau}\cdot\boldsymbol{\pi})\psi_N. \tag{23}$$

Under the transformation (10) the meson fields transform according to

$$(\boldsymbol{\tau}\cdot\boldsymbol{\pi})' = \exp[i(\boldsymbol{\tau}\cdot\boldsymbol{\lambda})](\boldsymbol{\tau}\cdot\boldsymbol{\pi})\exp[-i(\boldsymbol{\tau}\cdot\boldsymbol{\lambda})], \tag{24}$$

where the masses of the π^+, π^-, and π^0 mesons must be identical.

We have thus arrived at the conventional isobaric-invariant theory of the interaction between π mesons and nucleons.

4. FREE K MESONS

The conventional representation of the extended Lorentz group for bosons is exhausted by the π mesons. We shall describe the K mesons by the projective representation in which

$$I^2 = 1, \quad C^2 = 1, \quad T^2 = -1,$$
$$IT = TI, \quad IC = CI, \quad TC = CT. \tag{25}$$

The simplest irreducible representation consistent with these commutation relations is two-dimensional, $\varphi = \begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}$. The operators I, T, and C have the form

$$\begin{aligned} I:&\quad \varphi' = -\varphi,\\ T:&\quad \varphi' = i\tau_2\varphi,\\ C:&\quad \varphi_c = \varphi^*. \end{aligned} \tag{26}$$

We identify the K^+ meson with φ_1, the K^0 meson with φ_2, the K^- meson with φ_1^*, and the $\bar{K}^0$ meson with φ_2^*. The conservation law for the electric charge is then connected with the transformation

$$E:\quad \varphi' = \exp\left[\tfrac{1}{2}(1+\tau_3)\lambda\right]\varphi. \tag{27}$$

Indeed, with this transformation,

$$K^{+\prime} = \exp(i\lambda)K^+; \quad K^{-\prime} = \exp(-i\lambda)K^-,$$
$$K^{0\prime} = K^0, \quad \bar{K}^{0\prime} = \bar{K}^0. \tag{28}$$

The conservation law for the hypercharge corresponds to the transformation $\varphi' = \exp(i\tau_3\lambda)\varphi$, so that

$$\begin{aligned} K^{+\prime} &= \exp(i\lambda)K^+, & K^{0\prime} &= \exp(i\lambda)K^0,\\ K^{-\prime} &= \exp(-i\lambda)K^-, & \bar{K}^{0\prime} &= \exp(-i\lambda)\bar{K}^0. \end{aligned} \tag{29}$$

5. INTERACTION OF K MESONS WITH NUCLEONS. Λ AND Σ PARTICLES

We now turn to the investigation of the interaction of K mesons with nucleons. Since both the K mesons and the nucleons transform according to the projective representation of the extended Lorentz group, and since the baryonic charge is conserved, an additional baryon must necessarily participate in the interaction. This requirement inevitably leads to the law of associated production of strange particles. We first consider the case of a neutral baryon. We already said earlier that the relative spatial parities of all baryons are, for definiteness, assumed to be identical:

$$I:\quad Y'_0 = \gamma_4 Y_0. \tag{30}$$

For the nucleon transformation T there are two possibilities for the unknown baryon:

$$T:\quad Y'_0 = -\gamma_4\gamma_5 Y_{0c}, \tag{31}$$
$$Y'_0 = \gamma_4\gamma_5 Y_{0c}. \tag{32}$$

We have to introduce the antibaryon Y_{0C} into equations (31) and (32), since the transformation T for the nucleons anticommutes with the transformation[9] related to the conservation of the baryonic charge.

If we choose (31) for T, then the only form of the Lagrangian invariant under the transformations of the extended Lorentz group and the transformations (9) and E is

$$L = ig\,[\bar{\psi}(1\times\gamma_5 - \tau_1\times 1)(1\times 1 + \tau_3\times 1)\varphi\times Y_0 - \bar{\psi}_c(1\times\gamma_5 + \tau_1\times 1)(1\times 1 - \tau_3\times 1)\varphi^*\times Y_0] + \text{Herm. conj.} \tag{33}$$

In going from ψ and φ to the operators of the nucleon and K meson fields, we obtain the usual form for the Lagrangian for the interaction of nucleons with Λ_0 particles:

$$L = ig_\Lambda\,(\bar{p}\gamma_5\Lambda_0K^+ + \bar{n}\gamma_5\Lambda_0K^0) + \text{Herm. conj.} \tag{34}$$

