Considering in mesonic atoms only levels with values of n such that $r_Z/R_{nZ} \ll 1$, but to which perturbation theory is still applicable (n bounded from above), considering in the continuous spectrum such slow mesons for which $kr_Z \ll 1$, and using furthermore Eqs. (3) and (4), we obtain in lieu of (1) and (2)

$$\Delta E_{nl} = \frac{(n+l)!}{2n(n-l-1)!\,[(2l+1)!]^2}\left(\frac{1}{R_{nZ}}\right)^{2l+3}\int_0^{r_Z} r^{2(l+1)}\,v(r)\,dr, \tag{5}$$

$$\tau_{hl} = -\frac{2\mu}{\hbar^2}\frac{C_l^2}{[(2l+1)!!]^2}k^{2l+1}\int_0^{r_Z} r^{2(l+1)}\,v(r)\,dr. \tag{6}$$

Comparison of formulas (5) and (6) shows a simple connection between the level shift in the mesonic atom and the phase of scattering of a low-energy meson by a nucleus. This connection is a particular case of the more general relation obtained by Byers.[3]

In what follows we shall treat the imaginary part of the interaction potential Im v, which leads to a broadening of the levels (particularly by the 1S level) in mesonic atoms and to the absorption of pions when scattered by nuclei. We shall assume the imaginary part of the potential to be independent of r (the results of the calculations depend little on the shape of the potential and depend on $r_Z$, the average radius of the nuclear matter). Then, using (5) and using for Im $\Delta E_{1S}$ the value $(0.45 \pm 0.07)$ kev* measured experimentally by West and Bradley[4] for beryllium-9, we obtain for the S state Im v $\approx$ 1.5 Mev. An analogous estimate can be made using the two-nucleon model of absorption of pions by nuclei.[5] In this case (see, for example, reference 3), we obtain for the S state

$$\mathrm{Im}\,\tau = \Gamma\frac{Z}{6\pi}\left(\frac{\mu c}{\hbar}\right)10^{-27}\,C_0^2\,k\ \mathrm{cm}^2, \tag{7}$$

where the parameter $\Gamma$ characterizes the probability of absorption of a pion by a pair of nucleons in the nucleus, referred to the absorption in deuterium. Comparing this expression with formula (6) at $l = 0$ and taking $\Gamma = \sim 5$ (see reference 6), we get Im v = $0.28\Gamma \approx 1.4$ Mev, which is in approximate agreement with the quantity obtained above. Thus the complex portion of the potential, responsible for the absorption of slow mesons, is small.

Knowing the width of the 1S level, we can estimate the lifetime of the negative pion at this level for nuclear capture, and compare it with the lifetime for decay. This is of interest because Fry and White[7] reported observed cases of decay of negative pions stopped in emulsion and gave an approximate value of $10^3$ for the ratio of the decay probability to the nuclear-absorption probability (for the light elements C, N, and O). Taken together with an approximate pion lifetime of $10^{-8}$ seconds, this gives approximately $10^{-11}$ seconds for the lifetime for nuclear capture. This contradicts, as noted by Gol'danskii and Podgoretskii,[8] the strong interaction between the pion and the nuclear matter. Since the time that the meson hits the 1S orbit of the mesonic atom amounts to approximately $10^{-13}$ seconds, the observed decay should occur obviously on this orbit. Yet the lifetime for nuclear capture, obtained by West and Bradley from a measurement of the level width ($\Delta t$ Im $\Delta E \approx \hbar$) is approximately $10^{-18}$ seconds, which contradicts the results of Fry and White.

I am grateful to A. M. Baldin for a discussion of the above problems.

*Reference 4 gives the total level width $\gamma = (8/3)$ Im $\Delta E_{1S} = (1.2 \pm 0.2)$ kev.

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## *EXCHANGE EFFECTS IN FERROMAGNETIC RESONANCE*

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In the present paper we find a single dispersion law for transverse electromagnetic waves and for spin waves, which takes into account both relativistic and exchange interactions. When the wavelength is shortened (at constant $\omega$) the relative role of the displacement currents becomes less and less; on the contrary, the contribution from the exchange forces increases, and instead of transverse electromagnetic waves we get spin waves.

1. We start with the usual equations of motion for the magnetization,[1]

$$dM/dt = \gamma \{(H_{ex}a^2/M_S)[M\Delta M] + [MH]\}, \quad (1)$$

where $H_{ex}$ is the effective field of the exchange forces, a the lattice constant, $M_S$ the saturation magnetization, H the magnetic field intensity in the sample, $\gamma$ the ratio of the magnetic moment of the electron to its spin: $\gamma = 2.8$ Mcs/Oe. We shall put $\mathbf{M} = \mathbf{M_S} + \mathbf{m}$, $\mathbf{H} = \mathbf{H_i} + \mathbf{h}$ ($\mathbf{M_S} \parallel \mathbf{H_i} \parallel OY$), where $\mathbf{H_i}$ is the internal magnetic field inside the specimen, while $\mathbf{h}$ and $\mathbf{m}$ are the high-frequency components of the magnetic field and the magnetization. If $|m| \ll |M_S|$, $|h| \ll |H_i|$ we get from (1) for the waves E, H $\sim$ $\exp i[k(y\cos\theta + x\sin\theta) + \omega t]$ the linear equations,

