where we have set $k_1a_1 + k_2a_2 + k_3a_3 = k_1a\cos\vartheta + k\sin\vartheta\cos\varphi$. We see that the factor obtained is similar. It must be noted that after the changes of variables $q'_z = q_z - q_\perp^2/2\epsilon_1$ or $q'_z = q_z - q_\perp^2/2\epsilon$, for bremsstrahlung and pair production, respectively, the factors (4) and (5) differ somewhat, but not to any important degree if $\delta \gg q_\perp^2/2\epsilon_1$, $\delta \ll q_\perp^2/2\epsilon_2$, and $\delta \ll q_\perp^2/2\epsilon_-$, $\delta \ll q_\perp^2/2\epsilon_+$ for the two respective cases.

We remark that when one takes into account thermal vibrations in the interference factor of the radiation these make the contribution[2] $q_\perp < h/(\overline{u^2})^{1/2}$, where $\overline{u^2}$ is the mean square deviation of the atoms from their equilibrium positions. Furthermore, on inserting the factor (5) under the integral in Eq. (2) (or in the corresponding formula for the case of pair production) we arrive at a formula for radiation or pair production in collisions with a chain of atoms obtained by the impact-parameter method (when thermal vibrations are included there is an additional factor $\exp\{-(k^2 + k_1^2)\overline{u^2}\}$).

If we now compare these formulas with the formulas for the inteference radiation obtained by perturbation theory [Eqs. (41) and (42) of reference 1], we see that they are completely identical. An analogous argument can also be carried through for the case of collisions of charged particles with atomic electrons (ionization losses). Here it can be shown that the exact quantum mechanical formulas go over into those obtained by the impact-parameter method when they are expanded in powers of the parameter $q_\perp R$, where R is the radius of the atom and $q_\perp$ is the change of momentum in the direction perpendicular to the motion.

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DIFFRACTION BREAKUP OF LIGHT NUCLEI

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REFERENCE 1 treated the processes of diffractive interaction of deuterons with nuclei. Obviously, diffraction processes (scattering, disintegration, and stripping) can also occur in the collision of other loosely bound light nuclei with heavy nuclei. The possible occurrence of diffraction phenomena must be taken into account in studying the interaction with heavy nuclei of beams of light nuclei which have been accelerated to high energy.

There are several light nuclei whose binding energies against two-particle breakup are small. Thus, for example, Li^6 may be regarded as being made up of a deuteron and an α particle (with binding energy $\epsilon = 1.53$ Mev), Li^7 is a triton plus an α particle ($\epsilon = 2.52$ Mev), Be^9 is Be^8 plus a neutron ($\epsilon = 1.64$ Mev), B^{10} is Li^6 plus an α particle ($\epsilon = 4.36$ Mev), etc. Diffraction processes in the interaction with heavy nuclei of light nuclei which are made up of two relatively weakly bound particles can obviously be described in the same way as the diffractive interaction of deuterons with nuclei. However, the results of reference 1 cannot be used without change, since it was assumed in reference 1 that the deuteron radius R_d is considerably smaller than the nuclear radius R.

In the present note we calculate the total cross sections for various processes of diffractive interaction of a deuteron with a black nucleus, assuming an arbitrary ratio of the radii R_d and R. The Coulomb interaction is neglected.

To simplify the calculations, the wave function of the deuteron ground state is chosen to be Gaussian

$$\varphi_0(r) = (2/\pi R_d)^{3/2}\exp\left\{-2r^2/\pi R_d^2\right\}, \tag{1}$$

in which the constants are determined from the conditions $\int\varphi_0^2(r)\,d\mathbf{r} = 1$ and $\int r\varphi_0^2(r)\,d\mathbf{r} = R_d$.

Using the general formulas of reference 1, one easily obtains the following expressions for the total cross section for all processes σ_t, the cross section σ_n for stripping off a neutron, the cross section σ_p for stripping off a proton, and the elastic scattering cross section σ_e:

$$\sigma_t = 4\pi R^2\left\{1 - \int_0^\infty e^{-\xi^2/q^2}\frac{J_1^2(\xi)}{\xi}\,d\xi\right\},$$

$$\sigma_n = \sigma_p = \pi R^2\left\{1 - 2\int_0^\infty e^{-\xi^2/q^2}\frac{J_1^2(\xi)}{\xi}\,d\xi\right\}, \quad (2)$$

$$\sigma_e = 2\pi R^2\int_0^\infty I^2(\zeta)\,\zeta\,d\zeta,$$

where we have used the notation $q = 4R/\sqrt{\pi}\,R_d$, and the function $I(\zeta)$ which enters into σ_e is defined by

$$I(\zeta) = \begin{cases} 1 - \frac{2}{\pi}\int_0^{\pi/2}\exp\{-q^2(\cos\psi + \sqrt{1-\zeta^2\sin^2\psi})^2\}\,d\psi, & \zeta < 1 \\ \frac{2}{\pi}\int_0^{\arcsin(1/\zeta)}[\exp\{-q^2(\zeta\cos\psi - \sqrt{1-\zeta^2\sin^2\psi})^2\} \\ \quad - \exp\{-q^2(\zeta\cos\psi + \sqrt{1-\zeta^2\sin^2\psi})^2\}]\,d\psi, & \zeta > 1. \end{cases}$$