The transformation law (32) for T leads to a Lagrangian that differs from expression (33) only by a plus sign between the two terms of expression (33). It corresponds to the Σ_0 particle:

$$L = ig_{\Sigma_0}(\bar{p}\gamma_5\Sigma_0K^+ - \bar{n}\gamma_5\Sigma_0K^0) + \text{Herm. conj.} \tag{35}$$

We thus arrive at the conclusion that the transformation laws for Λ_0 and Σ_0 corresponding to the transformation T for the nucleons, differ by their signs. We now consider the interaction of nucleons with charged baryons. We can find a Lagrangian which is invariant under charge conjugation, space inversion, and time reversal, and which is consistent with the conservation laws for the electric and baryonic charges, only if we require that, under T,

$$T:\quad \Sigma^+ \to -\gamma_4\gamma_5\Sigma_c^-, \quad \Sigma^- \to -\gamma_4\gamma_5\Sigma_c^+, \tag{36}$$

This implies that the masses of the Σ^+ and Σ^- particles are equal. The Lagrangian has the form

$$L = -ig\,[\bar{\psi}(1\times 1 - \tau_1\times\gamma_5)(1\times 1 - \tau_3\times 1)\varphi^*\times\Sigma^+ + \bar{\psi}_c(1\times 1 + \tau_1\times\gamma_5)(1\times 1 + \tau_3\times 1)\varphi\times\Sigma^-] + \text{Herm. conj.} \tag{37}$$

In conventional notation this can be written in the form

$$L = ig_{\Sigma\pm}\,(\bar{p}\gamma_5\Sigma^+K^0 + \bar{n}\gamma_5\Sigma^-K^+) + \text{Herm. conj.} \tag{38}$$

The charge symmetry is obvious.

If we now require that the interaction Lagrangian also be invariant under the Pauli-Gürsey type transformation (10), we obtain

$$g_{\Sigma\pm} = \sqrt{2}\,g_{\Sigma_0} = \sqrt{2}\,g_\Sigma; \tag{39}$$

The masses of Σ^+, Σ^-, and Σ^0 must be equal.

We arrive at the usual isobaric-invariant Lagrangian

$$L = ig_{\Sigma}[(\bar{p}\gamma_5\Sigma_0 K^+ - \bar{n}\gamma_5\Sigma_0 K^0) + \sqrt{2}\,\bar{p}\gamma_5\Sigma^+ K^0 + \sqrt{2}\,\bar{n}\gamma_5\Sigma^- K^+] + \text{Herm. conj.} \quad (40)$$

6. Ξ PARTICLES

Still another possibility remains within the framework of these representations. In the projective representation (5), we can change the sign between the terms 1×1 and $\tau_1 \times \gamma_5$ in the transformation (14) connected with the conservation law for the electric charge:

$$E:\ \psi' = \exp\left[\frac{i}{2}(1 \times 1 - \tau_1 \times \gamma_5)\lambda\right]\psi. \quad (41)$$

At the same time we retain the transformation (9) connected with the conservation law for the baryonic charge. In all formulas we then simply have to replace p by Ξ^0, n by Ξ^-, K^+ by $\bar{K}^0$, and K^0 by K^-. We again have charge symmetry. If we require invariance under the Pauli transformation even in the case of interaction, we obtain the usual isobaric-invariant Lagrangians.

7. CONCLUSION

We carried out the program which we set ourselves in the beginning of this paper. We introduced the new irreducible projective representation of the extended Lorentz group. We showed that the existence of charge multiplets, charge symmetry, and associated production of strange particles are consequences of the standard conservation laws. The isobaric invariance follows from the invariance under the Pauli-Gürsey type transformation for free nucleons. This transformation is applicable, since the number of components of wave functions transforming according to projective representations necessarily had to be increased.

We did not discuss the weak interactions from this point of view. This task is much more difficult and less unambiguous due to the violation of the parity conservation laws.

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[1] I. M. Gel'fand and M. L. Tsetlin, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 1107 (1956), Soviet Phys. JETP **4**, 947 (1957).

[2] J. C. Taylor, Nucl. Phys. **3**, 606 (1957). J. A. McLennan, Jr., Phys. Rev. **109**, 986 (1958).

[3] J. Schwinger, Phys. Rev. **91**, 713 (1953).

[4] W. Pauli, Nuovo cimento **6**, 204 (1957).

[5] F. Gürsey, Nuovo cimento **7**, 411 (1958).

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