THE TWO-NUCLEON **LS** *POTENTIAL IN NONRELATIVISTIC MESON THEORY*

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The potential between nucleons is calculated up to and including terms linear in the velocity (with recoil taken partially into account) without using perturbation theory. A potential is found which decreases as e^{-2R} and depends both on the coupling constant and on the cross section for π-N scattering.

ANALYSIS of experimental data shows[1] that it is impossible to explain nucleon-nucleon scattering even at an energy of 90 Mev without a potential which depends on the velocities. Also, the **LS** potential between nucleons has been considered earlier only in perturbation theory.[2] Therefore, it is of interest to clarify the procedure of obtaining a potential that depends on the velocities from meson theory without using perturbation theory.

In this article, the **LS** potential is calculated within the framework of a meson theory which is nonrelativistic with respect to the nucleons. In general, a number of objections have been raised against such a theory at one time or other: (a) The region of high virtual energies may be essential in the interaction of low-energy nucleons. (b) The interaction Hamiltonian is not completely defined within such a theory (this is connected with the problem of a proper calculation of the role of nucleon-antinucleon pairs). However, in the case of the two-nucleon problem, one can consider the nonrelativistic approach to be well founded. The favorable situation here arises because of the resonance character of π-N scattering. Because of this, the center of gravity of the matrix elements for π-N scattering lies in the nonrelativistic region, and just this matrix element enters into the velocity-dependent two-nucleon potential. As regards the interaction Hamiltonian, the method of calculating the potential developed below is not connected essentially with the type of interaction. We consider in this article only the potential that comes from pseudovector coupling.

Instead of the perturbation-theory expansion, an expansion in rate of fall-off with respect to the distance R between two nucleons will be carried out in this article. The n'th-order potential falls off as $\exp(-nR)$. (We use a system of units $\hbar = c = \mu = 1$, where μ is the mass of the meson.) We calculate below potentials of type $\exp(-R)$ and $\exp(-2R)$. (It would seem that potentials of higher order are not meaningful within the nonrelativistic approximation.) It turns out that only the potential $\exp(-2R)$ depends on the velocity. This potential contains, in addition to a term obtained in perturbation theory, a term which depends on the cross section σ_{tot} for π-N scattering.

With the present state of knowledge, this cross section σ_{tot} must be taken from experiment; in this way, the potential obtained might be called semi-phenomenological. In the static case, such a semi-phenomenological potential has been considered earlier by Klein and McCormick,[3] Miyazawa,[4] and one of the authors.[5]

The calculation in this article is based on the methods of a theory of the scattering of "dressed" particles, which was given in reference 6 by one of the authors (we denote reference 6 by I). We consider the meson cloud of both nucleons to be the same as it would be in the absence of interaction between the nucleons. The interaction arises both from meson exchange and from distortion of the meson clouds.

Renormalization causes considerable difficulty, in general, in calculations other than perturbation theory ones. In this work, this difficulty does not arise, because we consider only the approximation linear in velocity, in which the problem of renormalization is the same as in the static case.

In the classification of terms with respect to velocity, some arbitrariness arises, in the general case, in the definition of the potential[2,7] since, on the one hand, terms of the type of the kinetic energy $p^2/2M$ are equivalent to static potentials of higher order and, on the other hand, as Levy showed,[7] some terms in the velocity do not contribute to the scattering. The former is not important in our case, since we consider only potentials of the first two orders, $\exp(-R)$ and $\exp(-2R)$, where this arbitrariness is connected with the well-investigated static potential of fourth order in perturbation theory.[2,7,8] Concerning the latter, those terms in the velocity which do not influence the scattering, have a characteristic form and are easily identified.

INITIAL FORMULAE FOR THE POTENTIAL

1. We denote by W_{ba} the matrix element of the potential, in momentum representation, between states a and b of the system of two free nucleons. (Two-nucleon states will be labelled by letters a, b, c....) The initial formula for W_{ba} has the following form [see I(28)]:

$$W_{ba}^{I} + W_{ba}^{II} = W_{ba} = (\Phi_b (H - E_a) \Phi_a) - \sum_{\substack{n, c \\ \mathbf{p}_c = -\mathbf{q}_n}}' \frac{(\psi_{nc}^{(-)}, (H - E_b) \Phi_b)^* (\psi_{nc}^{(-)}, (H - E_a) \Phi_a)}{E_n + E_c - E_a - i\varepsilon}. \quad (1)$$

Here $\psi_{nc}^{(-)}$ is the eigenvector of the complete Hamiltonian H for a state with n mesons and energy E_n and two nucleons c with energy E_c; $\mathbf{q}_n$ is the total momentum of the n mesons. Summation over n in Eq. (1) begins with $n = 1$ (there are no states in the sum which do not contain real mesons). In comparison with I(28), a factor $(2\pi)^3 \delta(\mathbf{P}_a - \mathbf{P}_b)$ has been taken out, where $\mathbf{P}$ is the total momentum, and specialization has been made to the center-of-mass system (c.m.s.).

In the c.m.s., the Hamiltonian H for two nucleons, interacting with the meson field, is

$$H = -\frac{1}{M} \nabla_R^2 + \frac{u^2}{4M} + H_\pi + U_1(\mathbf{r}_1) + U_2(\mathbf{r}_2);$$

$$U_i(\mathbf{r}_i) = \sum_q \{V_{iq} e^{i\mathbf{q}\cdot\mathbf{r}_i} a_q + \text{H.c.}\}; \quad V_{iq} = if(\sigma_i \cdot \mathbf{q}) \tau_{iq} / (2\omega_q)^{1/2}; \quad (2)$$

$$\mathbf{r}_1 = -\mathbf{r}_2 = \mathbf{R}/2; \quad \omega_q = (1 + q^2)^{1/2}; \quad \mathbf{u} = \sum \mathbf{q} a_q^+ a_q.$$

The operators σ_i and τ_i relate to the i-th nucleon; q denotes all quantum numbers of the meson. The term $u^2/4M$ appears in H from elimination of the center-of-mass coordinates.

The functionals Φ_a and Φ_b describe the asymptotic states of a system of two "dressed" nucleons; both Φ_a and Φ_b are solutions of the Schrödinger equation $(H - E_c)\Phi_c = 0$, with the full Hamiltonian H, for $R \to \infty$. The choice of Φ_a and Φ_b depends on the type of approximation in which the potential is calculated.

