

*(d, p) STRIPPING REACTIONS IN THE SILICON ISOTOPES*

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Submitted to JETP editor August 5, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 52-59 (January, 1959)

Thick emulsions were used to measure the angular distribution of protons from the ground and first excited states in the reactions  $\text{Si}^{29,30}(\text{d}, \text{p})\text{Si}^{30,31}$  at a deuteron energy of 4.3 Mev. The orbital angular momenta  $l$  of the captured neutrons were determined, as well as the absolute differential reaction cross sections and the reduced neutron widths of levels in the final nuclei. The experimental proton angular distributions were compared with the theoretical curves of Butler and Bhatia. The spin and parity of the ground and first excited states in  $\text{Si}^{31}$  confirm the calculations of Goldhammer.

THE angular distributions of protons or of neutrons in stripping reactions of the type  $(\text{d}, \text{p})$  or  $(\text{d}, \text{n})$  have become a generally accepted tool of nuclear spectroscopy for investigating the quantum numbers of states in the product nucleus.<sup>1-3</sup> As is well known, the orbital angular momentum  $l$  of the neutron captured by the nucleus in the  $(\text{d}, \text{p})$  reaction is determined by the position of the peak in the angular distribution of the emitted protons, and can be used to find the parity  $\pi$  and a set of possible spins  $I_f$  of the final nucleus. The absolute value of the cross section for the stripping reaction is proportional to the probability that a particle with orbital angular momentum  $l$  will be captured on the surface of the nucleus, provided that the core remains unexcited. This probability is proportional to the reduced widths  $\gamma_l^2$  and  $\Theta_l^2$  for the captured particle<sup>4</sup> or to the nuclear matrix element  $\Lambda_l$ .<sup>2</sup> If the states of the final nucleus can be described by the shell model, then the values of these parameters should be relatively large; they should at least be of the same order of magnitude for the excited states as they are for the ground states.

$(\text{d}, \text{p})$  reactions on enriched Silicon isotopes  $\text{Si}^{29}$  and  $\text{Si}^{30}$  were studied at deuteron energies of 4.3 Mev.<sup>5</sup> The energy spectra were calibrated using magnetic field measurements of the proton spectra<sup>6</sup> from the reactions  $\text{Si}^{28,29,30}(\text{d}, \text{p})\text{Si}^{29,30,31}$ .

## DESCRIPTION OF THE EXPERIMENT

A 4.3-Mev deuteron beam from the 72 cm cyclotron of the Institute of Nuclear Physics, Moscow State University was electromagnetically focused and, passing through a system of collimating slits, fell at  $45^\circ$  on the target, placed at the center of a

cylindrical camera. The energy spread in the beam had been determined earlier in previous experiments<sup>7</sup> and was less than 40 kev; the beam did not diverge by more than  $20'$ . Since the deuteron energy varied from one bombardment to another, the value of  $E_d$  used was obtained from the proton group of longest range and from a reaction with known  $Q$ . The protons emitted from the target were detected by type YA-2 NIKFI emulsions 100 and  $200 \mu$  thick and placed about 10 cm from the target at angles of  $6.5$  to  $150^\circ$  with the incident deuteron beam. Aluminum absorbers were placed between the target and the emulsions to suppress elastically-scattered deuterons and low-energy protons. The beam current was measured with a Faraday cylinder and integrator.

The targets for reactions on the enriched isotopes were made by depositing finely divided  $\text{SiO}_2$  powder on a piece of gold tinsel  $0.15 \text{ mg/cm}^2$  thick stretched over a frame. The silicon dioxide was enriched with  $\text{Si}^{29}$  or  $\text{Si}^{30}$  (see table I). The thickness of the target was found by weighing and turned out to be  $0.30 \text{ mg/cm}^2$  for  $\text{Si}^{29}$  and  $0.34 \text{ mg/cm}^2$  for  $\text{Si}^{30}$ . The  $\text{Si}^{28}$  target was made by evaporating a silicon sample with the natural isotopic abundances in vacuum. Only chemically pure materials were used in making the targets. Bombardments carried out as controls showed that there was a negligible amount of impurities in the targets and underlying tinsel foil.

