$h\kappa$ is the momentum of the quantum. $l = 1$ corresponds to right-circular polarization (spin directed parallel to the motion) and $l = -1$ to left-circular polarization (spin opposite to motion) of the γ-ray quantum.

In the case of a π meson at rest we get the following expression for the decay probability:

$$dW = \frac{e^2 g^2}{h^2 c\pi} \frac{k_\mu^2 \, dk_\mu \sin\theta \, d\theta}{k_{0\pi} K_\mu} (f_1 \pm s_\mu s_\nu f_2 \pm s_\mu l f_3 + s_\nu l f_4). \tag{2}$$

Here the upper sign is for decay occurring with the emission of a neutrino, and the lower sign is for decay with emission of an antineutrino. The notations used are:

$$\begin{aligned} f_1 &= k_\mu^2 \frac{q(1-\cos^2\theta)}{Q-K_\mu} + \frac{Q-K_\mu}{k_{0\pi}-\gamma}\gamma k_{0\pi}, \\ f_2 &= -\left\{k_\mu (1-\cos^2\theta)\left(K_\mu + \frac{k_\mu^2}{Q-K_\mu}\right) + \frac{Q-K_\mu}{k_{0\pi}-\gamma}(k_\mu - K_\mu\cos\theta)\right\}, \\ f_3 &= -\frac{K_\mu}{k_{0\pi}-\gamma}\left\{k_\mu(1-\cos^2\theta)\left(k_{0\pi} - \frac{k_{0\mu}^2}{K_\mu}\right) - (Q-K_\mu)k_{0\pi}\cos\theta + \frac{k_{0\pi}(Q-K_\mu)}{K_\mu(k_{0\pi}-\gamma)}k_\mu^2\cos\theta\right\}, \\ f_4 &= -\frac{k_{0\pi}}{k_{0\pi}-\gamma}\{\gamma^2 - \gamma(Q+K_\mu) + k_{0\mu}^2\}, \end{aligned} \tag{3}$$

where hk_μ is the momentum and hcK_μ the energy of the μ meson, $hck_{0\pi}$ is the rest energy of the π meson, θ is the angle between the directions of motion of the μ meson and the γ-ray quantum, and

$$Q = (k_{0\pi}^2 + k_{0\mu}^2)/2k_{0\pi}, \quad q = (k_{0\pi}^2 - k_{0\mu}^2)/2k_{0\pi},$$
$$\gamma = K_\mu - k_\mu \cos\theta.$$

In the expression (2) for the decay probability the last three terms are due to parity nonconservation, i.e., the longitudinal polarizations of the μ meson, the neutrino, and the γ-ray quantum. If in Eq. (2) we carry out a summation over the directions of polarization of the μ meson and the γ-ray quantum, we get the well known expression[1] for the decay probability of the π meson.

Summation only over the spin states of the μ meson leads to the result of Bund and Ferreira.[2]

To simplify the analysis of the formula (2) we suppose that the momentum of the μ meson is very small (close to zero); then the momenta of the γ-ray quantum and the neutrino will be antiparallel. In this limit ($k_\mu \to 0$)

$$\begin{aligned} f_1 = f_4 &= k_{0\mu}(k_{0\pi} - k_{0\mu})/2, \\ f_2 = f_3 &= {}^1\!/_2\, k_{0\mu}(k_{0\pi} - k_{0\mu})\cos\theta, \end{aligned} \tag{4}$$

where θ is the angle between the spin vector of the μ meson and the direction of motion of the γ-ray quantum. The analysis leads to the following results:

(a) if the spin of the μ meson is directed opposite to the motion of the γ-ray quantum ($s_\mu = -1$ and $\cos\theta = -1$), then the decay probability is different from zero only in the case $s_\nu = 1$ and $l = 1$, i.e., when the decay involves emission of a neutrino and the quantum emitted has right-circular polarization;

(b) if the spin of the μ meson is directed along the direction of motion of the γ-ray quantum ($s_\mu = 1$), then we must permit decay of the π meson with emission of an antineutrino ($s_\nu = -1$) and a γ-ray quantum with left-circular polarization ($l = -1$). We note that in this limit the probabilities of the two types of decay are equal.

From the above it follows that if the π meson decays with emission of a neutrino, then for small momenta of the μ meson its spin must make an angle close to 180° with the direction of the quantum. In the case of antineutrino decay this angle is close to zero. Obviously this conclusion can be checked by measurement of the μ-γ correlation.

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266

ON THE THEORY OF THE "SECOND MOMENT" IN THE NUCLEAR MODEL OF LANE, THOMAS, AND WIGNER

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THE "second moment" was first introduced in a paper of Lane, Thomas, and Wigner[1] as a quantitative criterion for the error committed when the nuclear Hamiltonian is replaced by the Hamiltonian of the shell model.