The connection between the desired potential W and the quantity W_{ba} is established in the following way. We represent W_{ba} as

$$W_{ba} = \int e^{i(\mathbf{p}_a - \mathbf{p}_b)\cdot\mathbf{R}} W_{\gamma'\gamma}(\mathbf{R}, \mathbf{v}_a, \mathbf{v}_b) d^3R. \quad (3)$$

Here $\mathbf{v}_a = \mathbf{p}_a/M$, $\mathbf{v}_b = \mathbf{p}_b/M$ are the nucleon velocities of the initial and final state in the c.m.s.; γ' and γ denote the totality of spin-charge variables of the nucleons, corresponding to states a and b. We calculate the quantity $W_{\gamma'\gamma}(\mathbf{R}, \mathbf{v}_a, \mathbf{v}_b)$ in the approximation linear in v and with neglect of terms from meson recoil of the type u^2/M. Here, as we shall show below, we can set $\mathbf{v}_a = \mathbf{v}_b = \mathbf{v}$ in the integrand in Eq. (3); i.e., in the calculation of $W_{\gamma'\gamma}$, the nucleon velocity can be considered as a parameter. The desired potential $W(\mathbf{R}, \mathbf{v})$, which depends on the velocity, is then determined by the formula

$$(\chi_{\gamma'}, W(\mathbf{R}, \mathbf{v}) \chi_\gamma) = W_{\gamma'\gamma}(\mathbf{R}, \mathbf{v}, \mathbf{v}), \quad (3a)$$

where $\chi_{\gamma'}$ and χ_γ are the spin-charge functions for the two-nucleon system in the phenomenological treatment. In other words, the potential calculated in our approximation (linear approximation with respect to velocity, and neglect of the term u^2/M) is, in essence, a potential between nucleons surrounded by meson clouds and moving with constant velocities $\mathbf{v}$ and $-\mathbf{v}$. We calculate this potential, discarding terms which fall off as $\exp(-3R)$ and faster.

A consistent procedure of calculation of $W(\mathbf{R}, \mathbf{v})$ should start from a relativistic equation for the scattering amplitude. But no relativistic theory of the π-N interaction exists, and, therefore, we choose another way for the construction of W, based on the success of the static-meson theory in the low-energy range. Since the renormalized constants are quadratic in velocity, the renormalization will be carried out automatically in the linear approximation, if the meson cloud of the nucleon, described by the functional F, has the same character as in the static case. This situation makes it possible to base the approximation, linear with respect to velocity, on the solution of the static problem for the nucleon. The functional $F_{\alpha v}(i)$ for the i-th nucleon, moving with velocity v, can be obtained from the functional for the fixed nucleon $F_{\alpha 0}(i)$ by transition to a moving system of reference (α represents all of the quantum numbers of the nucleon aside from the velocity v).

In changing to the moving system, we make a Galilean transformation of the state vector of the dressed nucleon. The functional $F_{\alpha v}(i)$ has the form

$$F_{\alpha v}(i) = \exp[i(\mathbf{p} - \mathbf{u})\cdot\mathbf{r}_i] \mid i\alpha v\rangle, \quad (4)$$

where $i\alpha v$ does not depend on $\mathbf{r}_i$, and the one-nucleon Hamiltonian

$$H_{iv}(0) = \{-(\mathbf{v}\cdot\mathbf{u}) + H_\pi + U_i(0)\} \quad (5)$$

differs from the static Hamiltonian $H_{i0}(0) = H_\pi + U_i(0)$ by the substitution of ω_k in H_π by $\omega_k - (\mathbf{v}\cdot\mathbf{k})$. Thus, we have

$$H_{iv}(0)\,|\,i\alpha v\rangle = 0. \qquad (6)$$

We note that in so far as the Hamiltonian for the fixed nucleon $H_{i0}(0)$ does not contain recoil terms of the type u^2/M, these terms are not in the Hamiltonian (5). Thus, the term u^2/M in the Hamiltonian (2) will also be omitted.

With the aid of the formulae (4), (5), and (6) for the one-nucleon function $F_{\alpha v}$, we can write the explicit form of the vector Φ which enters into Eq. (1) [see I(8)]:

$$\Phi_a = Z_{aa}^{-1/2}\sum_{\alpha\beta} C^a_{\alpha\beta} F_{\alpha v_a}(1,\ a^+)\, F_{\beta,-v_a}(2,\ a^+)\,\Lambda_0 = Z_{aa}^{-1/2} e^{i\mathbf{p}_a\cdot\mathbf{R}}\,|\,av), \qquad (7)$$

where the velocities of nucleons 1 and 2 have been set equal to $\mathbf{v}_a$ and $-\mathbf{v}_a$, respectively. Here

$$Z_{ba}(\mathbf{R}) = (bv_b\,|\,av_a) \qquad (8)$$

normalizes the amplitude Φ_a to unity (In all matrix elements of the type $(bv\,|\,L\,|\,av)$, the integration over R is not carried out.).

2. We shall show that, in the approximation linear with respect to $\mathbf{v}$ and with neglect of recoil terms of the type u^2/M, we can set $\mathbf{v}_a = \mathbf{v}_b$ in $W_{\gamma'\gamma}(\mathbf{R}, \mathbf{v}_a, \mathbf{v}_b)$ [see Eq. (3)]. To do this, we investigate the dependence of $W_{\gamma'\gamma}(\mathbf{R}, \mathbf{v}_a, \mathbf{v}_b)$ on $\mathbf{v}_a$ and $\mathbf{v}_b$. We represent $W_{\gamma'\gamma}(\mathbf{R}, \mathbf{v}_a, \mathbf{v}_b)$ in the form

$$W_{\gamma'\gamma} = \int d^3k e^{i\mathbf{k}\cdot\mathbf{R}} w'_{\gamma'\gamma}(\mathbf{k},\ \mathbf{v}_a,\ \mathbf{v}_b) + \int d^3q d^3k e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{R}} w''_{\gamma'\gamma}(\mathbf{k},\mathbf{q},\mathbf{v}_a,\mathbf{v}_b), \qquad (9)$$

where w' and w'' correspond to the potentials which fall off as $\exp(-R)$ and $\exp(-2R)$ respectively.