The tracks were counted using an MBI-2 microscope with magnification  $1.5 \times 90 \times 5$ . In the histograms below, the scale along the abscissa is in units of the scale in the ocular of the microscope. In the angular distributions shown below, the errors are statistical; the dotted lines show the isotropic

TABLE I

Isotope	Natural Abundance, %	Composition of the Si <sup>29</sup> sample, %	Composition of the Si <sup>30</sup> sample, %
Si <sup>28</sup>	92.28	34.6	35.3
Si <sup>29</sup>	4.67	63.9	12.5
Si <sup>30</sup>	3.05	1.5	52.2

part of the angular distribution, which is presumably due to formation of the compound nucleus.

## RESULTS

Si<sup>29</sup> (d, p) Si<sup>30</sup>. Figure 1 shows the proton spectra at angles  $\theta_{\text{lab.}} = 6.5^\circ$  and  $40^\circ$ . According to Dzheleпов and Peker,<sup>8</sup> Si<sup>30</sup> has an excited state at 1.250 mev. It does not appear in our experiment. The dotted line in Fig. 1 shows the position of the proton group which should correspond to such a state. If this state is formed its intensity of formation is 0.3% of the corresponding quantity for the ground state of Si<sup>30</sup> at  $\theta_{\text{lab.}} = 6.5^\circ$  and less than 0.6% for  $\theta_{\text{lab.}} = 40^\circ$ . The energy resolution of the nuclear emulsions is insufficient to distinguish between the proton groups Si<sup>30</sup> (1) corresponding to reactions on the first excited state of Si<sup>30</sup>, and the proton group Si<sup>29</sup> (0) corresponding to reactions

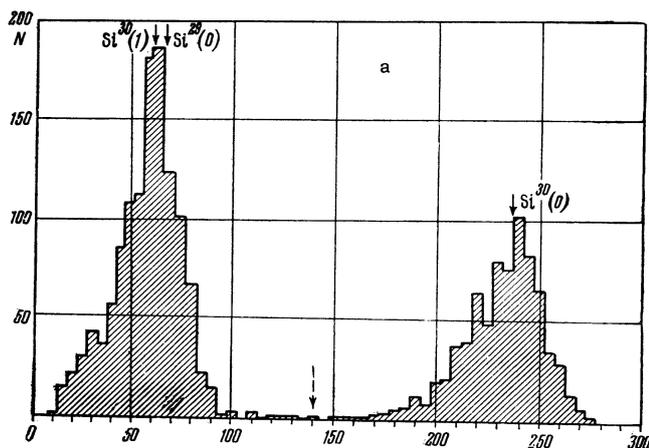


FIG. 1a. Proton spectrum at  $\theta_{\text{lab.}} = 6.5^\circ$  from the reaction Si<sup>29</sup> (d, p) Si<sup>30</sup>.

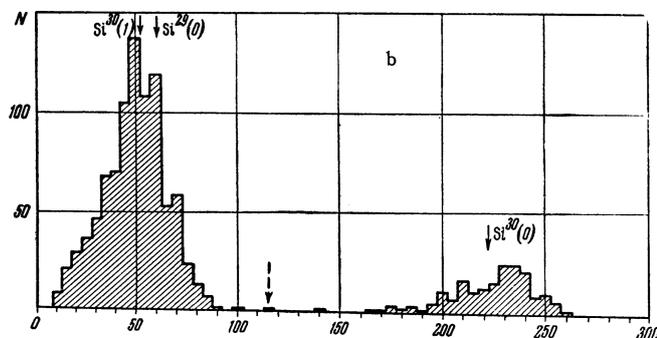


FIG. 1b. Proton spectrum at  $\theta_{\text{lab.}} = 40^\circ$  from the reaction Si<sup>29</sup> (d, p) Si<sup>30</sup>.