Let H be the nuclear Hamiltonian and H_0 the shell model Hamiltonian. Then $H = H_0 + H_1$, where H_1 is an operator which gives rise to correlations between single-particle states of the shell model. Further, let $H\Phi_b = E_b\Phi_b$, where the Φ_b are Slater determinants describing the various nuclear states in the shell model. The wave function of the nucleus for energy E, which satisfies the equation $H\psi_E = E\psi_E$, can be represented as an expansion in the complete orthogonal function system Φ_b:

$$\psi_E = \sum_b C_b(E)\,\Phi_b. \quad (1)$$

Let us assume that the state ψ_E corresponds to a state of the compound nucleus in the Bohr model, in which the wave functions in different channels do not interfere. This is equivalent to assuming that the phases of the coefficients $C_b(E)$ are random (cf. reference 1 and 2). Thus the description of the state ψ_E in the Bohr model is incomplete, and the state can be characterized by means of a density matrix. In the state ψ_E, the average value of H_0 is

$$(\psi_E,\ H_0\psi_E) = \sum_b |C_b(E)|^2 E_b. \quad (2)$$

Since according to the assumptions of the model[1] the function $|C_b(E)|^2$ has a maximum at $E_b \approx E$,*

$$(\psi_E,\ H_0\psi_E) \approx E. \quad (3)$$

Comparing (2) and (3), we conclude that the state of the compound nucleus in the Bohr picture is a "mixed" state which can be characterized by the statistical matrix:

$$W_{bb} = |C_b(E)|^2, \quad W_{bb'} = 0, \quad b \neq b', \quad (4)$$

where, according to (3) the nuclear energy E is the average energy of the system having the Hamiltonian H_0.

In order to characterize the compound nucleus with excitation energy E by a temperature $T(E)$, as was done by N. Bohr[3] and Ya. I. Frenkel,[4] it is sufficient to assume that

$$|C_b(E)|^2 \sim \exp(-E_b/kT), \quad (5)$$

where the nuclear temperature T is related to the total energy of excitation of the nucleus, E, by the equation of state of a Fermi gas at low temperature: $E = aT^2$.

On the basis of assumption (5) it is easy to determine the "second moment" of the model,[1] which, by definition, is equal to

$$W^2(E) = \sum_b (E_b - E)^2 |C_b(E)|^2. \quad (6)$$

Going over from a sum to an integral in (6) and using for the level density $\rho(E_b)$ the expression found by Landau,[5]

$$\rho(E_b) = \lambda^{-1}(E_b)\exp(S(E_b)/k),$$

where $S(E_b)$ is the entropy of the nucleus on the shell model, and $\lambda(E_b)$ is a smoothly varying function, we get

$$W^2(E) = \int_0^\infty \frac{1}{\lambda(E_b)}(E - E_b)^2 \exp\left[\frac{S(E_b)}{k} - \frac{E_b}{kT}\right] dE_b. \quad (7)$$

Expanding the exponent in powers of $(E - E_b)$ and stopping at quadratic terms, we find†

$$W^2(E) = A\int_0^\infty (E - E_b)^2 \exp[-(E - E_b)^2/\Omega^2(E)]\,dE_b, \quad (8)$$

where

$$1/A(E) = \int_0^\infty \exp[-(E - E_b)^2/\Omega^2(E)]\,dE_b,$$

$$\Omega^2(E) = \frac{4}{\sqrt{a}}E^{3/2}.$$

Performing the integration in (8), we find

$$W^2(E) = \frac{\Omega^2(E)}{2(1+\Phi(x))}\left\{1 + \Phi(x) - \frac{2x}{\sqrt{\pi}}e^{-x^2}\right\}, \quad (9)$$

$$x = E/\Omega(E), \quad \Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt.$$

If $x > 1$,

$$W^2(E) \approx \Omega^2(E)/2. \quad (10)$$

Formulas (9) and (10) give the dependence of the "second moment" on the excitation energy E and the nucleon number A ($a \approx 0.07A$).[7] Thus, for $a = 10\ \text{Mev}^{-1}$ and E = 8 Mev, W(E) = 3.8 Mev; if we take E = 16 Mev, W(E) = 10.8 Mev. It is interesting to note that the value found for the "second moment" for E = 16 Mev coincides with the value which had to be postulated in a paper of the authors[8] in order to give a satisfactory description of the parameters of the "giant resonance" in photonuclear reactions.

A straightforward quantum-mechanical computation of the second moment" has been made in several papers. In a paper of Vogt[9] the value W ~ 4.5 Mev was found, and in a paper of Brown et al.[10] the value W ~ 7 Mev, etc. However none of the above-mentioned papers enables one to determine the value of the excitation energy corresponding to their calculated values of the "second moment." This makes it difficult to give a quantitative comparison of the energy dependence of the "second moment" found in the present paper with the calculations of these earlier authors.

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*Relation (3) would be exact if the equation $(\psi_E, H_1\psi_E) = 0$ were satisfied.

†The approximation in which relation (8) is valid is equivalent to the assumption that

$$|C_b|^2\rho(E_b) = A\exp[-(E - E_b)^2/\Omega^2(E)],$$

which was used in reference 6.

