SHOWERS PRODUCED BY POSITRONS IN 100 to 400 Mev RANGE

Yu. D. PROKOSHKIN and T'ANG HSIAO-WEI

Joint Institute for Nuclear Research

Submitted to JETP editor April 23, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 10-16 (January, 1959)

Cascade curves for 101, 224, and 407 Mev positrons in lead and copper were measured. The curves are in agreement with shower curves computed by the Monte Carlo method. Functions approximating the cascade curves were determined.

INTRODUCTION

THE passage of fast electrons through matter is accompanied by the production of electron-photon showers. Although this fact has been known for a long time,[1] no theory has so far been developed which could adequately describe the development of showers produced by electrons with energy E_e comparable with the critical energy ϵ (7 Mev in lead). The cascade theory[2] existing presently is correct for large energies, of a few orders of magnitude greater than the critical. Difficulties arise, however, even in the description of showers in heavy substances, since the cross section for pair production is then dependent on the energy, and because of the scattering of shower particles. The asymptotic formula for cross sections which is used in the cascade theory becomes incorrect with decreasing energy, which invalidates the results of the theory for $E_e < 1$ Bev, i.e., exactly in that energy range which has now become accessible to experimental investigation thanks to powerful particle accelerators. This gap has recently been filled by the work of Belen'kii and Ivanenko,[3] who calculated the cascade curves and the spectrum of shower particles using the moment method. However, the experimental data which would enable us to test the accuracy of these calculations (especially the most interesting ones concerning the spectrum) is still lacking. In connection with the above, the study of showers at $E_e < 1$ Bev is at present of considerable interest. Data concerning the cascade curves,* fluctuations, etc., in the above energy range have also an application which has increased in importance in recent years owing to the construction of spectrometers based on the detection of Cerenkov light pulses[4] associated with showers. In these spectrometers the magnitude of the light pulse is proportional to the area under the cascade curves $N(E_e, \epsilon, t) = n(t)$. The dependence of $n(t)$ on the electron energy E_e in the 50 to 500 Mev range was calculated by Wilson,[5] using the Monte-Carlo method. The purpose of the present work is to determine experimentally the cascade curves $n(t)$ in the above range, and to compare them with the calculations of Wilson.

*The cascade curves describe the number of electrons and positrons $N(E_0, E, T)$ with energy greater than E, contained in the shower at the depth t below the shower initiation point as a function of t. E_0 is the energy of the primary particle.

2. EXPERIMENTAL METHOD

The experiments were carried out with a beam of monoenergetic positrons. A collimated beam of high energy photons (produced by π^0-meson decay) from the phasotron chamber at the Joint Institute for Nuclear Research, was directed towards a lead target (1 mm thick) placed before the magnet (see Fig. 1). The positrons produced in the target were deflected in the magnetic field by an angle of 20° and then collimated by a counter telescope (a) and a lead diaphragm 5 cm in diameter. The energy of the positrons could be varied from 50 to 500 Mev by varying the intensity of the magnetic field. The energy spread amounted to 2 or 3%. Behind the last counter of the telescope (a), a 20×20 cm^2 copper or lead absorber was placed in the way of the positron beam and the number of fast electrons emerging from the absorber $n(t)$ was measured for different thicknesses t of the absorber.

Before the construction of the detector used for the measurement of $n(t)$ is discussed, it should be noted that in showers produced by electrons (here and in the following discussion we do not distinguish between electrons and positrons) of several hundred Mev energy there is a high probability that several electrons with a small angle of divergence will be produced simultaneously, especially at small thicknesses t. One can measure

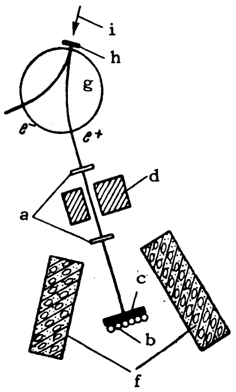

FIG. 1. Experimental set-up. a – telescope, b – tray of low-efficiency counters, c – absorber of thickness t, d – lead diaphragm, f – shield, e^+ – positron beam, g – magnet, h – lead target, i – photon and neutron beam.