leading to the ground state of Si<sup>29</sup>. The Q's for these reactions differ only by  $\Delta Q = 0.098$  mev. The sum of the angular distributions has been plotted for these groups. Figure 2 shows the measured angular distributions together with the theoretical curve of Bhatia.<sup>2</sup> When the neutron was captured with orbital angular momentum  $l = 0$ , the radius of the target nucleus was taken to be  $R = R_0 + 2 \times 10^{-13}$  cm, while  $R = R_0 + 1 \times 10^{-13}$  cm was used for  $l = 2$ .  $R_0$  was calculated as  $R_0 = (1.7 + 1.22A^{1/3}) \times 10^{-13}$  cm. In Fig. 2a the angular distribution of the group Si<sup>30</sup> (0) agrees satisfactorily with theoretical curve for  $l = 0$ . The angular

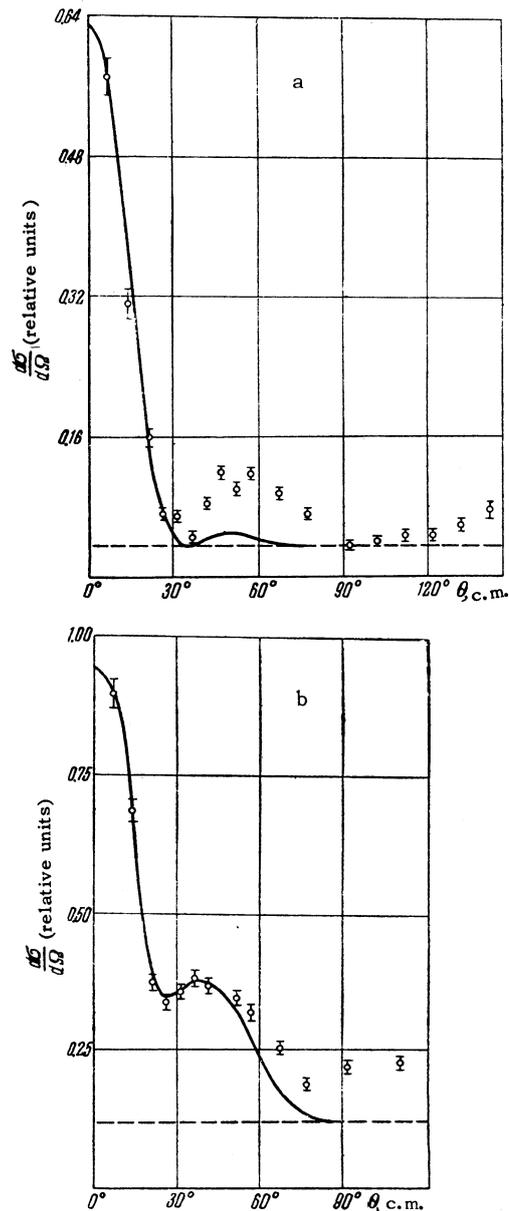


FIG. 2. Angular distribution of protons from the reaction Si<sup>29</sup> (d, p) Si<sup>30</sup>: a - ground state of Si<sup>30</sup>,  $l = 0$ ; b - first excited state of Si<sup>30</sup>  $l = 2$  and the ground state of Si<sup>29</sup>,  $l = 0$ .  $d\sigma/d\Omega$  is in units of 7.30 mbn/sterad.

