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POLARIZATION EFFECTS IN Σ^-- HYPERON CAPTURE BY DEUTERONS

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A phenomenological treatment is given of Σ^--hyperon capture by deuterons with formation of Λ^0-particles. A study of the polarization correlation of the strange particles yields information on the polarization of the Σ^--hyperon.

A phenomenological study of Σ^--hyperon capture by protons was recently given by Pais and Treiman.[1] The process is of interest for determination of the relative parity of Λ and Σ particles and for determination of the degree of polarization of the Σ particle using the decay process as analyzer. (As is well known, experiments on the "up-down" asymmetry coefficient in the associated production process give drastically different results for Λ and Σ particles which makes a determination of Σ polarization important.)

In this note we consider the capture of a polarized Σ^- hyperon by a deuteron

$$\Sigma^- + d \rightarrow 2n + \Lambda^0. \quad (1)$$

We assume that the spin of Λ and Σ particles is $1/2$. A quantitative study of this process indicates that it is of definite interest as a source of additional information on the question of the degree of polarization of the Σ^- particle.

We investigated the capture process in the impulse approximation (cf., e.g., references 2 and 3). In this approximation the amplitude for the process has the form[4]

$$T_d = J_{12}T(1,2) + J_{13}T(1,3), \quad (2)$$

where the index 1 refers to the strange particles, and 2 and 3 refer to the nucleons composing the deuteron;

$$J_{1l} = \int \Psi_f^+(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)\Psi_i(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)\,\delta(\mathbf{r}_1 - \mathbf{r}_l)\,d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 \quad (l = 2,3),$$

Ψ_i and Ψ_f are the initial and final wave functions of the three particle system; and T is the amplitude introduced by Pais and Treiman, for Σ^--hyperon capture by a proton.

Proceeding in a manner analogous to that of reference 4 we find the following formula for the polarization of the Λ particle produced in the capture of a polarized Σ^- hyperon by a deuteron:

$$R_d\mathbf{p}_\Lambda = \frac{1}{3}\{|F^-|^2\,\mathrm{Sp}[\Pi_t(2,3)T(1,2)\rho_\Sigma\Pi_t(2,3)T^+(1,2)\sigma_1] + |F^+|^2\,\mathrm{Sp}[\Pi_s(2,3)T(1,2)\rho_\Sigma\Pi_t(2,3)T^+(1,2)\sigma_1]\}. \quad (3)$$

Here the trace is over the spin variables of all three particles, $\rho_\Sigma = [1 + \mathbf{p}_\Sigma \cdot \boldsymbol{\sigma}_1]/2$ is the spin density matrix of Σ^- particles of polarization $\mathbf{p}_\Sigma$,

$$\Pi_t(2,3) = \frac{1}{4}[3 + \sigma_2\cdot\sigma_3], \qquad \Pi_s(2,3) = \frac{1}{4}[1 - \sigma_2\cdot\sigma_3],$$

$$F^\pm = \int \Phi_g^{\pm *}\Phi_d e^{i\varkappa\cdot\rho}d\rho,$$

Φ_g is the wave function of two neutrons with relative momentum $\mathbf{g}$, Φ_d is the deuteron wave function, 2κ is the momentum transferred to the nucleons;

$$R_d = \frac{1}{3}\{|F^-|^2\,\mathrm{Sp}[\Pi_t(2,3)T(1,2)\rho_\Sigma\Pi_t(2,3)T^+(1,2)] + |F^+|^2\,\mathrm{Sp}[\Pi_s(2,3)T(1,2)\rho_\Sigma\Pi_t(2,3)T^+(1,2)]\}. \quad (4)$$

As in reference 1, it is easy to obtain the following relation between $\mathbf{p}_\Lambda$ and $\mathbf{p}_\Sigma$:

$$R_d\mathbf{p}_\Lambda = \alpha\mathbf{p}_\Sigma + \beta(\mathbf{p}_\Sigma\cdot\mathbf{N})\,\mathbf{N}, \quad (5)$$

where $\mathbf{N}$ is a unit vector in the direction of motion of the Λ particle.

The values of R_d, α and β are determined by the type of transition from the initial to the final state and depend on the dynamics of the Σ^--proton interaction leading to capture. We shall consider the various transition types separately.