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267

ON THE LIMITS OF APPLICABILITY OF THE IMPACT-PARAMETER METHOD

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In studies of radiative processes, use is made of ordinary perturbation theory, and also of the impact-parameter method (Weizsäcker-Williams method). The opinion is sometimes expressed that the latter gives insufficiently accurate results.[1] But let us compare the results given by these methods.

We consider deceleration radiation in collisions with atoms.

Following Überall[1] we write the bremsstrahlung cross section in the form

$$\sigma_t = \frac{Z^2 r_0^2}{137}\,\frac{2d\varepsilon}{\varepsilon_1^2}\int_0^\infty dq_\perp^2 \int_{\delta+q_\perp^2/2\varepsilon_1}^{q} dq_z\,\frac{[1-F(q)]^2}{q^4}$$
$$\times\left\{-\frac{1}{(q_z-q_\perp^2/2\varepsilon_1)^2}-\frac{q_z+q_\perp^2(\varepsilon-\varepsilon_2)/2\varepsilon_1\varepsilon_2}{[(q_z-q_\perp^2/2\varepsilon_2)^2+4\delta^2 q_\perp^2]^{3/2}}\right. \tag{1}$$
$$\left.+\frac{(1+\varepsilon\delta)\,q_\perp^2+2}{(q_z-q_\perp^2/2\varepsilon_1)[(q_z-q_\perp^2/2\varepsilon_2)^2+4\delta^2 q_\perp^2]^{1/2}}\right\}.$$

Here ϵ_1, ϵ_2 are the initial and final energies of the electron, ϵ is the energy of the photon (denoted by k in reference 1), $\delta = \epsilon/2\epsilon_1\epsilon_2$, $\hbar = m = c = 1$. On the other hand, the impact-parameter method gives[2,3]

$$\sigma_t = \frac{Z^2 r_0^2}{137}\,\frac{d\varepsilon}{\varepsilon_1^2}\int_0^{\sim 1} dk^2 \int_\delta^{\sim 1} \frac{k^2 dk_1}{k_1^2(k_1^2+k^2+R^{-2})^2}$$
$$\times\left[\frac{\varepsilon_1}{\varepsilon_1-\varepsilon}+1-\frac{\varepsilon}{\varepsilon_1}-\frac{2\varepsilon}{k_1\varepsilon_1(\varepsilon_1-\varepsilon)}+\frac{\varepsilon^2}{k_1^2\varepsilon_1^2(\varepsilon_1-\varepsilon)^2}\right]; \tag{2}$$

here $k^2 = k_2^2 + k_3^2$ (ϵ is the energy of the photon). In Eq. (1) let us replace the variable q_z by $q_z' = q_z - q_\perp^2/2\epsilon_1$ and expand the expression in square brackets in powers of $(q_\perp/mc)^2$:

$$\frac{1}{q_z'^2}\left(1+\varepsilon\delta-\frac{2\delta}{q_z'}+\frac{2\delta^2}{q_z'^2}\right)q_\perp^2$$
$$+\left[\frac{\delta(1+\varepsilon\delta)}{q_z'^3}-\frac{9\delta^2+2\varepsilon\delta^3}{q_z'^4}+\frac{24\delta^3}{q_z'^5}-\frac{18\delta^4}{q_z'^6}\right]q_\perp^4\cdots \tag{3}$$

Comparing Eqs. (3) and (2) we see that the first term of the expansion of the exact formula gives the result of the impact-parameter method (we have here $q_z' = k_1$, $q_\perp^2 = k^2$), and the second term is only a correction if $q_\perp^2 \ll 1$. Consequently, Eq. (2) agrees with the exact formula (1) only in the region $q_\perp^2 \ll 1$ (which corresponds to values of the impact parameter larger than $\hbar/mc$). A similar treatment can be given for pair production.

Let us now turn our attention to radiation and pair production in periodic structures. In reference 1 a problem of this sort is solved for a chain of atoms [Eqs. (33) and (23)]. These formulas differ from the corresponding formulas for collisions with a single atom (for example, Eq. (1)) by the factor:

$$\frac{2\pi}{a}N\sum_h 2\Big/\sqrt{q_\perp^2\vartheta^2-\left(q_z-\frac{2\pi}{a}h\right)^2}, \tag{4}$$

which gives the effect of interference in radiation or pair production in collisions with a chain of atoms. It is easy to obtain analogous formulas by the impact-parameter method. To do this we set $\mathbf{r}_i = h\mathbf{a}$ in Eq. (4) of reference 2, where h is an integer and $\mathbf{a}$ is the direction vector of the chain of atoms. Integrating the crystalline factor with respect to ψ [Eq. (4) of reference 2] we get

$$\int d\psi\left|\sum_i e^{i(\mathbf{k}\cdot\mathbf{r}_i)}\right|^2 = \frac{2\pi}{a}N\sum_h 2\Big/\sqrt{k^2\vartheta^2-\left(k_1-\frac{2\pi}{a}h\right)^2}, \tag{5}$$