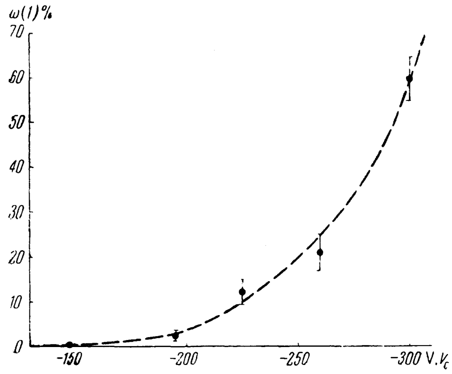

FIG. 2. Dependence of the probality w(1) on the value of the constant V_0 in the pulsed operation of the STS-8 counter. Delay of the high-voltage pulse equal to 0.6 μsec.

the number of these electrons, say, by means of a scintillation counter used together with a pulse-height analyzer. Such a method is in general complicated, and in our case, the difficulties are increased because of the large cross-section of the positron beam. For the measurement of the number of shower electrons, we used a tray of halogen counters (see Fig. 1) placed behind the absorber. The counters were fed by a triggered pulsed power supply.[6] The tray measured 15×15 cm and consisted of STS-8 counters of identical characteristics. Whenever a particle passed through the telescope (a), a high-voltage pulse (-1000 V) of 1.5 μsec duration with a rise time of 0.15 μsec, and delayed for 0.6 μsec with respect to the time of the passage of the particle, was applied to the cathodes of the counters, which were held at a constant voltage V_0. Since the high voltage pulse was delayed, the detection efficiency of the counters for single fast electrons w(1) was much smaller than unity for small values of V_0 (see Fig. 2).

The probability w(n) that a discharge will occur in the tray when n electrons pass simultaneously through it is given by the expression

$$w(n) = 1 - e^{-an}, \qquad (1)$$

where a is the mean number of ions produced by a single fast electron, which remain in the counter till the moment of the arrival of the high-voltage pulse. The dependence of the efficiency on n was measured, and the result is in good agreement with formula (1).

The necessary small value of w(1) which is determined by the condition

$$w(1)\, n \ll 1 \qquad (2)$$

can be obtained by adjusting V_0. When this condition is satisfied, the detector has almost linear counting characteristics

$$w(n) \approx an \approx w(1)\, n. \qquad (3)$$

Thus, for an energy of 400 Mev ($n \lesssim 3$), it is sufficient to choose $w(1) = 0.04$, in order that the maximum correction due to the nonlinearity of the characteristics amounts only to 6% of the counting rate of the tray.

When counters of such small efficiency are used, the problem of stability of the efficiency with respect to time becomes very serious, since small changes of the circuit parameters can change the value of w by factor of several times (from its normal value of $w \approx 1$). The stability of the array used was satisfactory.

The low efficiency of halogen counters used in the described method of operation is due not to the narrow zone of sensitivity, but to an almost identical decrease in the efficiency all over the counter volume.[6] The latter decrease is essential, since the diameter of the counters used (20 mm) is comparable with the dimensions of the positron beam. If the decrease of the efficiency were due to narrowing of the sensivity zone, the measurement would depend on the position of the tray with respect to the center of the beam. The radial dependence of the probability w(1) has been measured. The STS-8 counter was placed between two small counters which selected a beam of electrons 3 mm in width. The efficiency w(1), defined as the ratio of the counting rate of the investigated counter to the counting rate of the coincidences of the master counters, was measured for various distances between the anode of the counter and the electron beam. The measurements were carried out for several values of w(1), in the range of 3 to 25%. The results indicate that the sensitivity of the whole volume of the halogen counters is uniform, which is in agreement with Vishnyakov and Tyapkin.[6]

$E_e = 101 \pm 2$ **Mev**

t mm Pb	0	5	8	10	15	20	25	30
$w(n)$, %	4.35	6.11	4.85	5.50	3.17	1.83	1.60	1.60
$\pm \Delta w(n)$, %	0.09	0.31	0.24	0.28	0.16	0.11	0.14	0.20

$E_e = 294 \pm 10$ **Mev**

t mm Pb	0	5	10	13	15	20	30	35
$w(n)$, %	5.80	13.5	14.8	15.1	12.8	9.4	6.9	4.4
$\pm \Delta w(n)$, %	0,17	0.6	0.6	0.6	0.5	0.4	0.3	0.2

$E_e = 407 \pm 10$ **Mev**

t mm Pb	0	5	10	15	20	25	30	35	40	50
$w(n)$, %	4.8	8.5	12.5	12.9	11.9	9.4	6.4	6.5	4.2	1.6
$\pm \Delta w(n)$, %	0.3	0.9	0.8	0.6	1.0	1.0	0.3	0.7	0.4	0.6

RESULTS OF MEASUREMENTS AND DISCUSSION

The measuring apparatus was placed in the immediate vicinity of the neutron beam (see Fig. 1). However, thanks to the fact that the resolving time of the whole array amounted to $\sim 1\ \mu$sec, the background of chance coincidences was less than 1% for a counting rate of the tray equal to 200 pulses per minute (for the measurements of the number of chance coincidences, the triggering pulse was delayed for 2.5 μsec.).

