# *EFFECTS OF A RESONANT PION-PION INTERACTION IN FERMI'S STATISTICAL THEORY OF MULTIPLE PARTICLE PRODUCTION*

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The statistical theory of Fermi is applied to $\pi^-$-p scattering at 4.5 Bev, including effects of a possible resonant interaction between two pions. Production of strange particles is also included. The results are compared with experiment.

SEVERAL authors[1-4] have suggested that two pions have a resonant interaction. In a statistical theory, the effect of such a resonant interaction can be consistently included by treating the resonant state as a virtual particle.[5,6]

Nikishov[7] made a calculation along these lines, treating the resonant pion-nucleon interaction as an "isobar" effect, and thereby improved the agreement between the theory and experiment.* However, the ratio of strange-particle production to pion production remained considerably too high.[6] To be consistent, one should also include effects of a possible pion-pion resonance.

At present there are no direct experiments on pion-pion scattering, and it is hard to imagine such experiments being done in the near future. But indirect evidence can be used. For example, we see from the work of Dyson[1] and Takeda[2] that the second

**TABLE I.** Charge distribution of the products of $\pi^-$—p collisions for processes involving K-particle and hyperon production

| Type of process | Charge state | Statistical weight of the charge state | Type of process | Charge state | Statistical weight of the charge state |
|---|---|---|---|---|---|
| $\Lambda\Theta$ | $\Lambda^0\Theta^0$ | 1.000 | $\Sigma K1$ | $\Sigma^0 K^+\pi^-$ | 0.189 |
| | | | | $\Sigma^- K^+\pi^0$ | 0.189 |
| $\Sigma K$ | $\Sigma^- K^+$ | 0.555 | $\Sigma K\Pi$ Takeda | $\Sigma^0\Theta^0\Pi^0$ | 0.211 |
| | $\Sigma^0\Theta^0$ | 0.445 | | $\Sigma^+\Theta^0\Pi^-$ | 0.233 |
| $\Lambda\Theta1$ | $\Lambda^0\Theta^0\pi^0$ | 0.445 | | $\Sigma^-\Theta^0\Pi^+$ | 0.178 |
| | $\Lambda^0 K^+\pi^-$ | 0.555 | | $\Sigma^0 K^+\Pi^-$ | 0.189 |
| $\Lambda\Theta\Pi$ Takeda | $\Lambda^0\Theta^0\Pi^0$ | 0.445 | | $\Sigma^- K^+\Pi^0$ | 0.189 |
| | $\Lambda^0 K^+\Pi^-$ | 0.555 | $\Sigma K\Pi$ Dyson | $\Sigma^0\Theta^0\Pi^0$ | 0.445 |
| $\Lambda\Theta\Pi$ Dyson | $\Lambda^0\Theta^0\Pi_0$ | 1.000 | | $\Sigma^- K^+\Pi^0$ | 0.555 |
| $\Sigma K1$ | $\Sigma^0\Theta^0\pi^0$ | 0.211 | $NK\tilde{K}$ | $N\Theta^0\tilde{\Theta}^0$ | 0.278 |
| | $\Sigma^+\Theta^0\pi^-$ | 0.233 | | $NK^+K^-$ | 0.278 |
| | $\Sigma^-\Theta^0\pi^+$ | 0.178 | | $PK^-\Theta^0$ | 0.444 |
| | | | $\Xi KK$ | $\Xi^- K^+\Theta^0$ | 1.000 |

**TABLE II.** Statistical weights of $\pi^-$—p collision processes at 4.5 Bev.

| Number of secondary mesons | | 0 | 1 | | | 2 | | | | 3 | | | | | 4 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Type of process | | $N1$ | $N2$ | $N'1$ | $N\Pi$ | $N3$ | $N'2$ | $N\Pi1$ | $N'\Pi$ | $N4$ | $N'3$ | $N'\Pi1$ | $N\Pi2$ | $N\Pi\Pi$ | $N5$ | $N'4$ | $N'\Pi2$ | $N'\Pi\Pi$ | $N\Pi3$ |
| Statistical weight | Takeda version | 0 | 3 | 1 | 0 | 5 | 5 | 6 | 1 | 2 | 6 | 13 | 14 | 2 | 0 | 2 | 17 | 3 | 9 |
| | Dyson version | 1 | 5 | 1 | 0 | 7 | 8 | 5 | 0 | 4 | 9 | 9 | 10 | 1 | 1 | 3 | 11 | 1 | 6 |