Since the entire dependence of w' and w'' on $\mathbf{v}_a$ and $\mathbf{v}_b$ occurs through the one-nucleon matrix elements of the type

$$\langle\alpha' v_b|\ V_k\ |\alpha v_a\rangle;\qquad \langle\alpha' v_b\,|\,a_k\,|\,\alpha v_a\rangle;$$
$$\langle\alpha' v_b\,|\,a_k a_q\,|\,\alpha v_a\rangle;\qquad \langle\alpha' v_b\,|\,V_k a_q\,|\,\alpha v_a\rangle, \qquad (10)$$

we should first elucidate the way in which the quantities (10) depend upon $\mathbf{v}_a$ and $\mathbf{v}_b$. Here we employ the formulae

$$a_k F_{\alpha v} = -[H_v + \omega_k(v)]^{-1} V_k^+ \exp(-i\mathbf{k}\cdot\mathbf{r})\, F_{\alpha v},$$

$$a_k a_q F_{\alpha v} = \exp[-i(\mathbf{k}+\mathbf{q})\cdot\mathbf{r}]\,[H_v + \omega_q(v) + \omega_k(v)]^{-1} \times \{V_k^+[H_v + \omega_q(v)]^{-1}V_q^+ + V_q^+[H_v + \omega_k(v)]^{-1}V_k^+\}\, F_{\alpha v},$$

$$\omega_q(\pm v) = \omega_q^{\pm} = \omega_q \mp (\mathbf{q}\cdot\mathbf{v}), \qquad (11)$$

which go over into the well-known relations of the static theory for $v = 0$. Using Eqs. (11), the matrix elements (10) can be expressed through matrix elements of the form

$$\langle\alpha' v_b\,|\,V_q \frac{1}{H_v(0) + \omega_q(v)} \cdots \frac{1}{H_v(0) + \omega_{q'}(v)} \cdots V_{q'}\,|\,\alpha v_a\rangle, \qquad (12)$$

containing the product of operators V and $[H_v(0) + \omega(v)]^{-1}$ in various orders. Here the v without a suffix in Eq. (12) is equal to one of the vectors $\mathbf{v}_a$, $\mathbf{v}_b$. Using the unitary inversion operator I of the pseudoscalar meson field[9]

$$I = \exp\Big\{-\frac{i\pi}{4}\sum_q (a_q^+ - a_{-q}^+)(a_q + a_{-q})\Big\}, \qquad (13)$$

$$I a_q I^+ = -a_{-q};\qquad I a_q^+ I^+ = -a_{-q}^+, \qquad (14)$$

we find

$$I H_0(0) I^+ = H_0(0);\qquad I\mathbf{u}I^+ = -\mathbf{u};$$
$$I H_v(0) I^+ = H_{-v}(0). \qquad (15)$$

In addition

$$I\,|\,\alpha,\ v\rangle = |\,\alpha,\ -v\rangle, \qquad (16)$$

since the state $\alpha, \pm v$ relates, for fixed velocity v, to the lowest eigenvalue of $H_{\pm v}(0)$, which is nondegenerate (not considering the degeneracy with respect to α). With Eqs. (14), (15), and (16) we can obtain the following relations for (12).

$$\langle\alpha' v_b\,|\,I^+ I\, V_q \frac{1}{H_v(0) + \omega_q(v)} \cdots \frac{1}{H_v(0) + \omega_{q'}(v)} \cdots V_{q'} I^+ I\,|\,\alpha v_a\rangle = \langle\alpha',\ -v_b\,|\,V_q \frac{1}{H_{-v}(0) + \omega_q(v)} \cdots \frac{1}{H_{-v}(0) + \omega_{q'}(v)} \cdots V_{q'}\,|\,\alpha,\ -v_a\rangle. \qquad (17)$$

If follows from Eq. (17) that the matrix element (12) does not change if v is changed to $-v$ throughout, except in $\omega(v)$. Consequently, the entire matrix element (12) depends on $\mathbf{v}_a$ and $\mathbf{v}_b$ only through $\mathbf{v}_a^2$, $\mathbf{v}_b^2$ and $(\mathbf{v}_a\mathbf{v}_b)$, aside from the dependence through $\omega(v)$. Therefore, in the approximation linear in velocity, we can set $\mathbf{v}_a = \mathbf{v}_b = 0$ everywhere in Eq. (12) except in $\omega(v)$. From this it follows that the dependence of the quantities (12) and, through them, also w' and w'', is completely through $\omega(v)$, i.e., through scalar products $(\mathbf{v}_a\cdot\mathbf{k})$, $(\mathbf{v}_b\cdot\mathbf{q})$ etc.

As can be seen from Eqs. (3) and (7), the multiplication of $W(\mathbf{R}, \mathbf{v}_a, \mathbf{v}_b)$ by $(\mathbf{v}_a - \mathbf{v}_b)$ is equivalent to multiplication of the Fourier transforms w' and w'' by $-\mathbf{k}/M$ and $-(\mathbf{k}+\mathbf{q})/M$, respectively. Therefore, terms $(\mathbf{v}_a - \mathbf{v}_b, \mathbf{q})$ and $(\mathbf{v}_a - \mathbf{v}_b, \mathbf{k})$ in w' and w'' are equivalent to terms $-k^2/M$ and $-(\mathbf{k}+\mathbf{q}, \mathbf{k})/M$, which are characteristic of the meson recoil of type u^2/M. Consequently, if we neglect the quadratic recoil terms u^2/M, we can set $\mathbf{v}_a = \mathbf{v}_b$ in the potential $W(\mathbf{R}, \mathbf{v}_a, \mathbf{v}_b)$.

