Since $Y^{jM}_{\alpha_1\alpha_2}(-\mathbf{n}) = (-)^{s_1+s_2-j} Y^{jM}_{-\alpha_1-\alpha_2}(\mathbf{n})$, the condition that the matrix element be invariant under reflection requires that

$$A'^{j}_{\alpha_1\alpha_2\alpha'_1\alpha'_2} = A^{j}_{-\alpha_1-\alpha_2,\,-\alpha'_1-\alpha'_2}.$$

It is easily seen that the coefficients

$$A'^{j}_{\alpha_1\alpha_2\alpha'_1\alpha'_2} = \lambda_{\alpha_1\alpha_2}\lambda_{\alpha'_1\alpha'_2} A^{j}_{\alpha_1\alpha_2\alpha'_1\alpha'_2}, \qquad (2)$$

where each of the λ's are either +1 or −1, and satisfy the same unitary and symmetry conditions as do the $A^j_{\alpha_1\alpha_2\alpha'_1\alpha'_2}$.[5]

If, in addition, $\lambda_{-\alpha_1-\alpha_2} = k\lambda_{\alpha_1\alpha_2}$, where $k = \pm 1$ (if s_1 and s_2 are integers, $k = -1$ must be excluded, since the combinations of α's include one in which $\alpha_1 = \alpha_2 = 0$), the A' coefficients can be used to construct the scattering matrix. It is seen from (1) that this matrix, although different from M, gives the same scattering cross section. It is easily shown that $M'(\mathbf{n}_f\mathbf{n}_i) = A(\mathbf{n}_f)M(\mathbf{n}_f\mathbf{n}_i)A^*(\mathbf{n}_i)$, where $A(\mathbf{n})$ is diagonal and has matrix elements $\lambda_{\alpha_1\alpha_2}$ in the coordinate system whose third axis lies along $\mathbf{n}$. Let us consider double scattering given by

$$\rho_{f2}(\mathbf{n}_{f2}\mathbf{n}_{i2}) = M(\mathbf{n}_{f2}\mathbf{n}_{i2})\,\rho_{i2}(\mathbf{n}_{i2})\,M^*(\mathbf{n}_{f2}\mathbf{n}_{i2}).$$

The M matrix is given on the center-of-mass coordinate system, so that the density matrix $\rho_{i2}(\mathbf{n}_{i2})$ of the initial state before the second scattering must also be given in this system, since it is obtained from the density matrix of the particle after the first scattering by the transformation $\rho_{i2}(\mathbf{n}_{i2}) = S\rho_{f1}(\mathbf{n}_{f1})S^*$. Here S is some rotation in the plane of the first scattering,[6] and $\mathbf{n}_{i2}$ and and $\mathbf{n}_{f1}$ are vectors related in the usual way. Now consider the density matrices ρ' obtained from the scattering matrix by

$$M' = A(\mathbf{n}_f)M(\mathbf{n}_f\mathbf{n}_i)A^*(\mathbf{n}_i),$$
$$\rho'_{2f} = M'\rho'_{2i}M'^*, \quad \rho'_{2i} = S\rho'_{1f}S^*,$$
$$\rho'_{1f}(\mathbf{n}_{f1}) = A(\mathbf{n}_{f1})\rho_{f1}(\mathbf{n}_{f1})A^*(\mathbf{n}_{f1}).$$

The cross section for double scattering is then given by

$$\sigma_2 \sim \mathrm{Sp}(MS\rho_{1f}S^*M^*); \quad \sigma'_2 \sim \mathrm{Sp}(MS'\rho_{f1}S'^*M^*),$$

where $S' = A^*(\mathbf{n}_{i2})SA(\mathbf{n}_{f1})$.

If the spins of the particles involved are no greater than $\frac{1}{2}$, the transformations of Eq. (2) contain one for which $S'\rho_{f1}S'^* = \rho_{f1}$.[7] If at least one of the particles has spin greater than $\frac{1}{2}$, however, there is no such transformation among those given by (2). Thus for processes involving particles of spin greater than $\frac{1}{2}$, there is in general no arbitrariness such as that given by (2). An obvious exception is scattering by an infinitely heavy target, when $\mathbf{n}_{i2} = \mathbf{n}_{f1}$ and S = 1; in this case there is no multiplicity of scattering which can be used to differentiate between the set of phases given by (2).

In conclusion, I express my gratitude to Professor M. A. Markov, Professor Ia. A. Smorodinskii, R. M. Ryndin, M. I. Shirokov, and Chou Kuang-Chao for discussion of the work and valuable comments.