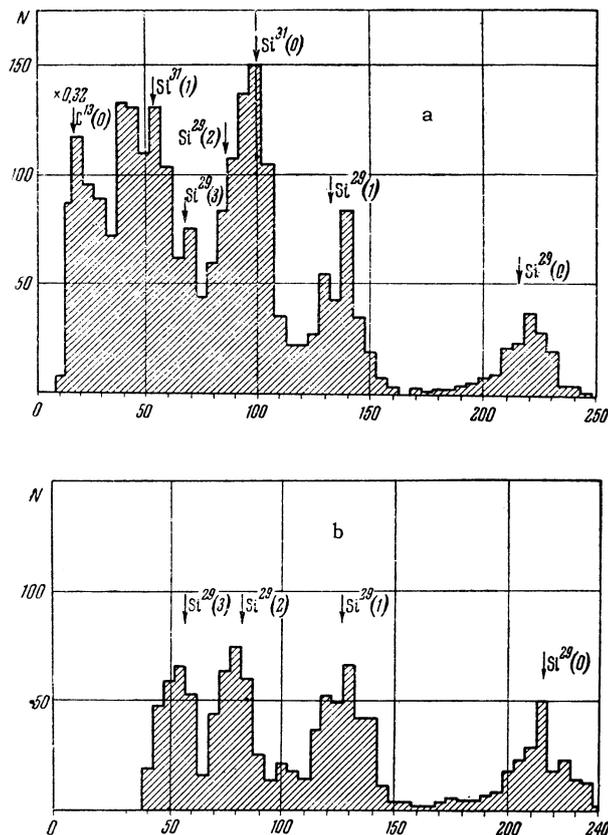


FIG. 3. Proton spectra: a - from the reactions  $\text{Si}^{30}(\text{d}, \text{p})\text{Si}^{31}$  and  $\text{Si}^{28}(\text{d}, \text{p})\text{Si}^{29}$  at an angle  $\theta_{\text{lab.}} = 40^\circ$ ; b - from the reaction  $\text{Si}^{28}(\text{d}, \text{p})\text{Si}^{29}$  at an angle  $\theta_{\text{lab.}} = 40^\circ$ .

distribution shown in Fig. 2b for the two groups  $\text{Si}^{30}(1)$  and  $\text{Si}^{29}(0)$  agrees with the sum of the curves for  $l = 2 + 0$ . Since the neutron is captured by the ground state of  $\text{Si}^{29}$  into an  $l = 0$  state,<sup>9,10</sup> the angular distribution of the proton group  $\text{Si}^{30}(1)$  corresponds to neutron capture with  $l = 2$ .

$\text{Si}^{30}(\text{d}, \text{p})\text{Si}^{31}$ . The proton spectra from the reactions  $\text{Si}^{30}(\text{d}, \text{p})\text{Si}^{31}$  and  $\text{Si}^{28}(\text{d}, \text{p})\text{Si}^{29}$  are shown in Fig. 3. The energy resolution is not enough to separate the proton groups  $\text{Si}^{31}(0)$  from  $\text{Si}^{29}(2)$  or  $\text{Si}^{31}(1)$  from  $\text{Si}^{29}(3)$  (the  $Q$ 's for these reactions differ only by 0.144 and 0.212 mev). Figure 4 shows the sum of the angular distributions for these two groups. The bottom part of the figure shows the angular distributions of the proton groups  $\text{Si}^{29}(2)$  and  $\text{Si}^{29}(3)$ , drawn to a scale which shows the contributions these groups make to the total angular distribution. A correction was made for the variation in deuteron energy ( $\Delta E_d = 0.4$  mev), using the data of Tobocman and Kalos.<sup>11</sup> The correction never exceeded 0.5%. The angular distribution of the  $\text{Si}^{29}(3)$  group is isotropic, while the  $\text{Si}^{29}(2)$  protons correspond to capture of a neutron with  $l = 2$  (the theoretical curve is drawn using Bhatia's formula with  $R = R_0 + 1 \times 10^{-13}$  cm). These results agree with those of Holt and Mar-