3. Starting from Eq. (1), we introduce formulae for the potential $W(\mathbf{R}, \mathbf{v})$. The first term in Eq.

(1) pertains to a number of matrix elements, methods of calculation of which were considered in I. In this case, additional terms arise because of the factor Z, which was not taken into account in the derivation of I(20). With this in mind, we find as we go from Eq. I(20) to separate coordinates for the meson clouds, that with the choice of the functional Φ_a and Φ_b in the form Eq. (7) (and in our approximation), the term $(\Phi_b, (H-E_a)\Phi_a)$ leads to the potential W^I with matrix elements

$$W^{I}_{\gamma'\gamma} = \langle bv \,|\, (1+\hat{N})\,[U_1^{(+)}(a_2) + U_2^{+}(a_1)]\,|\, av\rangle (Z_{aa}Z_{bb})^{-1/2} + i\,(\mathbf{v}\cdot\nabla Z_{aa})\,Z_{ab}\,(Z_{bb}Z^3_{aa})^{-1/2}; \qquad (18)$$
$$Z_{ab} = \langle av\,|\,(1+\hat{N})\,|\,bv\rangle.$$

Here and throughout $|av\rangle$ denotes the two-nucleon state vector with separate coordinates:

$$|av\rangle = \sum_{\alpha\beta} C^a_{\alpha\beta} e^{-i\mathbf{p}_a\cdot\mathbf{R}} F_{\alpha v}(1, a_1^+)\, F_{\beta,-v}(2, a_2^+)\,\Lambda_0 = \sum_{\alpha\beta} C^a_{\alpha\beta} e^{-i\mathbf{p}_a\cdot\mathbf{R}} e^{-i(\mathbf{u}_1-\mathbf{u}_2)\cdot\mathbf{R}/2}\,|1, \alpha v\rangle\,|2, \beta\, -v\rangle, \qquad (19)$$

where the functional of the i-th nucleon $|i\alpha v\rangle$, defined by Eqs. (4) and (6), depends only on the operators a_i^+; $\mathbf{u}_i = \sum_q q a_{iq}^+ a_{iq}$.

To obtain the potential of order $\exp(-2R)$, it is necessary to insert into the term $(\mathbf{v}\cdot\nabla Z)$ in Eq. (18) the first two terms of the expansion I(22) for the operator $\hat{N}$:

$$\hat{N} = \sum_q (a_{1q}^+ a_{2q} + a_{2q}^+ a_{1q}) + \frac{1}{2}\sum_{kq} (a_{2q}^+ a_{2k}^+ a_{1q} a_{1k} + a_{1q}^+ a_{1k}^+ a_{2q} a_{2k} + 2a_{2q}^+ a_{1k}^+ a_{1q} a_{2k}). \qquad (20)$$

In the calculation of the first term in Eq. (18), the first term in Eq. (20) is sufficient.

We transform the second term of Eq. (1). We introduce the notation

$$\Gamma_a = (H - E_a)\,\Phi_a. \qquad (21)$$

As shown above, Γ falls off as $\exp(-R)$. Therefore, the second term in Eq. (1) in which Γ occurs twice, can be written in the following form:

$$-W^{II}_{ba} = (\Gamma_b\,|\,(H-E_a)^{-1}\,|\,\Gamma_a) - \sum_{\substack{c \\ p_c=0}} (\Gamma_b\,|\,\Phi_c)\,(E_c - E_a - i\varepsilon)^{-1}\,(\Phi_c\,|\,\Gamma_a). \qquad (22)$$

Here we have substituted the vector Φ_c for $\psi_{0c}^{(-)}$, since the terms discarded here lead to a potential $\exp(-3R)$.

We adopt the convention that the double line in the functional

$$\|\, G(a_1, a_2, a_1^+, a_2^+)\,\Lambda_0)$$

means that we should first calculate the expression $G\Lambda_0$ to the right of the double line, that is, we should first free it from all annihilation operators a_1, a_2, moving them to the right to the vacuum Λ_0, taking account of commutation relations, and then set $a_1^+ = a_2^+ = a^+$.

For transformation of the first term in Eq. (22), it is convenient to use Eq. (11) to represent Γ_a in the form $\Gamma_a = \|\,\Gamma_a(a_1^+, a_2^+)\,\Lambda_0)$, where

$$\Gamma_a(a_1^+, a_2^+)\,\Lambda_0 = \sum_{\mathbf{k},\alpha\beta} C^a_{\alpha\beta}\, g(\mathbf{k}+\mathbf{p}_a, \mathbf{R})\,\hat{M}(\mathbf{k}, \mathbf{v}, 0)\,|\alpha v\rangle\,|\beta, -v\rangle;$$
$$\hat{M}(\mathbf{k}, \mathbf{v}, \mathbf{R}) = e^{i\mathbf{k}\cdot\mathbf{R}}\Big\{-(\mathbf{v}\cdot\mathbf{k})\,Z_{aa}(\mathbf{k}) + \Big[\frac{1}{H_{1v}+\omega_k(-v)} + \frac{1}{H_{2,-v}+\omega_k(-v)}\Big] V_{1k}V_{2k}\Big\}; \qquad (23)$$
$$g = \exp\Big\{i\Big[\frac{\mathbf{u}_2-\mathbf{u}_1}{2} + \mathbf{p}_a + \mathbf{k}\Big]\cdot\mathbf{R}\Big\}; \quad Z_{aa}(\mathbf{R}) = \sum_{\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{R}} Z_{aa}(\mathbf{k}).$$

The quantity $(H-E_a)^{-1}\Gamma_a$ in Eq. (22) can be transformed in the following way:

$$[H_\pi + U_1 + U_2 - E_a]^{-1}\,\|\Gamma_a) = \|\,[H_{10} + H_{20} - E_a]^{-1}\Gamma_a(a_1^+, a_2^+)\,\Lambda_0) - [H_\pi + U_1 + U_2 - E_a]^{-1}\,\|\,(U_1^{(+)}(a_2) + U_2^{(+)}(a_1)) \times [H_{10} + H_{20} - E_a]^{-1}\,\Gamma_a(a_1^+, a_2^+)\,\Lambda_0); \qquad (24)$$
$$H_{iv} = H_\pi(a_i^+, a_i) + U_i(a_i^+, a_i) - (\mathbf{v}\cdot\mathbf{u}_i).$$

The second term on the right-hand side falls off as $\exp(-2R)$ and can be discarded. With the aid of Eq. (24), the following expression for the potential corresponding to the first term in Eq. (22) is obtained

$$\sum_{\mathbf{k}} \langle bv\,|\,[U_1^{(-)}(a_2^+) + U_2^{(-)}(a_1^+) - i\,(\mathbf{v}, \nabla Z_{bb})]\,[H_{1,v+v_k} + H_{2,-v-v_k} + 2E(p+k) - 2E(p) - i\varepsilon]^{-1}\,\hat{M}(\mathbf{k}, \mathbf{v}, \mathbf{R})\,|\,av\rangle,$$
$$\mathbf{v}_k = \mathbf{k}/M. \qquad (25)$$