1. S → S Transition

In this case the amplitude for Σ^- capture by a proton has the form

$$T = a_1\Pi_t(1,2) + a_2\Pi_s(1,2), \quad (6)$$

where a_1 and a_2 are the amplitudes for the transitions ${}^3S_1 \rightarrow {}^3S_1$ and ${}^1S_0 \rightarrow {}^1S_0$ of the strange particle-nucleon system respectively (we use standard spectroscopic notation).

R_d, α, and β are given by the following expressions:

$$R_d = \frac{1}{16}\{(11|a_1|^2 + 3|a_2|^2 + 2\mathrm{Re}\,a_1a_2^*)|F^-|^2 + (|a_1|^2 + |a_2|^2 - 2\mathrm{Re}\,a_1a_2^*)|F^+|^2\}, \quad (7a)$$

$$\alpha = \frac{1}{48}\{(25|a_1|^2 + |a_2|^2 + 22\,\mathrm{Re}\,a_1a_2^*)|F^-|^2 - (|a_1|^2 + |a_2|^2 - 2\mathrm{Re}\,a_1a_2^*)|F^+|^2\}, \quad (7b)$$

$$\beta = 0. \quad (7c)$$

For $\kappa \approx 0$ (i.e., for small momentum transfer to the nucleons) one has $F^- \approx 0$ and $\mathbf{p}_\Lambda = -\mathbf{p}_\Sigma/3$, independent of any assumption about the coherence or incoherence of the states with a_1 and a_2.

Consequently, in this case it is possible to obtain definite information about the Σ^- polarization by studying the asymmetry in the Λ-decay for both the instance where the Σ^- was captured from the continuum and where it came from an S-orbit.

In the case of capture by protons the inequality $p_\Lambda \leq {}^2\!/_3 p_\Sigma$ is obtained only for transitions from S-orbits.[1]

2. S → P Transitions

$$T = ({}^3\!/_2)^{1/2} b_1 \mathbf{N}\cdot\mathbf{S} + \sqrt{3}b_2\mathbf{N}\cdot\mathbf{S}'\Pi_t + b_3\mathbf{N}\cdot\mathbf{S}'\Pi_s,$$

where

$$\mathbf{S} = (\sigma_1 + \sigma_2)/2, \qquad \mathbf{S}' = (\sigma_1 - \sigma_2)/2;$$

b_1, b_2 and b_3 are the amplitudes for the transitions ${}^3S_1 \to {}^3P_1$, ${}^3S_1 \to {}^1P_1$ and ${}^1S_0 \to {}^3P_0$ respectively.

$$R_d = \frac{1}{8}\Big\{\Big(5|b_1|^2 + \frac{9}{2}|b_2|^2 + \frac{3}{2}|b_3|^2 + \sqrt{2}\,\mathrm{Re}\,b_1b_2^* + \sqrt{\frac{2}{3}}\,\mathrm{Re}\,b_1b_3^* + \frac{1}{\sqrt{3}}\,\mathrm{Re}\,b_2b_3^*\Big)|F^-|^2 + \Big(|b_1|^2 + \frac{3}{2}|b_2|^2 + \frac{1}{2}|b_3|^2 - \sqrt{2}\,\mathrm{Re}\,b_1b_2^* - \sqrt{\frac{2}{3}}\,\mathrm{Re}\,b_1b_3^* - \sqrt{\frac{1}{3}}\,\mathrm{Re}\,b_2b_3^*\Big)|F^+|^2\Big\}, \quad (8a)$$

$$\alpha = \frac{1}{8}\Big\{-\Big(|b_1|^2 + \frac{1}{2}|b_2|^2 + \frac{1}{6}|b_3|^2 + 5\sqrt{2}\,\mathrm{Re}\,b_1b_2^* + 5\sqrt{\frac{2}{3}}\,\mathrm{Re}\,b_1b_3^* + \sqrt{\frac{1}{3}}\,\mathrm{Re}\,b_2b_3^*\Big)|F^-|^2 + \Big(|b_1|^2 + \frac{1}{2}|b_2|^2 + \frac{1}{6}|b_3|^2 - \sqrt{2}\,\mathrm{Re}\,b_1b_2^* - \sqrt{\frac{2}{3}}\,\mathrm{Re}\,b_1b_3^* + \sqrt{\frac{1}{3}}\,\mathrm{Re}\,b_2b_3^*\Big)|F^+|^2\Big\}, \quad (8b)$$

$$\beta = \frac{1}{4}\Big\{\Big(3|b_1|^2 + \frac{1}{2}|b_2|^2 + \frac{1}{6}|b_3|^2 + 3\sqrt{2}\,\mathrm{Re}\,b_1b_2^* + \sqrt{6}\,\mathrm{Re}\,b_1b_3^* + \frac{5}{\sqrt{3}}\,\mathrm{Re}\,b_2b_3^*\Big)|F^-|^2 + \Big(-\frac{1}{2}|b_2|^2 - \frac{1}{6}|b_3|^2 + \frac{1}{\sqrt{3}}\,\mathrm{Re}\,b_2b_3^*\Big)|F^+|^2\Big\}. \quad (8c)$$

Let us again consider the case $\kappa \approx 0$ (experimentally this corresponds to the observation of a Λ particle with energy larger than a given E_0).