The cascade curves n(t) were measured for positrons with energy equal to 101 ± 2, 294 ± 10, and 407 ± 10 Mev. The positron energy was determined by the method of current-carrying wire.[7] We limited ourselves to an energy of 400 Mev, in view of the fact that for larger energies the background of chance coincidences increased sharply, and the intensity of the positron beam decreased. Simultaneously, the mixture of protons in the positron beam increased, which caused a fast drop of the counting rates with and without the target.

The value of n(t) was determined by comparing the counting rate of the tray (b) and of the telescope (a) [the "tray-counting rate" to "telescope counting rate" ratio is equal to w(n); for $t = 0$, the ratio is equal to w(1)]. The measured probabilities w(n), related to the values of an by Eq. (1), are given in the table. The errors given are statistical.

Before comparing the cascade curves with the results of calculations of Wilson,[5] it was necessary to determine the energy threshold of the tray (b). For that purpose, an aluminum absorber was placed before the tray in order to cut off slow electrons. The threshold of the tray was found to be close to the critical energy $\epsilon = 7$ Mev, which made it possible to compare our results directly with the calculations of Wilson. It can be seen, by comparing the integrals J of the measured cascade curves n(t) with the value of the ratio E_e/ϵ, that the apparatus does not detect soft electrons of the shower. The ratio $J/(E_e/\epsilon)$ is equal to 0.35. It follows that the detector detects roughly one-third of the total number of electrons in the shower.

The cascade curves calculated by Wilson and those obtained by us differ in the fact that the thickness of the lead absorber t is measured in different units: in g/cm^2 (present work) and in radiation units in lead, r.u.Pb (Wilson). A comparison of the data contained in the table and the cascade curves of Wilson makes it therefore possible to determine the value of one radiation unit in lead in units of g/cm^2, which is of interest since the values of the radiation unit in lead calculated by various authors[2,3,8] are considerably different from each other. The value of radiation unit in lead was found by the least-squares method to be

$$1 \text{ r.u.Pb} = 5.6 \pm 0.2 \text{ g/cm}^2,$$

which is in agreement with the value given in reference 8, if the latter is corrected according to the latest data on the energy dependence of the pair production cross-section in lead.[9]

The cascade curves obtained in the course of the present work (measured in radiation units) are compared with the curves of Wilson in Figs. 3 and 4. It can be seen from these figures that the two results are in agreement.

To test the present cascade theory, we measured the cascade curves n(t) for lead and copper (see Figs. 3 and 4). It is well known that the theory predicts the following relation for the position of the maxima (t_{max}) of the cascade curves[8] in the

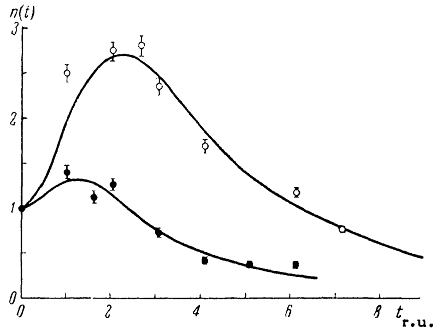

FIG. 3. Cascade curves n(t), measured for positrons: ● – with energy (101 ± 2) Mev, o – (294 ± 10) Mev in lead. 1. r.u. in lead = 5.6 g/cm². The curves shown in the figure were calculated by Wilson[5] for electrons with energy of 100 and 300 Mev.