Integrating over the second term in Eq. (22), we find the following expression for the potential corresponding to this term

$$\sum_{\mathbf{k}, c} \frac{\langle bv\,|\,[U_1^{(-)}(a_2^+) + U_2^{(-)}(a_1^+) - i\,(\mathbf{v}, \nabla Z_{bb})\,|\,c, v + \frac{k}{M}\rangle\langle v + \frac{k}{M}, c\,|\,\hat{M}(\mathbf{k}, \mathbf{v}, \mathbf{R})\,|\,av\rangle}{2E(k+p) - 2E(p) - i\varepsilon}, \qquad (26)$$

where the label c in Eq. (26) denotes summation only over spin-charge indices of the two-nucleon state. Unifying Eqs. (25) and (26), we find the potential W^{II} to be

$$-W^{II}_{Y'Y}(\mathbf{R},\ \mathbf{v})$$
$$=\sum_k \langle b,\ v \,|\, [U_1^{(-)}(a_2^+) + U_2^{(-)}(a_1^+) - i(\mathbf{v},\ \nabla Z_{bb})] \quad (27)$$
$$\times [H_{1,v+v_k} + H_{2,-v-v_k} - 2(\mathbf{k}\cdot\mathbf{v})]^{-1}$$
$$\times (1-P_0)\,\hat{M}(\mathbf{k},\ \mathbf{v},\ \mathbf{R})\,|\,av\rangle,$$

where P_0 is the projection operator on the ground state of the Hamiltonian $H_{1,v+v_k} + H_{2,-v-v_k}$ (not containing real mesons). In Eq. (27) we have set $E(p+k) - E(p) \approx (\mathbf{k}\cdot\mathbf{v})$. Neglecting, as earlier, terms of the type u^2/M, one can take $k = 0$ in the operators H_1, H_2 and P_0 in Eq. (27). Terms with $(\mathbf{v}\cdot\nabla Z_{aa})$ and $(\mathbf{v}\cdot\nabla Z_{bb})$ do not then contribute to Eq. (27).

The total potential is the sum of W^I and W^{II} [Eqs. (18) and (27)]. Direct calculation from Eqs. (18) and (27) leads, in general, to a complicated dependence on velocity. In the approximation linear in the velocity, which we consider, the only parts which are meaningful are W_{st} (static potential) and the part W_{LS}, which is linear in velocity, of the sum $W(\mathbf{R}, \mathbf{v}) = W^I + W^{II}$:

$$W_{st} = W(\mathbf{R},\ 0);\quad W_{LS} = (\mathbf{v},\ [\nabla_v W(\mathbf{R},\ \mathbf{v})]_{v=0}). \quad (28)$$

THE POTENTIAL

In Eqs. (18) and (27) for the potential, the meson variables relating to the clouds of nucleons 1 and 2 are completely separated, so that the calculation of (18) and (27) reduces to the calculation of one-nucleon matrix elements. Here, as follows from the form of H_{iv} and the discussion in connection with Eq. (17), in the approximation linear in the velocity, the one-nucleon matrix elements differ from the static matrix elements only in the substitution of ω_k by $\omega_k \pm (\mathbf{k}\cdot\mathbf{v})$ in the energy denominators, so that the results of the static theory can be used for their calculation. We calculated these matrix elements, which have the form (12) for $v_a = v_b$, using an expansion in terms of the system of functions of the one-nucleon Hamiltonian. Because of this, the two-nucleon potential turned out to be connected with the π-N scattering, as in the static case.[3-5] In summing over the intermediate states, the following formula[3] was employed:

$$\langle\alpha'\,|V_{ik}\,G(H_i)\,V_{iq}^+|\,\alpha\rangle = \sum_\nu \langle\alpha'\,|V_{ik}|\,\nu\rangle\langle\nu\,|V_{iq}^+|\,\alpha\rangle\,G(E_0)$$
$$+\sum_{I,J}(2J+1)^{-1}P_{kq}^{(i)}(2I)\,F_{kq}^{(i)}(2J)\,\frac{4\,kq}{(\omega_k\omega_q)^{1/2}}$$
$$\times\int d\omega_n\,\frac{G(\omega_n)}{k_n}\,\sigma_{2I,\,2J}(\omega_n), \quad (29)$$

where $\sigma_{2I,2J}$ is the total cross section for π-N scattering in the state with isotopic spin I and angular momentum J; P(2I) and F(2J) are projection operators on the state of isotopic spin I and angular momentum J (matrix elements $P_{kq} \equiv (k\,|\,P\,|\,q)$ and $F_{kq} \equiv (k\,|\,F\,|\,q)$ in the notation of reference 3); $G(H_i)$ is some function of the one-nucleon Hamiltonian H_i. In Eq. (29) for W^{II}, the function G depends both on H_1 and H_2. In this case, the result of applying Eq. (29) to the calculation of matrix elements between states of the first nucleon [first part of Eq. (29)] will be functions of the Hamiltonian of the second nucleon. This function $\widetilde{G}(H_2)$ should be considered the starting point for the application of Eq. (29) to summation over the states of the second nucleon.