The cross sections for diffraction breakup σ_d and absorption of the deuteron σ_a can be found from the relations[1]

$$\sigma_d + \sigma_e = \sigma_t/2, \qquad \sigma_a + \sigma_n + \sigma_p = \sigma_t/2. \quad (3)$$

Figure 1 shows the dependence of the various cross sections on the ratio R/R_d.

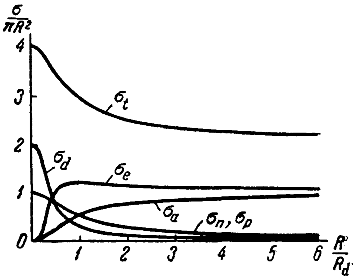

FIG. 1

In the limit of large values of the parameter q ($q \gg 1$), the following asymptotic formulas are valid:

$$\sigma_e = \pi R^2 + (1 - 1/\sqrt{2})\,\pi R R_d, \qquad \sigma_d = \tfrac{1}{2}(\sqrt{2} - 1)\,\pi R R_d,$$

$$\sigma_n = \sigma_p = \pi R R_d/2, \qquad \sigma_a = \pi R^2 - \pi R R_d/2,$$

$$\sigma_t = 2\pi R^2 + \pi R R_d. \quad (4)$$

We note that for $R/R_d \sim 2$, the cross section values given by the asymptotic formulas (4) are already practically the same as the exact values found by numerical integration.

Relations (2) and (3) also describe processes of diffractive interaction of weakly bound light nuclei with heavy nuclei if R_d is interpreted to be the average distance between the constituents of the light nucleus.

The region $R/R_d < 1$ in Fig. 1 corresponds to the process of interaction of π mesons (or nucleons) with deuterons at high energies, to which the diffraction model is also applicable. R should then be interpreted to be the radius of interaction between the π meson and the nucleon. According to reference 2, for $E_\pi = 1.4$ Bev this radius is $R = 1.18 \times 10^{-13}$ cm, i.e., $R/R_d \approx 0.5$. Assuming that $q \ll 1$, one can get the following approximate formulas:

$$\sigma_e = 2\pi R^2 q^2, \qquad \sigma_d = 2\pi R^2(1 - \tfrac{9}{8}q^2),$$

$$\sigma_n = \sigma_p = \pi R^2(1 - q^2/4), \qquad \sigma_a = \tfrac{1}{4}\pi R^2 q^2,$$

$$\sigma_t = 4\pi R^2(1 - q^2/8). \quad (5)$$

The cross section σ_e corresponds to elastic scattering of the π meson by the deuteron, σ_d to scattering of the π meson accompanied by breakup of the deuteron, σ_n and σ_p to processes of inelastic interaction of the π meson with the neutron or the proton in the deuteron, and σ_a to the process of inelastic interaction of the π meson with the neutron and proton. As a result of the diffraction the total interaction cross section of a fast π-meson with a deuteron is less than the sum of the total cross sections for interaction of the π meson with a free neutron and proton (eclipse effect[3,4]). The diffraction also has the effect that the scattering of the π meson by the deuteron occurs mostly with simultaneous breakup of the deuteron ($\sigma_d \gg \sigma_e$).

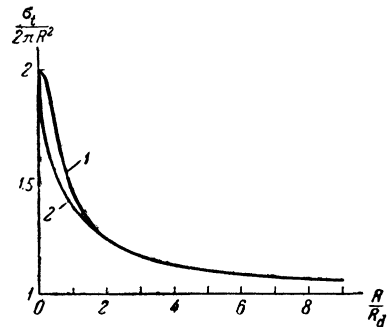

FIG. 2

It should be pointed out that the total cross sections in the region $R < R_d$ are strongly dependent on the choice of the wave function $\varphi_0(r)$ of the deuteron ground state. Figure 2 shows the total cross section σ_t for two choices of $\varphi_0(r)$. Curve 1 is for the Gaussian of Eq. (1) and curve 2 for $\varphi_0(r) = \sqrt{\alpha/2\pi}\,e^{-\alpha r}/r$, where $\alpha = 1/2R_d$. On the other hand, in the region $R > R_d$ the cross section values are practically independent of the form of the wave function of the deuteron ground state.