| Number of secondary mesons | | Strange particles | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Type of process | | $\Lambda\Theta$ | $\Sigma\Theta$ | $\Lambda\Theta1$ | $\Lambda\Theta\Pi$ | $\Sigma K1$ | $\Sigma K\Pi$ | $\Xi KK$ | $NK\tilde{K}$ |
| Statistical weight | Takeda version | 0 | 0 | 2 | 1 | 4 | 3 | 0 | 1 |
| | Dyson version | 0 | 1 | 3 | 2 | 6 | 4 | 0 | 2 |

*There seems to be an error in the calculation of the frequency of stars with various numbers of prongs in reference 7. The relative probabilities for elastic non-diffractive scattering, 2-prong, 4-prong and 6-prong stars should be respectively 0.75, 45, 28.5, and 0.75 instead of 1, 49, 24.5, and 0.5.

**TABLE III.** Charge distribution of $\pi^- - p$ collision products, including effects of $\Pi$-particle production

| Type reaction | Reaction products | Statistical weights of charge states | |
|---|---|---|---|
| | | Dyson version | Takeda version |
| $N'\Pi$ | $P+--$ | 0.222 | 0.081 |
| | $P-00$ | 0,111 | 0.193 |
| | $N+-0$ | 0.445 | 0.726 |
| | $N\,000$ | 0.222 | — |
| $N\Pi 1$ | $P+--$ | 0,371 | 0.189 |
| | $P-00$ | 0.185 | 0,189 |
| | $N+-0$ | 0.296 | 0.622 |
| | $N\,000$ | 0.148 | — |
| $N\Pi\Pi$ | $P+--0$ | — | 0,378 |
| | $N++--$ | 0.444 | 0.155 |
| | $N+-00$ | 0.445 | 0,467 |
| | $N\,0000$ | 0.111 | — |
| $N'\Pi 1$ | $P+--0$ | 0.183 | 0.355 |
| | $P-000$ | 0,092 | 0,071 |
| | $N++--$ | 0.375 | 0.181 |
| | $N+-00$ | 0.296 | 0.393 |
| | $N\,0000$ | 0,054 | — |
| $N\Pi 2$ | $P+--0$ | 0.252 | 0.465 |
| | $P-000$ | 0.126 | 0.074 |
| | $N++--$ | 0.311 | 0.130 |
| | $N+-00$ | 0.259 | 0,331 |
| | $N\,0000$ | 0.052 | — |
| $N'\Pi\Pi$ | $P++---$ | 0,148 | 0.032 |
| | $P+--00$ | 0.148 | 0.395 |
| | $P-0000$ | 0,037 | — |
| | $N++--0$ | 0.296 | 0.425 |
| | $N+-000$ | 0.296 | 0.148 |
| | $N\,00\,000$ | 0,075 | — |
| $N'\Pi 2$ | $P++---$ | 0.173 | 0.096 |
| | $P+--00$ | 0.201 | 0,324 |
| | $P-0000$ | 0.054 | 0.025 |
| | $N++--0$ | 0.339 | 0.364 |
| | $N+-000$ | 0,212 | 0.191 |
| | $N\,00000$ | 0.021 | — |
| $N\Pi 3$ | $P++---$ | 0,144 | 0,057 |
| | $P+--00$ | 0.180 | 0,306 |
| | $P-0\,000$ | 0.054 | 0,015 |
| | $N++--0$ | 0,385 | 0,428 |
| | $N+-000$ | 0.222 | 0.194 |
| | $N\,00\,000$ | 0.015 | — |

maximum in the $\pi^-$-p scattering around 1 Bev might be explained by a resonant interaction between two pions. Using the Fermi statistical theory, we can handle such an interaction by supposing that a new "virtual particle" $\Pi$ is being produced, having mass $\mu = 0.47$,* ordinary spin $S = 0$ and isotopic spin $T = 0$[1] or $T = 1$.[2]

By comparing the results of the theory with experiment, we can find out which of the two isotopic spin values is more correct, within the limits of the statistical model.† We calculate here the effect of the pion-pion interaction on $\pi^-$-p scattering at 4.5 Bev.