We shall not dwell on the details of the calculation, in view of the analogy with the calculation of the potential in the static case.[5] Since the influence of the factor Z is of interest, we first give the result for $Z_{ab} \neq \delta_{ab}$, and then indicate the change introduced by $Z_{ab} \neq \delta_{ab}$. Calculation in this case gives the following expression for the operator $W(\mathbf{R}, \mathbf{v})$ from which — according to Eq. (28) — both the static part W_{st} of the potential and the part depending on the velocity, W_{LS}, can be obtained

$$W(\mathbf{R},\ \mathbf{v}) = W_0^{(1)}(\mathbf{R},\ \mathbf{v}) + W_0^{(2)}(\mathbf{R},\ \mathbf{v})$$
$$+ W_{\sigma 0}(\mathbf{R},\ \mathbf{v}) + W_{\sigma\sigma}(\mathbf{R},\ \mathbf{v}). \quad (30)$$

The first term in Eq. (30), which falls off as $\exp(-R)$, comes from the term $\langle bv\,|\,[U_1^{(+)}(a_2) + U_2^{(+)}(a_1)]\,|\,av\rangle$ in Eq. (18):

$$W_0^{(1)} = -2\,(2\pi)^{-3}\int d^3k e^{i\mathbf{k}\cdot\mathbf{R}}\,V_{1k}^+ V_{2k}/\,\omega_k^- . \quad (31)$$

The remaining part $W(\mathbf{R},\mathbf{v})$ falls off as $\exp(-2R)$. It can be conveniently expressed through the functions h_1, h_2 and h_3:

$$h_1(\omega_t,\ \omega_s) \quad (32)$$
$$= \frac{1}{\omega_k^- + \omega_q^-}\left[\frac{1}{\omega_t + \omega_q^+}\,\frac{1}{\omega_s + \omega_k^-} + \frac{1}{\omega_t + \omega_k^-}\,\frac{1}{\omega_s + \omega_q^+}\right],$$

$$h_2(\omega_t,\ \omega_s) = \frac{1}{\omega_k^- + \omega_q^-}\,\frac{1}{\omega_t + \omega_q^+}\,\frac{1}{\omega_s + \omega_q^t}$$

$$+ \frac{1}{\omega_t + \omega_q^+}\,\frac{1}{\omega_t + \omega_k^-}\,\frac{1}{\omega_s + \omega_q^-} + \frac{1}{\omega_t + \omega_k^-}\,\frac{1}{\omega_s + \omega_k^+}\,\frac{1}{\omega_s + \omega_q^-}$$

$$+ \frac{1}{\omega_k^- + \omega_q^-}\,\frac{1}{\omega_t + \omega_k^-}\,\frac{1}{\omega_s + \omega_k^+}, \tag{33}$$

$$h_3(\omega_t,\ \omega_s) = \frac{1}{\omega_s + \omega_t + 2\,(\mathbf{k}\cdot\mathbf{v})}\left[\frac{1}{\omega_s + \omega_q^+}\,\frac{1}{\omega_s + \omega_k^-}\right.$$

$$+ \frac{1}{\omega_t + \omega_q^+}\,\frac{1}{\omega_s + \omega_k^-} + \frac{1}{\omega_s + \omega_q^+}\,\frac{1}{\omega_t + \omega_k^-}$$

$$\left. + \frac{1}{\omega_t + \omega_q^+}\,\frac{1}{\omega_t + \omega_k^-}\right]. \tag{34}$$

Here the terms in the operators $W_0^{(2)}$, $W_{\sigma 0}$, $W_{\sigma\sigma}$ which contain h_1 and h_2 come from the term $\langle bv\,|\,N\,[U_1^{(+)}(a_2) + U_2^{(+)}(a_1)]\,|\,av\rangle$ in Eq. (18), and the terms with h_3 come from the operator W^{II}. The operator $W_0^{(2)}$, which does not depend on the π-N scattering cross section, is of the form

$$W_0^{(2)} = -(2\pi)^{-6}\int d^3q\,d^3k\,e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{R}}\{V_{1q}V_{1k}V_{2q}V_{2k}h_1(0,\ 0)$$

$$+ V_{1q}V_{1k}V_{2k}V_{2q}\,h_2(0,\ 0)\}. \tag{35}$$

This operator leads to a potential of fourth order in perturbation theory. The operators $W_{\sigma 0}$ and $W_{\sigma\sigma}$ depend on the cross section $\sigma_{2I,2J}$ for π-N scattering. $W_{\sigma 0}$ is linear in $\sigma_{2I,2J}$:

$$W_{\sigma 0} = -4\,(2\pi)^{-6}\sum_{IJ}(2J+1)^{-1}\int d^3k\,d^3q\,e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{R}}\int_0^\infty dt\,\frac{qk\sigma_{2I,2J}(t)}{\omega_t\,(\omega_q\omega_k)^{1/2}}$$

$$\times\{P_{qk}^{(1)}(2I)\,F_{qk}^{(1)}(2J)\,V_{2q}^+V_{2k}\,[h_1(\omega_t,\ 0) + h_3(\omega_t,\ 0)]$$

$$+ V_{1q}^+V_{1k}P_{qk}^{(2)}(2I)\,F_{qk}^{(2)}(2J)\,[h_1(0,\ \omega_t) + h_3(0,\ \omega_t)]$$

$$+ P_{qk}^{(1)}(2I)\,F_{qk}^{(1)}(2J)\,V_{2k}^+V_{2q}h_2(\omega_t,\ 0)$$

$$+ V_{1q}^+V_{1k}P_{kq}^{(2)}(2I)\,F_{kq}^{(2)}(2J)\,h_2(0,\ \omega_t)\}. \tag{36}$$

The operator $W_{\sigma\sigma}$ depends quadratically on the cross section $\sigma_{2I,2J}$:

$$W_{\sigma\sigma} = -16\,(2\pi)^{-6}\sum_{I,\,I',\,J,\,J'}(2J+1)^{-1}(2J'+1)^{-1}$$

$$\times\int d^3q\,d^3k\int_0^\infty dt\,ds\,\sigma_{2I,\,2J}(t)\,\sigma_{2I',\,2J'}(s)\,\frac{q^2k^2}{\omega_q\omega_k\omega_t\omega_s}\,e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{R}}$$

$$\times\{P_{qk}^{(1)}(2I)\,F_{qk}^{(1)}(2J)\,P_{qk}^{(2)}(2I')\,F_{qk}^{(2)}(2J')[h_1(\omega_t,\ \omega_s) + h_3(\omega_t,\ \omega_s)]$$

$$+ P_{qk}^{(1)}(2I)\,F_{qk}^{(1)}(2J)\,P_{kq}^{(2)}(2I')\,F_{kq}^{(2)}(2J')\,h_2(\omega_t,\ \omega_s)\}. \tag{37}$$

In Eqs. (31) to (37), the dependence on the velocities enters only through the quantities $\omega^\pm$ in the denominators of the functions h_1, h_2 and h_3. The potential W_{st} calculated from Eqs. (30) to (37) coincides with the potential obtained by Klein and McCormick.[3]