*We note that the cross section and tensor moments are easily expressed in terms of the parameters $A^j_{\alpha_1\alpha_2\alpha'_1\alpha'_2}$ and generalized spherical functions[4] without the use of Racah coefficients.

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COMPUTING THE SPECTRA OF FISSION NEUTRONS

V. P. KOVALEV, V. S. STAVINSKII

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USACHEV,[1] Watt,[2] Fraser,[3] and Gurevich and Mukhin[4] have studied the theoretical interpretation of fission neutron spectra. These authors used a model in which the neutrons are assumed to be evaporated from the moving fission fragments. In the center-of-mass system the spectrum of neutrons evaporated from a fragment whose excitation energy is E_0 is of the form

$$n(\varepsilon) \sim \sigma(\varepsilon, E_0)\,\varepsilon\omega(E_0 - \varepsilon).$$

Here ϵ is the kinetic energy of the neutron, $\sigma(\epsilon, E_0)$ is the cross section for capture of a neutron with energy ϵ by fragments whose energy is $E_0 - \epsilon$, and $\omega(E_0 - \epsilon)$ is the energy level density of a fragment at an excitation energy of $E_0 - \epsilon$. In the works cited above the level density was assumed proportional to $\exp(-\epsilon/\tau)$, where τ is the temperature of the fragment. Satisfactory agreement with experiment was attained by properly choosing the value of τ, Watt[2] and Gurevich and Mukhin[4] assuming that $\sigma(\epsilon, E_0) \sim 1/\sqrt{\epsilon}$, while Usachev[1] and Fraser[3] assume $\sigma(\epsilon, E_0)$ to be independent of both ϵ and E_0. Such a choice of $\sigma(\epsilon, E_0)$ is valid, however, only when the neutrons are captured by unexcited nuclei.

In the present note we calculate the fission neutron spectra taking account of the energy dependence of the neutron capture cross section. The energy dependence of $\sigma(\epsilon, E_0)$ was calculated by the complex-potential method,[5] which gives a satisfactory description of the total cross section and capture cross section for low energy neutrons interacting with nuclei. It was assumed that the imaginary part of the potential increases when ϵ is decreased, since to low values of ϵ correspond large excitation energies of the target nucleus, so that the Pauli principle, which weakens the energy exchange between the incident neutrons and the target nuclei, plays a lesser role. Further, since the fragments are so overloaded with neutrons, we may assume that the imaginary part of the potential for fission fragments is greater than the experimentally observed value for ordinary nuclei. Therefore $\sigma(\epsilon, E_0)$ may be evaluated by using the model of a "black" nucleus.[6]

In calculating $\sigma(\epsilon, E_0)$ we must make sure that the potential varies smoothly at the boundary of the nucleus. This can be done by using the approximate expression $\sigma(\epsilon) = \sigma_0(\epsilon) T_d(\epsilon)/T_0(\epsilon)$. Here $\sigma(\epsilon)$ is the cross section we wish to obtain, $\sigma_0(\epsilon)$ is the neutron capture cross section by a "black" nucleus with a sharp potential change[6] at the boundary, $T_0(\epsilon)$ is the penetration transmission coefficient of the neutron wave for this potential, and $T_d(\epsilon)$ is this coefficient for a smoothly varying potential of the form

$$V(x) = -V_0/[1 + \exp(-2x/d)],$$

where $-\infty < x < \infty$, $v_0 = 42$ Mev, and $d = 1.4 \times 10^{-13}$ cm.

The table gives $T_d(\epsilon)/T_0(\epsilon)$ and $\sigma(\epsilon)\epsilon^{1/2}$, which characterizes the deviation of $\sigma(\epsilon)$ from the $1/\sqrt{\epsilon}$ law.

It is interesting to note that the energy dependence obtained for the inverse process is close to the $1/\sqrt{\epsilon}$ law up to an energy of about 1 Mev, which is a qualitative verification of this ϵ dependence chosen by Usachev[1] and Fraser.[3]

ε, Mev	$T_d(\varepsilon)/T_0(\varepsilon)$	$\sigma(\varepsilon)\varepsilon^{1/2}$ barn-Mev$^{1/2}$
0.1	2.55	1.83
0.2	2.50	1.83
0.3	2.40	1.99
0.4	2.30	2.05
0.5	2.20	2.12
0.75	2.02	2.28
1.0	1.89	2.44
1.5	1.74	2.59
2.0	1.63	2.68
2.5	1.55	2.94
3.0	1.49	3.19
5.0	1.34	4.03
10.0	1.18	5.70

In calculating the spectra it was assumed that the most probable fission mode occurs, that the neutrons are emitted isotropically in the center-of-mass system of the moving fragments,[1-4] and that the level density obeys Weisskopf's law $\omega(\epsilon) \sim \exp(2\sqrt{aE})$ over the whole interval of excitation energy,[6] with $a = 0.1\,A$, where A is the atomic weight of the fragment.