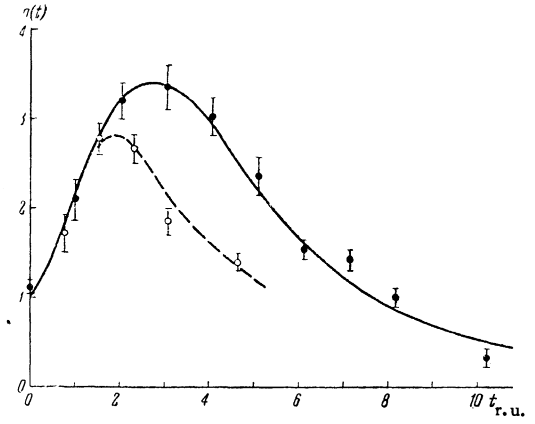

FIG. 4. Cascade curves n(t), measured for positrons of 407 ± 10 Mev in lead(●) and copper(o). 1 r.u. Pb = 5.6 g/cm², 1 r.u. Cu = 11.5 g/cm². The solid line was obtained for E_0 = 407 Mev by an interpolation of the results of Wilson.[5]

approximation B:

$$t_{max\,Pb}/t_{max\,Cu} = b_1 \ln(E_e/\varepsilon_{cr\,Pb})/\ln(E_e/\varepsilon_{cr\,Cu}). \quad (4)$$

This ratio is equal to ≈ 1.7 for E_e = 407 Mev, which is in agreement with the experimentally found value of 1.5 ± 0.2.

The following relation holds for the maxima of cascade curves according to the cascade theory:

$$n_{max\,Pb}/n_{max\,Cu} = b_2(\varepsilon_{cr\,Cu}/\varepsilon_{cr\,Pb})\{\ln(E_e/\varepsilon_{cr\,Cu})/\ln(E_e/\varepsilon_{cr\,Pb})\}^{1/2}. \quad (5)$$

For E_e = 407 Mev, the ratio (5) is equal to ≈ 1.5, which is close to the ratio 1.20 ± 0.25 obtained by us. It should be noted that, if the ratio (5) is calculated neglecting the dependence of the pair production cross section on energy (assuming $b_2 = 1$), then the value obtained is considerably different from that measured in our work: for $b_2 = 1$, the relation (5) is equal to 2.5.

The cascade curves n(t) obtained for lead can

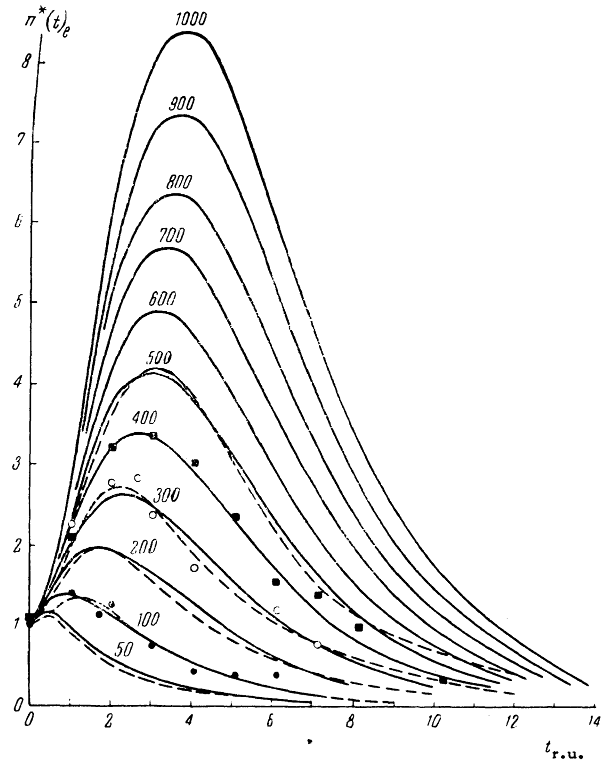

FIG. 5. Approximation of the cascade curves in lead (showers initiated by electrons). Solid curves – function $n^*(t)_e$, calculated according to formula (6). The numbers at the curves indicate the energy of the primary electron E_0 in Mev. Dashed curves were calculated by Wilson. ●, o, ■ – data of the present work (errors not shown).

be approximated by the very simple functions

$$n^*(t)_e = (1 + 1.08\,\alpha t^{0.6\alpha})\,e^{-0.6t}, \quad (6)$$

$$n^*(t)_\gamma = 1.08\,\alpha t^{0.6\alpha}\,e^{-0.6t} \quad (7)$$

(see Fig. 5), where $\alpha = 1\ \ln(E_0/25)$, E_0 is the energy of the primary particle in Mev; and t is measured in radiation units. The index e or γ indicates the nature of the particle initiating the shower. Equations (6) and (7) enable us to find the dependence of the position of shower maximum on the energy of the primary particle in an analytic form:

$$t_{max}(E_0)_\gamma = \alpha, \quad (8)$$

$$n^*_{max}(E_0)_\gamma = 3(\alpha/e)^{0.6\alpha+1} \approx 0.008\,E_0. \quad (9)$$

The functions $t_{max}(E_0)_e$ and $n_{max}(E_0)_e$ are similar to Eqs. (8) and (7) (see Fig. 6), but are more complicated.

