# CYCLIC ACCELERATION OF PARTICLES IN HIGH-FREQUENCY FIELDS*

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The feasibility of using high-frequency fields for controlling the motion of particles in cyclic accelerators is indicated.

In 1947 one of us directed attention to the possibility of cyclic acceleration of charged particles in rapidly varying magnetic fields. The basic idea of an accelerator of this kind is described below.

If a constant or slowly-varying (in time) magnetic field acts over sectors, rather than over an entire circular area, at an appropriate field value (the same in all sectors) a particle with constant energy will move along a curve which is a closed polygon with rounded vertices.

Instead of a fixed magnetic field or a slowly varying magnetic field, we can use a rapidly varying field; if the time in which the particle traverses one sector is a multiple of the period of the magnetic field, the particle will enter each magnetic sector at a time such that the field in the sector is at the desired value. The sectors can be rectangular wave guides with closed ends. A traveling-wave circular accelerator has also been considered.

Calculations carried out by Burshtein and Kolomenskii[1] indicated, however, that the variants cited above would not provide stable motion.

We wish to point out the possibility of cyclical acceleration by a rapidly varying electromagnetic field. As in the above examples, this field must act as a guide field.

The characteristic feature of the method proposed below is that the particle is not in the high-frequency electromagnetic field for a fraction of a period, as above, but rather for a large number of periods, or even for the entire time of acceleration. Consider the motion of a particle in a field produced by a plane standing wave $E_y = E_0 \cos kx \times \sin \omega t$ and $H_z = -H_0 \sin kx \cos \omega t$ where $E_0$ and $H_0$ are the amplitudes of the electric and magnetic fields respectively. Integrating the appropriate equations of motion we have

$$\frac{d}{dt} m\dot{x} = \frac{e^2}{mc^2}\frac{E_0 H_0}{2k} \sin 2kx \cos^2 \omega t. \qquad (1)$$

*This work was carried out in 1956.

Thus, on a particle in the field of a standing wave there is exerted a force which tends to localize the particle at the nodes of the electric field.* If we now assume that the nodal surface is a cylinder, it is easily shown that the particle will be confined by the varying field, executing oscillations about some closed orbit.

As an example, we consider the motion of a particle in a coaxial cylindrical resonator in which an axially symmetric TM mode is excited (the axis of the resonator is taken as the z axis). We are not concerned with the acceleration mechanism and shall consider only the stability of a particle with given energy.

We have the following equations of motion

$$\frac{d}{dt} m\dot{r} = mr\dot{\theta}^2 + eE_r - \frac{e}{c}\dot{z}H_\theta,$$
$$\frac{d}{dt} mr^2\dot{\theta} = 0, \quad \frac{d}{dt} m\dot{z} = eE_z + \frac{e}{c}\dot{r}H_\theta, \qquad (2)$$

where

$$E_r = \frac{h}{\lambda} A_0 r^{-1/2} \cos\lambda(r-a) \sin hz \sin\omega t,$$
$$E_z = -A_0 r^{-1/2} \sin\lambda(r-a)\cos hz \sin\omega t,$$
$$H_\theta = \frac{k}{\lambda} A_0 r^{-1/2}\cos\lambda(r-a)\cos hz\cos\omega t,$$
$$\lambda = n\pi/(b-a),\ h = m\pi/d,\ k^2 = \omega^2/c^2 = h^2 + \lambda^2,$$

a and b are the major and minor radii of the cylinder and d is the height. It is assumed that $\lambda r \gg 1$ and that the asymptotic expressions for the Bessel functions can be used.

In the general case the solution of these equations is extremely complicated; however, as an illustrative example, we may consider the case in which $n = 2$ and $m = 0$. Assuming that $r = R_0 + \rho$ where $R_0 = (a + b)/2$ and that $\rho/R_0 \ll 1$, to first order in $\rho/R_0$ we have:

*The problem of trapping particles in high-frequency fields has been considered by Gaponov and Miller.[2]

$$\frac{d}{dt} m\dot{\rho} = m(R_0 + \rho)\dot{\theta}^2 + \frac{eH_0}{c}\dot{z}\cos k\rho \cos\omega t,$$

$$mR_0(R_0 + 2\rho)\dot{\theta} = M_0, \qquad (3)$$

$$\frac{d}{dt} m\dot{z} = eE_0 \sin k\rho \sin\omega t - \frac{eH_0}{c}\dot{\rho}\cos k\rho\cos\omega t,$$

where $E_0 = H_0 = A_0 R_0^{-1/2}$. Whence we obtain the equation for $\rho$:

$$\frac{d}{dt} m\dot{\rho} = \frac{M_0^2}{mR_0^3} - \frac{3M_0^2}{mR_0^4}\rho - \frac{e^2}{mc^2}\frac{E_0 H_0}{2k}\sin 2k\rho\cos^2\omega t. \qquad (4)$$

Assuming now that $eE_0 < km_0c^2$, $k\rho \ll 1$ and $p_0^2 \ll p_H^2$, as is the case for large radii, we have

$$d^2\rho/d\tau^2 + p_H^2\rho\cos^2\tau = p_0^2 R_0, \qquad (5)$$

where

$$\tau = \omega t, \quad p_0 = \omega_0/\omega, \quad p_H = \omega_H/\omega,$$

$$\omega_0 = M_0/mR_0^2, \quad \omega_H = eH_0/mc.$$

Equation (5) is the Mathieu equation which, as is well known, has stable solutions. By choosing an appropriate frequency $\omega$ and high-frequency field $E_0$ we can achieve operation in a given stability region. Under these conditions, a particle moving along the circle will oscillate about $\bar{\rho} \sim 2R_0(p_0/p_H)^2$.

As the energy increases the representative point is shifted toward the origin of the stability diagram. Thus, if we do not operate in the first stability region, leaving other considerations aside, the maximum energy is determined by the value of $p_H$ which obtains at the boundary of a given stability region. The motion of a particle in a system of this kind is reminiscent, in some respects, of motion in a strong-focusing accelerator.

In the example considered above we have not considered stability in the $z$ direction. However, by appropriate choice of mode and resonator shape it is also possible to achieve vertical stability.*

An interesting feature of the mechanism considered here is the fact that the force is independent of the sign of the charge; thus a system of this type could be used for confining quasi-neutral bunches. In turn, this makes possible the use of a radiation acceleration method[3] for cyclic acceleration of quasi-neutral bunches.

[1] E. L. Burshtein and A. A. Kolomenskii, Reports of the Institute of Physics, Academy of Sciences, USSR (1947).

[2] A. V. Gaponov and M. A. Miller, J. Exptl. Theoret. Physics (U.S.S.R.) **34**, 242 (1958), Soviet Phys. JETP **7**, 168 (1958).

[3] V. I. Veksler, Proceedings of the CERN Symposium on High-Energy Accelerators, I. Geneva 1956.

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***By absence of stability here we mean that a state of neutral equilibrium obtains.**