If the Σ^- particle is captured from a discrete level the interference terms $b_1b_3^*$ and $b_2b_3^*$ vanish.[1] In addition, if the amplitude b_1 of the transition ${}^3S_1 \to {}^3P_1$ dominates the others the simple expression $\mathbf{p}_\Lambda \approx \mathbf{p}_\Sigma$ results. In the case when the amplitude b_2 or b_3 (or both)

$$\mathbf{p}_\Lambda = {}^1\!/_3\mathbf{p}_\Sigma - {}^2\!/_3(\mathbf{p}_\Sigma\cdot\mathbf{N})\,\mathbf{N}$$

is obtained.

3. P → S Transition

In this case the amplitude T has the same form as for S → P transitions with the unit vector **N** replaced by the unit vector **n** in the direction of the relative momentum of the $(\Sigma^- - p)$ system. In the final expressions for R_d, α and β an average over **n** was performed.

$$R_d = \frac{1}{8}\Big\{\Big(5|c_1|^2 + \frac{9}{2}|c_2|^2 + \frac{3}{2}|c_3|^2 + \sqrt{2}\,\mathrm{Re}\,c_1c_2^* + \sqrt{\frac{2}{3}}\,\mathrm{Re}\,c_1c_3^* + \sqrt{\frac{1}{3}}\,\mathrm{Re}\,c_2c_3^*\Big)|F^-|^2 + \Big(|c_1|^2 + \frac{3}{2}|c_2|^2 + \frac{1}{3}|c_3|^2 - \sqrt{2}\,\mathrm{Re}\,c_1c_2^* - \sqrt{\frac{2}{3}}\,\mathrm{Re}\,c_1c_3^* - \sqrt{\frac{1}{3}}\,\mathrm{Re}\,c_2c_3^*\Big)|F^+|^2\Big\}, \quad (9a)$$

$$\alpha = \frac{1}{8}\Big\{\Big(|c_1|^2 - \frac{1}{6}|c_2|^2 - \frac{1}{18}|c_3|^2 - 3\sqrt{2}\,\mathrm{Re}\,c_1c_2^* - \sqrt{6}\,\mathrm{Re}\,c_1c_3^* + \frac{7}{9}\sqrt{3}\,\mathrm{Re}\,c_2c_3^*\Big)|F^-|^2 + \Big(|c_1|^2 + \frac{1}{6}|c_2|^2 + \frac{1}{18}|c_3|^2 - \sqrt{2}\,\mathrm{Re}\,c_1c_2^* - \sqrt{\frac{2}{3}}\,\mathrm{Re}\,c_1c_3^* + \frac{5}{9}\sqrt{3}\,\mathrm{Re}\,c_2c_3^*\Big)|F^+|^2\Big\}, \quad (9b)$$

$$\beta = 0. \quad (9c)$$

Here c_1, c_2, c_3 are transition amplitudes for ${}^3P_1 \to {}^3S_1$, ${}^1P_1 \to {}^3S_1$ and ${}^3P_0 \to {}^1S_0$ respectively of the $(\Sigma^- - p)$ system.

Considering again the case $\kappa \approx 0$ and taking into account the fact that all three amplitudes c_1, c_2, and c_3 are incoherent (i.e., all interference terms vanish), we obtain the inequality

$$\tfrac{1}{9}p_\Sigma \leqslant p_\Lambda \leqslant p_\Sigma.$$

The limiting value $p_\Lambda \approx p_\Sigma$ occurs when the amplitude c_1 dominates the others; the other limiting value $p_\Lambda \approx \tfrac{1}{9}p_\Sigma$ is obtained when one of the amplitudes c_2, c_3 is dominant.

As was to be expected, these results could be obtained by averaging the equalities found in section 2.