$$i\omega m_x = \gamma\{-H_{ex}a^2k^2m_z + M_sh_z - H_im_z\};$$
$$i\omega m_z = \gamma\{H_{ex}a^2k^2m_x - M_sh_x + H_im_x\},$$

from which we find the components of the magnetic susceptibility tensor,

$$\mu_{xx} = \mu_{zz} = \mu_1 = 1 - \frac{(\sigma_0 + \alpha k^2)p}{1 - (\sigma_0 + \alpha k^2)^2};$$
$$\mu_{xz} = -\mu_{zx} = i\mu_2 = \frac{ip}{1 - (\sigma_0 + \alpha k^2)^2}, \quad (2)$$

where $\alpha = \gamma H_{ex}a^2/\omega$, $\sigma_0 = \gamma H_i/\omega$, $p = 4\pi M_S\gamma/\omega$. As $\alpha \rightarrow 0$ expression (2) goes over into the well known equations of Polder's.[2]

Absorption can be taken into account by substituting for $\omega$, $\omega - i\gamma\Delta H$, where $\Delta H$ is the half-width of the resonance curve.

2. Substituting (2) into the general Eq. (4) of reference 3, which describes the propagation of electromagnetic waves in gyrotropic media, we find a connection between $\omega$ and k, the dispersion law,

$$\omega^6 - \omega^4[(\gamma B + \beta k^2)^2 + 2\omega_0^2] + \omega^2\{(\gamma B + \beta k^2)[2(\gamma H + \beta k^2) + 4\pi M_S\gamma\sin^2\theta] + \omega_0^2\}\omega_0^2 - (\gamma H + \beta k^2 + 4\pi M_S\gamma\sin^2\theta)(\gamma H + \beta k^2)\omega_0^4 = 0 \quad (3)$$

($\omega_0 = ck/\sqrt{\epsilon}$, c is the velocity of light, $\beta = \alpha\omega$), which describes both transverse electromagnetic waves and spin waves. This equation is of the third degree in $\omega^2$ and has three roots, corresponding to three branches of the dispersion curve.

If we neglect the displacement currents (as $\omega\sqrt{\epsilon}/ck \rightarrow 0$), (3) goes over into the equation of the static approximation,

$$\omega^2 = (\gamma H + \beta k^2)(\gamma H + \beta k^2 + 4\pi M_S\gamma\sin^2\theta), \quad (4)$$

i.e., into the spin wave dispersion relation found in reference 1.

3. The character of the dispersion curves can most conveniently be observed from the particular cases $\theta = 0$ and $\theta = \pi/2$.

(1) $\theta = 0$. In this case (3) has three positive solutions,

$$k_1^2 = \frac{1}{2\beta}\left\{-\left(\omega + \gamma H - \frac{\omega^2}{c^2}\varepsilon\beta\right) + \left[\left(\omega + \gamma H - \frac{\omega^2}{c^2}\varepsilon\beta\right)^2 + 4\frac{\omega^2}{c^2}\beta\varepsilon(\omega + \gamma B)\right]^{1/2}\right\}, \quad (5)$$

$$k_{2,3}^2 = \frac{1}{2\beta}\left\{\left(\omega - \gamma H + \frac{\omega^2}{c^2}\varepsilon\beta\right) \pm \left[\left(\omega - \gamma H + \frac{\omega^2}{c^2}\varepsilon\beta\right)^2 + 4\frac{\omega^2}{c^2}\beta\varepsilon(\gamma B - \omega)\right]^{1/2}\right\}. \quad (6)$$

Expression (5) describes the ordinary wave, (6) the extraordinary one. The ($\omega$, k) plane can be divided into three regions: region A ($\omega < \gamma H$), region R ($\gamma H \leq \omega \leq \gamma B$), and region D ($\omega > \gamma B$). In region A only one of the extraordinary waves is propagated [the one corresponding to the plus sign before the radical in Eq. (6)], while the spin wave propagates in region R. In region D the spin wave branch is retained, but added to it is one branch of the transverse electromagnetic waves, the one corresponding to the minus sign in front of the square root in Eq. (6).

(2) $\theta = \pi/2$. In that case two waves are possible: type E and type H.[4] For waves of type E $k^2 = (\omega^2/c^2)\epsilon$. Dividing Eq. (3) by $k^2 - (\omega^2/c^2)\epsilon$ we get the dispersion law for waves of type H,

$$2\omega^2 = (\gamma B + \beta k^2)^2 + \omega_0^2 \pm \{[(\gamma B + \beta k^2)^2 + \omega_0^2]^2 - 4\omega_0^2(\gamma B + \beta k^2)(\gamma H + \beta k^2)\}^{1/2}. \quad (7)$$

The $\omega(k)$ curves for intermediate values of $\theta$ lie between the dispersion curves corresponding to $\theta = 0$ and $\theta = \pi/2$. The line $\omega^2 = \gamma^2 H(H + 4\pi M_S\sin^2\theta)$ is the lower boundary of the region R, and the line $\omega = \gamma B$ its upper boundary.


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