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THE SPLITTING OF A SMALL DISCONTINUITY IN MAGNETOHYDRODYNAMICS

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In 1926 Kochin[1,2] investigated the break-up of an arbitrary hydrodynamic plane discontinuity. In doing this he made essential use of the fact that either a single shock wave or a single self-similar rarefaction wave can be propagated in each direction from the initial discontinuity.

In magnetohydrodynamics a discontinuity breaks up, generally speaking, in a considerably more complicated manner: up to three waves (shock waves or self-similar waves) can be propagated in each direction from the initial discontinuity. This is connected with the fact that in magnetohydrodynamics there exist three different types of stable shock waves[3] (fast and slow magnetoacoustic waves and magnetohydrodynamic waves) and two types of self-similar waves[4] (fast and slow magnetoacoustic waves). Because the different speeds of propagation, up to three waves of the types enumerated above may be propagated in each direction from the initial discontinuity.

We note that the initial discontinuity is characterized by seven parameters — the discontinuities in the density $\Delta\rho$, in the entropy Δs, in the velocity ΔV and in the tangential component of the magnetic field ΔH_t. Since each wave is characterized by one parameter, the initial discontinuity breaks up into seven waves, of which three are propagated to the left, three are propagated to the right and one — a contact discontinuity — remains stationary. As has been shown by Akhiezer et al.,[3] two waves of the same type move in such a way that the wave in the rear overtakes the wave in front. Therefore waves of three different types must be propagated in each direction: in front there will be the fast magnetoacoustic (shock or self-similar) wave, followed by the Alfven shock wave, and finally, the slow magnetoacoustic (shock or self-similar) wave.

One should have in mind the fact that the self-similar wave is a rarefaction wave,[4] while the shock wave is a compression wave.[5]

The problem now consists of choosing the amplitudes of these seven waves in such a way as to make a transition from the state to the left of the initial discontinuity to the state to the right of the initial discontinuity. For the sake of simplicity, we shall restrict ourselves to the case when the initial discontinuity is very small. Then all the secondary discontinuities will also be small. The relation between the discontinuities in the magnetohydrodynamic quantities in the self-similar and the shock waves (in the case of low intensity) is the same as between the amplitudes of the corresponding linearized wave. We now state these relations:

(1) Magnetoacoustic waves (shock and self-similar waves)

$$\Delta_{\pm}^{(\varepsilon)} v_x = \varepsilon (u_{\pm}/\rho)\,\Delta_{\pm}^{(\varepsilon)}\rho,$$
$$\Delta_{\pm}^{(\varepsilon)} \mathbf{v}_t = -\,\varepsilon H_x \mathbf{H}_t u_{\pm} \Delta_{\pm}^{(\varepsilon)}\rho \,/\, 4\pi\rho^2 (u_{\pm}^2 - V_x^2),$$
$$\Delta_{\pm}^{(\varepsilon)} \mathbf{H}_t = u_{\pm}^2 \mathbf{H}_t \Delta_{\pm}^{(\varepsilon)}\rho \,/\, \rho (u_{\pm}^2 - V_x^2),$$
$$\Delta_{\pm}^{(\varepsilon)} s = 0, \qquad \Delta_{\mp}^{(\varepsilon)} p = c^2 \Delta_{\pm}^{(\varepsilon)}\rho,$$

where c is the speed of sound, $\mathbf{V}_t$ is the tangential component of the velocity of the liquid $\mathbf{V}$, and

$$\mathbf{V} = \mathbf{H}/\sqrt{4\pi\rho}, \quad u_{\pm}^2 = \tfrac{1}{2}\left[V^2 + c^2 \pm \sqrt{(V^2+c^2)^2 - 4c^2V_x^2}\right].$$

The plus sign corresponds to the fast magnetoacoustic wave, the minus sign corresponds to the slow one. For waves moving to the right $\epsilon = +1$; for waves moving to the left $\epsilon = -1$. The difference between the shock and the self-similar magnetoacoustic waves is that in the former the density increases, while in the latter it decreases.

(2) Alfven shock waves

$$\Delta_A^{(\varepsilon)} \mathbf{v}_t = -\,\varepsilon \Delta_A^{(\varepsilon)} \mathbf{H}_t / \sqrt{4\pi\rho},$$
$$\Delta_A^{(\varepsilon)} v_x = \Delta_A^{(\varepsilon)} p = \Delta_A^{(\varepsilon)} \rho = 0, \qquad \Delta_A^{(\varepsilon)} H_t^2 = 0.$$

(3) Contact discontinuity

$$\Delta_C v_x = \Delta_C v_y = \Delta_C v_z = \Delta_C p = \Delta_C H_y = \Delta_C H_z = 0,$$
$$\Delta_C \rho = (\partial\rho/\partial s)_p \Delta_C s, \quad H_x \neq 0.$$

The sum of the discontinuities of each magnetohydrodynamic quantity at the seven new waves is equal to the initial discontinuity. We thus obtain seven equations in seven unknowns, on solving