4. P → P Transitions*

$$T = (3/4)^{1/2}\Big\{\frac{1}{3}d_1[4\mathbf{N}\cdot\mathbf{n}\Pi_t - 3i\mathbf{S}\cdot[\mathbf{N}\times\mathbf{n}] - (\mathbf{n}\cdot\mathbf{S})(\mathbf{N}\cdot\mathbf{S})] + d_2[i\mathbf{S}\cdot[\mathbf{N}\times\mathbf{n}] + (\mathbf{n}\cdot\mathbf{S})(\mathbf{N}\cdot\mathbf{S})] + \sqrt{2}i\mathbf{S}'\cdot[\mathbf{N}\cdot\mathbf{n}](d_3\Pi_t + d_4\Pi_s) + 2d_5(\mathbf{N}\cdot\mathbf{n})\Pi_s + \frac{2}{3}d_6[(\mathbf{N}\cdot\mathbf{n})\Pi_t - (\mathbf{n}\cdot\mathbf{S})(\mathbf{N}\cdot\mathbf{S})]\Big\}.$$

The amplitudes d_1, d_2, d_3, d_4, d_5 and d_6 refer to the transitions $^3P_2 \to {}^3P_2$, $^3P_1 \to {}^3P_1$, $^3P_1 \to {}^1P_1$, $^1P_1 \to {}^3P_1$, $^1P_1 \to {}^1P_1$ and $^3P_0 \to {}^3P_0$ respectively of the $(\Sigma^- - p)$ system.

If the conditions for incoherence are satisfied we obtain (after averaging over $\mathbf{n}$):

$$R_d = \frac{1}{16}\Big\{\Big(\frac{35}{6}|d_1|^2 + \frac{19}{6}|d_2|^2 + 3(|d_3|^2 + |d_4|^2 + |d_5|^2) + |d_6|^2 + \frac{\sqrt{2}}{3}\operatorname{Re} d_2d_3^*\Big)|F^-|^2 + \Big(\frac{5}{6}|d_1|^2 + \frac{5}{6}|d_2|^2 + |d_3|^2 + |d_4|^2 + |d_5|^2 + \frac{1}{3}|d_6|^2 - \frac{\sqrt{2}}{3}\operatorname{Re} d_2d_3^*\Big)|F^+|^2\Big\}, \quad (10a)$$

$$\alpha = \frac{1}{48}\Big\{\Big(\frac{97}{9}|d_1|^2 + |d_2|^2 + |d_5|^2 + \frac{1}{9}|d_6|^2 - 4\sqrt{2}\operatorname{Re} d_2d_3^*\Big)|F^-|^2 + \Big(\frac{11}{9}|d_1|^2 - |d_2|^2 - |d_5|^2 - \frac{1}{9}|d_6|^2 - 2\sqrt{2}\operatorname{Re} d_2d_3^*\Big)|F^+|^2\Big\}, \quad (10b)$$

$$\beta = \frac{1}{24}\Big\{\Big(-\frac{97}{36}|d_1|^2 + \frac{7}{4}|d_2|^2 - \frac{1}{2}|d_3|^2 - \frac{1}{2}|d_4|^2 - \frac{1}{9}|d_6|^2 + \frac{3}{\sqrt{2}}\operatorname{Re} d_2d_3^*\Big)|F^-|^2 + \Big(-\frac{11}{36}|d_1|^2 + \frac{5}{4}|d_2|^2 + \frac{1}{2}|d_3|^2 + \frac{1}{2}|d_4|^2 + \frac{1}{9}|d_6|^2 + \frac{3}{\sqrt{2}}\operatorname{Re} d_2d_3^*\Big)|F^+|^2\Big\}. \quad (10c)$$

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Note added in proof (November 24, 1958). The amplitudes of Pais and Treiman[1] characterize the transition of the strange particle-nucleon system. On the other hand in the deuteron case it is necessary to consider transitions of the system strange particle — two nucleons. However it is clear that for $\kappa \approx 0$ the two definitions coincide.

*There are a number of misprints in reference 1. This is true in particular of formulas (7) and (9) and of the form of the P → P amplitude.

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[2] G. F. Chew, Phys. Rev. **80**, 196 (1950).

[3] I. Ya. Pomeranchuk, J. Exptl. Theoret. Phys. (U.S.S.R.) **21**, 1113 (1951).

[4] S. G. Matinyan and O. D. Cheishvili, Сообщ. АН ГрузССР (Reports, Acad. Sci., Georgian S.S.R.) (in press).

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