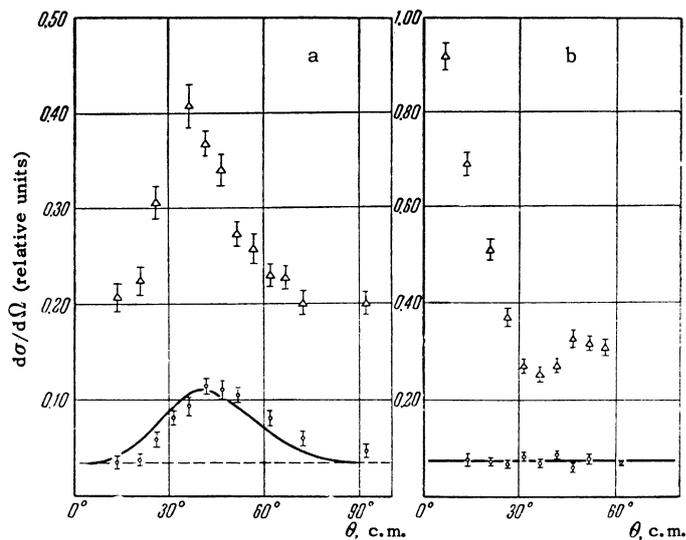


FIG. 4. Angular distributions of protons from the reactions  $\text{Si}^{30}(\text{d}, \text{p})\text{Si}^{31}$  and  $\text{Si}^{28}(\text{d}, \text{p})\text{Si}^{29}$ : a)  $\Delta$  - ground state of  $\text{Si}^{31}$ ,  $l = 2$ , and the second excited state of  $\text{Si}^{29}$ ,  $l = 2$ ;  $\circ$  - for the proton group  $\text{Si}^{29}(2)$ ; b)  $\Delta$  - first excited state of  $\text{Si}^{31}$ ,  $l = 0$ , and the third excited state of  $\text{Si}^{29}$  (isotropic).  $d\sigma/d\Omega$  is in units of 9.30 mbn/sterad.

sham,<sup>9</sup> taken at a deuteron energy of 8 Mev. The final angular distributions for the proton groups  $\text{Si}^{31}(0)$  and  $\text{Si}^{31}(1)$  (obtained from the difference between the angular distributions of Fig. 4) are shown in Fig. 5, together with theoretical curves calculated from the formulas of Bhatia (solid curves,  $R = R_0 + 1 \times 10^{-13}$  cm for  $l = 2$  and  $R = R_0 + 2 \times 10^{-13}$  cm for  $l = 0$ ) and Butler (dotted curves,  $R = R_0 + 1 \times 10^{-13}$  cm). The radius  $R$  was increased by  $1 \times 10^{-13}$  cm for  $l = 2$  in order to obtain better agreement with experiment. The change displaces the peak in the theoretical curve by 3 or 4° toward the smaller angles.

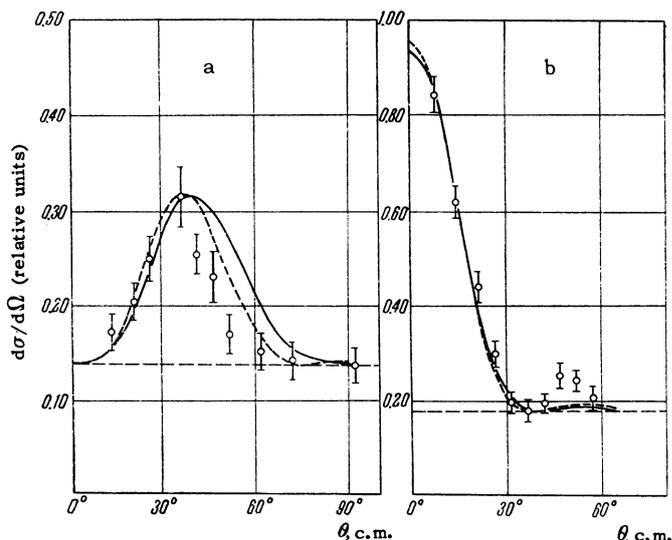


FIG. 5. Angular distribution of protons from the reaction  $\text{Si}^{30}(\text{d}, \text{p})\text{Si}^{31}$  a - ground state of  $\text{Si}^{31}$ ,  $l = 2$ ; b - first excited state of  $\text{Si}^{31}$ ,  $l = 0$ .  $d\sigma/d\Omega$  is in units of 9.30 mbn/sterad.

TABLE II

Reaction	Level position E Mev from reference 6	Level position E Mev from our data*	$l$	$I_f, \pi$	$d\sigma/d\Omega$ , mbn/steradian	$10^2 \cdot \gamma_l^2$ , mev-cm $\times 10^{-30}$	$\Theta_l^2$	Shell model configuration
$\text{Si}^{29}(d, p)\text{Si}^{30}$	0	0	0	$0^+, (1^+)$	4.57 ( $0^\circ$ )	0.88	0.029	$(2s_{1/2})^2$
	$2.239 \pm \pm 0.020$	$2.25 \pm \pm 0.09$	2	$2^+, (1^+, 3^+)$	1.55 ( $38^\circ$ )	0.57	0.019	$(2s_{1/2})^1 (1d_{3/2})^1$
$\text{Si}^{30}(d, p)\text{Si}^{31}$	0	0	2	$3/2^+, (3/2^+)$	1.66 ( $38^\circ$ )	0.31	0.011	$(2s_{1/2})^2 (1d_{3/2})^1$
	$0.757 \pm \pm 0.007$	$0.76 \pm \pm 0.08$	0	$1/2^+$	7.25 ( $0^\circ$ )	0.17	0.006	$(2s_{1/2})^1 (1d_{3/2})^2$