We take the statistical weights from the formula

$$s(n) = [\Omega / (2\pi)^3]^{n-1}\, dQ(E) / dE, \quad \Omega = \Omega_0 / \gamma_2', \quad \Omega_0 = {}^4/_3\, \pi R_0^3.$$

Here $R_0 = 1.4 \times 10^{-13}$ cm is the pion Compton wavelength, and $\gamma_2'$ is the nucleon energy in the $\pi$-p center-of-mass system. In all cases the statistical weights are calculated with conservation of energy, momentum, spin, isotopic spin and its components, and identity of particles. In the case of strange particles, we also take into account the conservation of strangeness.

The calculations for two and three particles were made with the exact formulae of reference 6. For four and five particles, we made the approximation of treating the nucleons and nucleon-isobars nonrelativistically, the pions and the $\Pi$-particles ultrarelativistically. We neglected processes in which more than four secondary pions are produced, such processes being certainly unimportant at these energies.

In calculating the production of K-particles and hyperons, we assumed the K-particles to have spin zero, the hyperons to have spin $\frac{1}{2}$. At 4.5 Bev, allowing for the conservation of strangeness, only the following processes are energetically possible: $\Lambda\Theta$, $\Sigma\Theta$, $\Lambda\Theta 1$, $\Sigma\Theta 1$, $\Lambda\Theta\Pi$, $\Sigma\Theta\Pi$, $\Xi\Theta\Theta$, $N\Theta\tilde{\Theta}$. Table I enumerates the charge-states arising from these processes.

Table II shows the statistical weights of the various processes. To compare the theoretical results with experiment, the charge-distribution of all the reaction products was calculated (Tables I and III). The charge-distribution of the products of $\pi^-$-p collisions, assuming only pions and isobars to be produced, is shown in Table II of reference 6. In our calculations we have assumed[6] that the isobar,

**TABLE IV.** Distribution by prong-number of charged products of $\pi^- - p$ collisions, normalized to 100%

| Type of collision | Experiment[7] | Statistical Theory | | | |
|---|---|---|---|---|---|
| | | Including isobars only*[7] | Including isobars and strange particles only | Takeda version** | Dyson version** |
| 2-prong inelastic | 60 | 61 | 72 | 48 | 51 |
| 4-prong inelastic | 38 | 38 | 27 | 49 | 45 |
| 6-prong inelastic | 2 | 1 | 1 | 3 | 3 |

*Corrected value, see first footnote of text.

**The Dyson and Takeda versions are calculated including isobars, strange particles and $\Pi$-particles, the latter possessing isotopic spin 0 according to Dyson, 1 according to Takeda.

*We use units such that $\hbar = c = M_{nucl} = 1$.

†This kind of treatment of a resonant interaction is also interesting, because it allows us indirectly to incorporate into the statistical theory the effects of a matrix element $H'_{if}$ in the expression $W_{i\to f} = \frac{2\pi}{\hbar}|H'_{if}|^2 \rho_E$. In the original Fermi theory such matrix elements were neglected.

which can have the $T_Z$ component of its isotopic spin equal to $\pm 3/2$ or $\pm 1/2$, decays from the state with $T_Z = 3/2$ into $p\pi^+$; from the state $T_Z = -3/2$ into $N\pi^-$; from the state $T_Z = 1/2$ with probability $2/3$ into $p\pi^0$ and with probability $1/3$ into $N\pi^+$; from the state $T_Z = -1/2$ with probability $1/3$ into $p\pi^-$ and with probability $2/3$ into $N\pi^0$. In the theory of Takeda ($T_\Pi = 1$), the $\Pi$-particle has three states with $T_Z = \pm 1, 0$. It decays from the state $T_Z = +1$ into $\pi^+\pi^0$; from the state $T_Z = -1$ into $\pi^-\pi^0$; from the state $T_Z = 0$ into $\pi^+\pi^-$. In the Dyson theory ($T_\Pi = 0$) the $\Pi$-particle has only one state $T_Z = 0$, and decays with probability $2/3$ into $\pi^+\pi^-$, with probability $1/3$ into $\pi^0\pi^0$.