We consider the dependence of the integrands on the velocity and on the vectors **k**, **q** and $\boldsymbol{\sigma}$, keeping the terms of the operator $W(\mathbf{R},\ \mathbf{v})$ [Eq. (30)] which are linear in velocity. These vectors occur here only in the combinations

$$(\mathbf{k}+\mathbf{q},\ \mathbf{v})(\mathbf{k}\cdot\mathbf{q})^2,\ (\mathbf{k}+\mathbf{q},\ \mathbf{v})(\sigma_1\cdot[\mathbf{k}\times\mathbf{q}])(\sigma_2\cdot[\mathbf{k}\times\mathbf{q}]), \tag{38}$$

$$(\mathbf{k}\cdot\mathbf{v})(\mathbf{k}\cdot\mathbf{q})(\mathbf{S}\,[\mathbf{k}\times\mathbf{q}]),\ (\mathbf{q}\cdot\mathbf{v})(\mathbf{k}\cdot\mathbf{q})(\mathbf{S}\cdot[\mathbf{k}\times\mathbf{q}]), \tag{39}$$

(aside from $\exp[i(\mathbf{k}+\mathbf{q})\cdot\mathbf{R}]$), where $\mathbf{S} = \boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2$. The combinations (38) and (39) come from the symmetric and antisymmetric terms (relative to k and q) entering into the part of the function h_i which is linear in velocity. Only the terms with the combinations (39) lead to our **LS** type potential, after integration over angles. This potential falls off as $\exp(-2R)$. Terms of the type $(\mathbf{k}+\mathbf{q})\cdot\mathbf{v}$ in the potential $\exp(-2R)$ and of the type $(\mathbf{k}\cdot\mathbf{v})$ in the potential $\exp(-R)$ correspond in the more exact treatment to an expansion of the differences of kinetic energies, $E(\mathbf{p}) - E(\mathbf{p}+\mathbf{k}+\mathbf{q})$ or $E(\mathbf{p}) - E(\mathbf{p}+\mathbf{k})$ in the denominators. But, as Levy showed,[7] terms involving the difference in kinetic energies of the initial and final states do not contribute to the scattering, so that terms of this character can be discarded.

We now consider the influence of the factor $Z_{ab} \neq \delta_{ab}$ on the potential. After taking this factor into account, the result calculated for the potential differs from the potential (30) in the following ways.*

1. The potential W_{st} coincides in this case with the potential calculated by Miyazawa and one of the authors[5]† and differs from the potential of reference 3 by a factor Z^{-1}.

2. No part linear in the velocity will occur in the operator $W_0^{(1)}$.

3. In the terms linear in the velocity in the functions h_i [Eqs. (32) to (34)], parts symmetric in the

*It is convenient to choose the states a and b such that $Z_{ab}^{(1)} = Z_{aa}^{(1)}\delta_{ab}$, where $Z^{(1)}$ comes from the first term in Eq. (20).

†In reference 5 the term $W_{\sigma\sigma}(\mathbf{R},\ 0)$ was not calculated and the elastic, rather than total, cross section was considered. We note the following misprints and inaccuracies in reference 5: in Eq. (39), the square bracket in the last term should come before the round one, that is, the expression should be:

$$(\tau_1\tau_2 - 1)\,[2(\mathbf{k}\cdot\mathbf{k}')^2 - (\boldsymbol{\sigma}_1\cdot[\mathbf{k}\times\mathbf{k}']\ (\boldsymbol{\sigma}_2\cdot[\mathbf{k}\times\mathbf{k}'])]\,(2k_0q_0 + 2k_0^2 + k_0k_0')\}.$$

Further, in Eq. (44) a factor $\exp(i\mathbf{q}\cdot\mathbf{R})$ was left out before the group of terms beginning with $\frac{7}{6}\,(\boldsymbol{\sigma}_1\cdot\mathbf{k}')\,(\boldsymbol{\sigma}_2\cdot\mathbf{k})(\mathbf{k}\cdot\mathbf{k}')$. These terms lead to a potential of the type $\exp(-3R)$.

variables q and k will not be present, except for the symmetric part of the term resulting from taking into account the $2(\mathbf{k}\cdot\mathbf{v})$ in the first factor of Eq. (34). The antisymmetrical part here will be unchanged.

Thus, when one takes into account $Z_{ab} \neq \delta_{ab}$, all terms with $(\mathbf{k}+\mathbf{q})\cdot\mathbf{v}$ and $(\mathbf{k}\cdot\mathbf{v})$ disappear from the potential $W(R, v)$, with the exception noted above, whereas the terms with **LS** are unchanged. In other words, normalization of the asymptotic functions does not change the potential W_{LS}. In both cases the potential W_{LS} depends on the velocity only through $(\mathbf{LS})$, as it should.

If we retain only the largest cross section σ_{33} out of all the cross sections $\sigma_{2I,2J}$, then we obtain the following expression for the potential W_{LS}:

$$W_{LS} = \frac{(\mathbf{LS})}{MR^3}\{w_{00} + w_{\sigma 0} + w_{\sigma\sigma}\}, \qquad w_{00} = \frac{f^4}{2(2\pi)^3}[3 - 2(\boldsymbol{\tau}^1\boldsymbol{\tau}^2)]\int_0^\infty \frac{dk\,dq\,(kq)^{7/2}}{\omega_k\omega_q} J_{5/2}(kR)J_{5/2}(qR)\,A_1(k, q, i, i),$$

$$w_{\sigma 0} = \frac{f^2}{3(2\pi)^4}\int_0^\infty dk\,dq\,dt\,\frac{(kq)^{7/2}\sigma_{33}(t)}{\omega_k\omega_q\omega_t}\{3A_3(k, q, t)(3 - (\boldsymbol{\tau}^1\boldsymbol{\tau}^2)) + [A_1(k, q, t, i) + A_2(k, q, t, i)](3 + (\boldsymbol{\tau}^1\boldsymbol{\tau}^2))\}\,J_{5/2}(kR)\,J_{5/2}(qR)_{5/2},$$

$$w_{\sigma\sigma} = -\frac{2}{9(2\pi)^5}[6 - (\boldsymbol{\tau}^1\boldsymbol{\tau}^2)]\int_0^\infty dk\,dq\int_0^\infty dt\,ds\,\frac{(kq)^{7/2}}{\omega_k\omega_q\omega_t\omega_s}\sigma_{33}(t)\,\sigma_{33}(s)\,\{A_1(k, q, t, s) + A_2(k, q, t, s)\}\,J_{5/2}(kR)\,J_{5/2}(qR). \qquad (40)$$