\*The indicated uncertainty is the root-mean-square error.

The fact that the experimental angular distribution is shifted toward the smaller angles and is somewhat narrower than the theoretical curve for  $R = R_0$  indicates that there is a nuclear interaction not accounted for in the theories of Butler and Bhatia. In our case the deuteron energy was 4.3 Mev, almost equal to the height of the Coulomb barrier for the nuclei investigated, so that Coulomb effects cannot be neglected; their effect is to shift the peak in a direction opposite to that given by the nuclear forces, that is, toward larger angles. The Coulomb barrier of the nucleus  $\text{Si}^{29}$  is 3.70 Mev, while for the nucleus  $\text{Si}^{30}$  it is 3.66 Mev. However, when  $E_d$  is about the same as the Coulomb barrier, the shape of the angular distribution is not deformed enough to give a wrong value for  $l$ , at least for nuclei up to  $Z = 19$  and for  $l = 2,^{12}$  even though the Coulomb forces, together with other nuclear interactions, can have a big effect on the magnitude of the stripping cross section.

## DISCUSSION

The results of the experiment are shown in Table II. The level spins were obtained from the selection rules corresponding to the formula  $I_f = I_i + l + \frac{1}{2}$ , the values in parentheses being less reliable. The values of the absolute differential cross sections were calculated without taking into account the isotropic part of the angular distributions at the maxima of the theoretical curves (for  $\theta = 0^\circ$  for  $l = 0$  and  $\theta = 36$  to  $38^\circ$  for  $l = 2$ ). The accuracy of  $d\sigma/d\Omega$  is about 40%. The reduced neutron widths  $\gamma_l^2$  and  $\Theta_l^2$  for levels of the final nuclei were calculated from the results of Friedman and Tobocman<sup>3</sup> and from the tables of reference 13, without taking into account the Coulomb and nuclear interactions.  $\Theta_l^2$  is given in units of the Wigner sum rule limit.

In principle, a measurement of the absolute value of  $\sigma_{\text{str}}$  gives the value of  $\gamma_l^2$  and the kind of nuclear excitation. However, it is not possible

to make a direct determination of the reduced width because one cannot take properly into account the nuclear interaction between the particles taking part in the reaction. The effects of the Coulomb forces can be calculated exactly, but such a calculation is very complicated. The reduced widths obtained without taking these interactions into account are too small. Hence the values of  $\gamma_l^2$  and  $\Theta_l^2$  quoted in table II should be regarded as only relative. From the calculations of Tobocman and Kalos, the neglected interactions should increase  $\gamma_l^2$  and  $\Theta_l^2$  by a factor  $N > 10$ . Calculations carried out by Neudachin and Teplov<sup>14</sup> for the reaction  $\text{Ca}^{40}(d, p)\text{Ca}^{41}$  for  $E_d \approx 4$  Mev give a value of  $N$  about 40 or 50. Since the nuclear interactions have but little effect on the cross section, one can carry out an approximate calculation of the Coulomb corrections. Such estimates were obtained by Sawicki<sup>15</sup> and Butler<sup>16</sup> but the corrections obtained were small compared with the numerical computations of Tobocman and Kalos. According to Butler, a correction factor  $\{R_\infty\}^2$  can be calculated in the limit of small  $l$  and for  $E_d \gg Ze^2/R$ , i.e., for relatively large angular momenta of the deuteron,  $\lambda_d$ , and proton  $L_p$ . (The correction factor  $\{R_\infty\}^2$  is applied to decrease the value of  $\sigma_{\text{stripping}}$  as calculated taking Coulomb corrections into account). The approximation improves with increasing  $\lambda_d$  and  $L_p$ , because of the short range of nuclear forces. For  $E_d \approx Ze^2/R$ , the factor  $\{R_\infty\}^2$  is too small, but still can be used to obtain a lower limit for  $\sigma_{\text{stripping}}$ . The factor  $\{R_\infty\}^2$  is almost exactly equal to  $\exp[-\pi(\eta_p + \eta_d)]$  where  $\eta$  is the Coulomb parameter,  $\eta = Ze^2m/\hbar^2k$  ( $k$  is the wave number). The second factor in  $\{R_\infty\}^2$  is usually little different from one ( $E_d \gtrsim Ze^2/R$ ,  $Q >$  binding energy of the deuteron).