For the comparison with experiment we have used references 8 and 9. Reference 9 concerns $\pi^-$-p scattering at 5 Bev, but the experimental results at this energy differ very little from the results at 4.5 Bev.

Table IV shows the distribution of the reaction products according to the number of prongs, calculated from Tables I to III. Comparison of these numbers with experiment shows that the number of two-prong stars is too small, especially in the Takeda theory. However, the Dyson theory gives better agreement than a calculation including only isobars and strange particles. It should also be remembered that the reaction $\Lambda\Theta 2$ will make some contribution to the statistical weights, and this will increase the number of two-prong stars.

The ratio between strange particles and observable pions is 0.08 to 0.09 in Takeda's theory, 0.14 in Dyson's theory, and 0.3 to 0.4 in the theory which includes only isobars. The experimental value of this ratio[9] is 0.04 to 0.05.

The average multiplicity is 2.7 in Takeda's theory, 2.4 in Dyson's theory and $2.3 \pm 0.1$ experimentally.[9]

From these results we draw the following conclusions:

1. The statistical theory, including only isobars, K-particles and hyperons, disagrees significantly with experiment, both in the distribution of numbers of prongs (Table IV) and in the ratio of strange particles to pions.

2. When a resonant pion-pion interaction, or $\Pi$-particle, is included in the statistical theory in addition to the isobar, the agreement with experiment is much improved so far as K-particle and hyperon production is concerned.

3. The Dyson theory agrees with experiment better than the Takeda theory, although there are some discrepancies in both cases. Such discrepancies are to be expected, since our calculations have ignored the conservation of angular momentum.

In conclusion, it is my pleasant duty to thank Professor Zh. S. Takibaev and his collaborators P. A. Usik and Ya. M. Granovskii for valuable advice and help, and A. Akhmedshin for his assistance with the calculations.

Note added in proof (October 15, 1958). After this paper was submitted for publication, we learned of the work of R. Gatto and M. A. Ruderman [Nuovo cimento 8, 775 (1958)], in which a similar treatment of a resonant pion-pion interaction is used in a calculation of proton-antiproton annihilation.

[1] F. J. Dyson, Phys. Rev. **99**, 1037 (1955).

[2] G. Takeda, Phys. Rev. **100**, 440 (1955).

[3] A. N. Mitra and R. P. Saxena, Phys. Rev. **108**, 1083 (1957).

[4] J. S. Kovacs, Phys. Rev. **101**, 397 (1956).

[5] S. Z. Belen'kiĭ, Nucl. Phys. **2**, 259 (1956); G. Uhlenbeck and H. A. Bethe, Physica, **3**, 729 (1936) and **4**, 195 (1937).

[6] Belen'kiĭ, Maksimenko, Nikishov, and Rozental', Usp. Fiz. Nauk, **62**, 1 (1959).

[7] A. I. Nikishov, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 990 (1956), Soviet Phys. JETP **3**, 783 (1956).

[8] Maenchen, Powell, Saphir, and Wright, Phys. Rev. **99**, 1619 (1955).

[9] Maenchen, Fowler, Powell, and Wright, Phys. Rev. **108**, 850 (1957).

Translated by F. J. Dyson
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## ERRATA TO VOLUME 9

| Page | Reads | Should read |
|---|---|---|
| 115, Col. 2, line 18 from top | R. Gatto and M. A. Ruderman, [Nuovo cimento 8, 775, (1958)] | T. Goto, Nuovo cimento 8, 625 (1958) |
| 294, Col. 2, line 4 from bottom | $N = N_{exp}(p, \theta)\, F(p, \theta)$ | $N = N_{exp}(p, \theta)\, 1 + F(p, \theta)$ |
| 462, Col. 1, line 8 from top | which are approximately 13Z ev | and approximately equal to 13Z ev |
| 646, Col. 1, line 3 from top | $\langle j_1' t_1' \alpha \| R^{J_2} \| j_1 t_1 \alpha_1 \rangle$ | $\langle j_1' t_1' \alpha \| R^{J_1} \| j_1 t_1 \alpha_1 \rangle$ |
| 661, Col 1, line 6 from top | $\lambda\!\!\!^- = 2.14 \times 10^{-13}$ | $\lambda\!\!\!^- = 1.04 \times 10^{-13}$ |