Here we have introduced the following notation

$$A_1(k, q, t, s) = \frac{(\omega_s + \omega_q)^2(\omega_t + \omega_k) + (\omega_t + \omega_q)^2(\omega_s + \omega_k)}{(\omega_s + \omega_q)^2(\omega_t + \omega_q)^2(\omega_k + \omega_q)(\omega_t + \omega_k)(\omega_s + \omega_k)},$$

$$A_2(k, q, t, s) = (\omega_t + \omega_s + 2\omega_k)\,\frac{[(\omega_t + \omega_q)^2 + (\omega_s + \omega_q)^2](\omega_t + \omega_s) + (\omega_t + \omega_s + 2\omega_q)(\omega_t + \omega_q)(\omega_s + \omega_q)}{(\omega_s + \omega_t)^2(\omega_s + \omega_q)^2(\omega_t + \omega_q)^2(\omega_s + \omega_k)(\omega_t + \omega_k)},$$

$$A_3(k, q, t) = \frac{\omega_t[(\omega_t + \omega_k)\omega_k + (\omega_k + \omega_q)(\omega_t + \omega_k + \omega_q)]}{\omega_q^2\omega_k(\omega_k + \omega_q)(\omega_t + \omega_q)^2(\omega_t + \omega_k)}.$$

The term w_{00} in Eq. (40), which is proportional to f^4, would be a potential of 4th order in perturbation theory.[2] It is well known that the sign of this term is opposite to that necessary to explain the experimental data.[1] The terms $w_{\sigma 0}$ and $w_{\sigma\sigma}$ depend on the total cross section for π-N scattering. From the form of the expression for W_{LS} and the asymptotic expansion (see below) one might think that the sum $w_{\sigma 0} + w_{\sigma\sigma}$ is of the same order of magnitude as w_{00}. In view of the complexity of Eqs. (40), the sign of the sum for $R \sim 1$ can be determined only through numerical integration. In calculating the potential W_{LS}, one should keep in mind the fact that it is necessary to introduce a cut off factor (nuclear form-factor) into the calculation of the potential W_{LS}; otherwise, the calculation within the framework of static meson theory is meaningless. Then it is clear that one can consider the potential W_{LS} to be satisfactory only for such distances R that the influence of the cut off is small. In the case of the static potential this region is $R \geq 0.4$, where the potentials of order e^{-R} and e^{-2R} are of main importance. One might hope that the LS potential, which falls off as e^{-2R} will also depend weakly on the cut off in this region.

ASYMPTOTIC EXPANSION FOR W_{LS}

In Eq. (40) for the **LS**-potential, asymptotic integration over q and k can be carried out by deforming the contour of integration to a contour C, starting from $+i\infty$, going around the point i, and returning to $+i\infty$ (both in the integration over q and over k). In addition, poles should be taken into account.

As a result, an asymptotic series in inverse integral and half-integral powers of R, with a common multiplier e^{-2R}/R^3, is obtained. Here the series for w_{00} contains only inverse integral powers of R and begins with a constant term. Series in inverse integral and half-integral powers of R are obtained for $w_{\sigma 0}$ and $w_{\sigma\sigma}$, with the series for $w_{\sigma 0}$ beginning with $R^{-1/2}$, and the series for $w_{\sigma\sigma}$ with $R^{-3/2}$. The coefficients of these terms can be expressed in terms of integrals of the form

$$L_n = \int_0^\infty \frac{\sigma_{33}(p)\,dp}{\omega_p^{n+1}}, \quad n = 1, 2, 3\ldots, \qquad (41)$$

which can easily be calculated from the experimental values of the cross section σ_{33}.

In this way we calculated the expansion up to terms of R^{-6} in the expressions for w_{00}, $w_{\sigma 0}$ and $w_{\sigma\sigma}$. In view of the complexity of these expansions, we give here only the asymptotic series up to terms in $R^{-3/2}$. They are

$$w_{00} = \frac{f^2}{2(2\pi)^3} e^{-2R} \left\{3 - 2\,(\boldsymbol{\tau}^1\boldsymbol{\tau}^2)\left(1 + \frac{15}{4} R^{-1}\right)\right\},$$

$$w_{\sigma 0} = \frac{f^2}{3(2\pi)^4} e^{-2R} \Big\{2\sqrt{\pi}\,(6 - (\boldsymbol{\tau}^1\boldsymbol{\tau}^2))\, L_1 R^{-1/2}$$

$$- 2\pi\,(3 - 2\,(\boldsymbol{\tau}^1\boldsymbol{\tau}^2))\, L_2 R^{-1}$$

$$+ R^{-3/2} \sqrt{\pi}\left(\frac{79}{8} L_1 + 3L_3\right)\left[6 - (\boldsymbol{\tau}^1\boldsymbol{\tau}^2)\right]\Big\},$$

$$w_{\sigma\sigma} = -\frac{2\sqrt{\pi}}{9(2\pi)^5} e^{-2R}\,[6 - (\boldsymbol{\tau}^1\boldsymbol{\tau}^2)] R^{-3/2} L_1 L_2. \qquad (42)$$

The coefficients L_n which enter into these formulae were calculated from the data of reference 10. The cross section for elastic scattering was used for the cross section σ_{33}. The numerical values of these coefficients were

$$L_1 = 3.2,\ L_2 = 1.8,\ L_3 = 1.0.$$

From Eqs. (42) it can be seen that the main term at large distances is w_{00}. It is well known that sometimes the asymptotic expressions are good approximations up to $R \sim 1$. If we assume that this is so in our case, then in the region $R \sim 1$, the terms w_{00} and $w_{\sigma 0}$ will be of the same order of magnitude, and $w_{\sigma\sigma}$, an order of magnitude smaller.

We note that the expressions obtained for the potential W_{LS}, together with the static potential W_{st} found earlier, permit one, for the first time, to judge the extent of validity of the nonrelativistic meson theory for the N-N scattering in the low-energy region.

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