For the reaction  $\text{Si}^{29}(d, p)\text{Si}^{30}$  at  $E_d = 4.30$  Mev, the correction decreases  $\sigma_{\text{str}}$  by a factor  $10^3$ , which is clearly wrong but does provide a lower limit to the reaction cross section. From the values of the reduced width  $\Theta_l^2$  for the ground

state of  $\text{Si}^{30}$  (see table II),  $\sigma_{\text{str}}$  cannot be decreased by more than a factor 30, since  $\{\Theta_2^2\}_{\text{Coul.}} = \Theta_2^2 / \{R_\infty\}^2$  cannot be more than unity ( $\{\Theta_2^2\}_{\text{Coul.}}$  is the reduced width taking Coulomb corrections into account). The reason for this discrepancy with experiment is not hard to find, since the orbital angular momentum of the incident deuteron is

$$\lambda_d = \hbar^{-1} R [2M_d(E_d - Ze^2/R)]^{1/2} \approx 1.$$

For the ground state of  $\text{Si}^{30}$ , the orbital angular momentum of the captured neutron is  $l = 0$ , i.e., the values  $\lambda_d = L_p = 1$  are too small. In the case of the reaction  ${}^9\text{F}^{19}(\text{d}, \text{p}){}^9\text{F}^{20}$ , proceeding from the ground state with  $E_d = 14.3$  Mev, the Butler correction is small compared to the result given by the calculations of Tobocman and Kalos. Here  $l = 2$ ,  $\lambda_d = 4$ ,  $E_d \gg Ze^2/R = 2.4$  Mev, but  $\{R_\infty\}^2$  is less than the result given by Tobocman and Kalos by a factor 7.5. We may conclude that the approximations made by Butler are too crude, at least for deuteron energies up to 14 Mev.

The nucleus  $\text{Si}_{16}^{30}$  is even-even and has isotopic spin  $T = 1$ . The spin and parity of the ground state are  $0^+$ . For even-even nuclei, the first excited state usually has spin and parity  $2^+$ , which for this nucleus means that one of the two neutrons outside a closed shell has undergone the transition  $(2s_{1/2}) \rightarrow (1d_{3/2})$ . The close agreement between the values of the reduced widths for the ground and first excited states suggests that these can be pictured as two neutrons interacting outside a closed shell.

The nucleus  $\text{Si}_{17}^{31}$  is even-odd, with  $T = 3/2$ . Possible values for the spin and parity of the ground state are  $3/2^+$ ,  $5/2^+$ ; according to Dzheleпов and Peker the ground state is  $3/2^+$ . The spin of the first excited state is determined uniquely by this experiment; it is  $1/2^+$ . The values  $3/2^+$ ,  $1/2^+$  for the spins and parities of the ground and first excited states agree with the calculations carried out by Goldhammer;<sup>17</sup> for three neutrons outside a spherical core, his calculations involved a coupling between nucleons in orbits of the same parity introduced by surface oscillations of the core.

In conclusion, I should like to thank S. S. Vasil'ev for his guidance. I should like also to express my gratitude to the cyclotron crew of the Nuclear Physics Institute of the Moscow State University and especially G. V. Koshelyaev, A. A. Danilov, S. M. Kulakova and A. D. Shcheglova for helping to scan the plates.

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Translated by R. Krotkov