One advantage of Eqs. (6) to (9) given above is that they contain only one parameter α which depends on the energy of the primary particle. Formulae (6) to (9) may be used for calculating total absorption spectrometers, and also in other problems, where it is not necessary to know exactly the

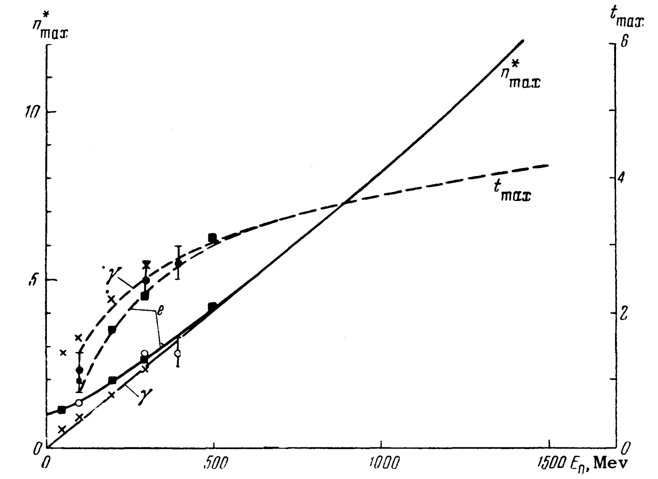

FIG. 6. Maxima of the showers in lead for various energies E_0. Solid curves – dependence of n^*_{max} on the energy of the incident electron (e) or photon (γ). Dashed lines – the same for t_{max}. •, o – data of the present work; ■, × – data of reference 5.

tails of the cascade curves ($t > 10$ r.u.). The agreement between the curves can also be attained in the region $t > 10$ r.u., if we introduce a term $\exp(-0.24\,t)$ into the approximating functions, describing the tail of the cascade curves for large thicknesses.[5] The approximating functions then become complicated. It is at present difficult to establish the degree of applicability of the relations (6) to (9) for higher energies than those used in the present work. The results of experiments carried out so far with cosmic rays[10] cannot be used for such a purpose, since the energy of the particles initiating the showers is not sufficiently well known.

[1] D. B. Skobel'tsyn, Z. Physik **54**, 686 (1929); B. Rossi, Physik. Z. **33**, 304 (1932).

[2] B. Rossi and K. Greisen, Revs. Modern Phys. **13**, 240 (1941).

[3] S. Z. Belen'kiĭ, J. Exptl. Theoret. Phys. (U.S.S.R.) **19**, 940 (1949). I. P. Ivanenko, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 491 (1957), Soviet Phys. JETP **5**, 413 (1957).

[4] C. E. Swartz, Nucleonics **14**, 4, 44 (1956); Cassels, Fidecaro, Wetherell, and Wormald, Proc. Phys. Soc. **A70**, 405 (1957).

[5] R. R. Wilson, Phys. Rev. **86**, 261 (1952).

[6] V. V. Vishnyakov and A. A. Tyapkin, Атомная энергия (Atomic Energy) **3**, 10, 298 (1957).

[7] J. J. Thomson, Phil. Mag. Ser. **6**, 13, 561 (1907); J. Loeb, Compt. rend. **222**, 488 (1957).

[8] S. Z. Belen'kiĭ, Лавинные процессы в космических лучах (Cascade Processes in Cosmic Rays) 1948.

[9] De-Wire, Ashkin, and Beach, Phys. Rev. **83**, 505 (1951); Davis, Bethe, and Maximon, Phys. Rev. **93**, 788 (1954).

[10] C. Y. Chao, Phys. Rev. **75**, 581 (1949); Butler, Rosser, and Barker, Proc. Phys. Soc. **A362**, 63, 145 (1950). Wang Kang-ch'ang, Ch'en Cheng-ch'ih and Li Ming, Chinese J. Phys. **11**, 421 (1955) (in Chinese).

Translated by H. Kasha
2