The mean excitation energy $E_{L,H}$ was calculated using the relation[7,8] $d\nu/dE = 0.12$ Mev^{-1} and Fraser's[3] relation $\nu_L/\nu_H = 1.3$. Here ν_L and ν_H are the mean numbers of neutrons emitted by the light and heavy fragments, respectively. The mean energy of the accompanying gammas was taken as 7.8 Mev, and was distributed equally between the two fragments.[9]

These assumptions were used to calculate the fission neutron spectra of U^{235} and Cf^{252} The spectra of the "first" and "second" neutrons from an individual fragment in the laboratory system were calculated using the formula

$$N_{L,H}(E) \sim \int_{E(1-\sqrt{\omega_{L,H}/E})^2}^{E(1+\sqrt{\omega_{L,H}/E})^2} \sigma_{L,H}(\varepsilon)\,\varepsilon^{1/2} \times \exp\{2\sqrt{a_{L,H}[\bar{E}_{L,H} - E^B_{L,H} - \varepsilon]}\}\,d\varepsilon.$$

Here $\omega_{L,H}$ is the mean kinetic energy per nucleon of a fragment, and $E^B_{L,H}$ is the neutron binding energy in the fragment.[9,10] The mean excitation energy of a fragment after emission of the first neutron is given by $\bar{E}'_{L,H} = \bar{E}_{L,H} - E^B_{L,H} - \bar{\epsilon}_{L,H}$, where $\bar{\epsilon}_{L,H}$ is the mean kinetic energy of the first neutron. The theoretical calculations for U^{235} are in good agreement with the experimental data.[8,11] Comparing our results with the curve given by $N(E) \sim \exp(-E/0.965)\sinh\sqrt{2.29E}$, which is the best approximation of the observed[8] fission neutron spectrum from U^{235}, we find no deviations greater than 7 per cent up to an energy of about 7 Mev. The agreement with experiment is also satisfactory for Cf^{252}, although the experimental accuracy is not high.[12] Similar results can be obtained assuming that $\omega(\epsilon) \sim \exp(-\epsilon/\tau_{L,H})$, where $\tau_{L,H}$ are the temperatures of the fragments which correspond to their mean excitation energies. These temperatures can be calculated using the formula

$$\tau_{L,H} = \left[\frac{\bar{E}_{L,H} - E^{B}_{L,H} - \bar{\varepsilon}_{L,H}}{a_{L,H}}\right]^{1/2},$$

and for U^{235} fragments we obtain $\tau_L \sim 1$ Mev and $\tau_H \sim 0.8$ Mev.

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THE POSSIBILITY OF ESTIMATING THE MEAN LIFETIME OF ALPHA PARTICLES WITHIN NUCLEI

Iu. I. SEREBRENNIKOV

Leningrad Polytechnic Institute

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IT has often been shown[1-6] that α-particle substructures and others exist within the nucleus. According to the references cited, these substructures participate in nuclear cascade processes and can be knocked out of a nucleus by a fast particle passing through it. Cuër, Combe, and Samman[1-5] assumed that these substructures are unstable in nuclei. Combe[4] considers their lifetimes to be probably of the order of 10^{-22} sec. If this is so, it may be possible to obtain experimental indications as to their mean lifetimes. Let us consider knockout of α particles from nuclei.

If the α particles are stable within the nucleus, then the energy spectrum with which they are knocked out will be given by $N(E) = f(E)P(E)$, where $f(E)$ is the recoil-energy distribution function of the α-particles within the nucleus for positive E, and $P(E)$ is the Coulomb barrier penetration factor for the α particles. If the α particles are unstable within the nucleus with a mean lifetime τ_α of the order of 10^{-22} sec, the expression for $N(E)$ should contain a factor which accounts for their disintegration during the time they move within the nucleus. If this disintegration can be described by an exponential law of the form $N = N_0 \exp(-t_{eff}/\tau_\alpha)$, the knockout α-particle energy spectrum will be of the form

$$N(E) = f(E)P(E) \exp\{-[m_\alpha/2(E+U)]^{1/2} l/\tau_\alpha\}, \tag{1}$$

where $t_{eff} = l/v$ is the time it takes an α-particle which attains the velocity v at the point of collision to move through the shortest distance l to the surface of the nucleus. This distance l should be chosen from the condition that in a spherical shell of thickness l low-energy recoil α particles can be produced efficiently and can leave the nucleus with the least possible losses due to disintegration. In the above equation m_α