that in this case (7) yields $v = 1$, which is the velocity of light in vacuo.

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*Here A_k is the four-vector whose components are A_x, A_y, A_z, and $i\varphi$, and $A_{k,l} = \partial A_k/\partial x_l$; we set $c = 1$.

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SELECTION RULES IN REACTIONS INVOLVING POLARIZED PARTICLES

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SIMON and Welton[1] and Shirokov[2] have obtained the selection rules for a reaction of the type $a + b \to c + d$ in the form of relations between the polarization vectors and tensors. They assume that the initial state is not polarized. The present communication gives a derivation of the selection rules for any arbitrarily-polarized initial state. We shall use Shirokov's notation[2] and assume that all the particles have nonvanishing rest mass.

Consider the statistical tensors of the final state in the $a + b \to c + d$ reaction,

$$\rho'(q_c, \tau_c, q_d, \tau_d;\ g_c g_a^{-1}), \tag{1}$$

which depend essentially on the parameters of the rotation $g_c g_a^{-1}$ which carries the z_a, y_a, x_a coordinate system into the z_c, y_c, x_c coordinate system. The first of these systems is associated with the initial state, the z_a axis being parallel to $\mathbf{n}_a$, and the y_a axis being perpendicular to the production plane of particle a. The second of these systems has z_c parallel to $\mathbf{n}_c$, and y_c in the direction of the cross product $\mathbf{n}_a \times \mathbf{n}_c$. Here $\mathbf{n}_i$ is the unit vector along the direction of motion of particle i, and q is the rank of the statistical tensors. The spin indices τ are defined in terms of these particular coordinate systems. With this choice of coordinate systems, the Euler angles for the rotation $g_c g_a^{-1}$ are $\{-\pi,\ \theta_c,\ \pi-\varphi_c\}$, where θ_c and φ_c are the spherical angles of the unit vector $\mathbf{n}_c$ in the z_a, y_a, x_a coordinate system (see Shirokov[2]).

Let the state obtained from the initial one by space reflection be characterized by the statistical tensor ρ'_I. Under the reflection the z_a and z_c axes, chosen along the momenta of particles a and c, change direction, while the y_a and y_c axes remain invariant. The spherical angles θ_{cI} and φ_{cI} of the reflected $-\mathbf{n}_c$ vector in the reflected $\{z_a, y_a, x_a\}_I$ coordinate system are

$$\vartheta_{cI} = \vartheta_c,\quad \varphi_{cI} = -\varphi_c. \tag{2}$$

The spin operators remain invariant under reflection. If θ_c and φ_c are replaced by θ_{cI} and φ_{cI} in Eq. (1), we obtain the ρ'_I statistical tensors from ρ'; the spin indices τ of the new ρ'_I tensors must be quantized with respect to the old nonreflected z_c, y_c, x_c system. Since the reflected $\{z_c, y_c, x_c\}_I$ coordinate system differs from the initial one only by rotation through an angle π about the y_c axis, the transformation properties of the statistical tensors[2] lead to the equations

$$\rho'_I(q_c, \tau_c, q_d, \tau_d;\ \vartheta_c, -\varphi_c) = \sum_{\tau'_c\tau'_d} D^{q_c}_{\tau_c\tau'_c}(0,\pi,0)\,D^{q_d}_{\tau_d\tau'_d}(0,\pi,0) \times \rho'(q_c, \tau'_c, q_d;\ \tau'_d;\ \vartheta_c, \varphi_c). \tag{3}$$

Here the spin indices τ are quantized with respect to their own proper coordinate systems. Since $D^q_{\tau\tau'}(0, \pi, 0) = (-1)^{q+\tau}\delta_{\tau,-\tau'}$ (see Shirokov[2] and Gel'fand and Shapiro[3]), Eq. (3) leads to

$$\rho'_I(q_c, \tau_c, q_d, \tau_d;\ \vartheta_c, -\varphi_c) = (-1)^{q_c+\tau_c+q_d+\tau_d}\rho'(q_c, -\tau_c, q_d, -\tau_d;\ \vartheta_c, \varphi_c). \tag{4}$$

The law of parity conservation may be stated in the following way: if the initial statistical tensors ρ are replaced by the reflected tensors ρ_I, the tensors ρ'' of the products of the reaction are the tensors ρ'_I which are obtained from ρ' by Eq. (4). In other words, if the statistical tensors ρ' are written $\rho' = F(\rho)$, then

$$\rho'_I = F(\rho_I). \tag{5}$$

Equations (4) and (5) together give the most general selection rules in the form of a relation between the statistical tensors.

Let us now consider some simple examples. If the initial state of the $a + b \rightarrow c + d$ reaction is unpolarized, then $\rho = \rho_I$. Then according to (4) and (5) $\rho' = \rho'_I$, or

$$\rho'(q_c, \tau_c, q_d, \tau_d; \vartheta_c) = (-1)^{q_c+\tau_c+q_d+\tau_d}\rho'(q_c, -\tau_c, q_d, -\tau_d; \vartheta_c). \tag{6}$$

In our case the ρ' tensors do not depend on φ_c. Equations (6) are the same selection rules as Simon and Welton obtained for $q = 1$ and the same as those obtained by Shirokov.*

Let us now consider a cascade of the form $a + b \rightarrow c + d$ followed by $c + e \rightarrow f + g$ (the incident beam a, the target b, and e are unpolarized). According to (6), $\rho = \rho_I$ in the initial state of the second reaction, and we obtain

$$\rho'(q_j, \tau_f, q_g, \tau_g; \vartheta_f, -\varphi_f) = (-1)^{q_f+\tau_f+q_g+\tau_g}\rho'(q_f, -\tau_f, q_g, -\tau_g; \vartheta_f, \varphi_f). \tag{7}$$

For the special case in which $q_f = q_g = 0$, Eq. (7) becomes

$$\sigma(\vartheta_f, -\varphi_f) = \sigma(\vartheta_f, \varphi_f). \tag{8}$$

Since φ_f is the azimuth angle of $\mathbf{n}_f$ in the coordinate system in which the y_c axis is directed along $\mathbf{n}_a \times \mathbf{n}_c$, Eq. (8) states the well known fact that the angular distribution is symmetric about the production plane of the incident particle in the second reaction of the cascade. Equations (7) may be regarded as a generalization of this assertion.

In conclusion, we remark that our selection rules can also be obtained by Shirokov's method, but the present approach is simpler.

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*We remark that the first and second selection rules given by Shirokov are actually two different ways of stating the same rule.

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ON THE QUESTION OF THE UNIQUENESS OF PHASE ANALYSIS

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MINAMI[1] has given a transformation of the scattering matrix which leaves the differential cross section invariant for the case in which the colliding particles have spins 0 and $\frac{1}{2}$. The present note gives an analog of this transformation for all spins s_1 and s_2 of the colliding particles.

We express the scattering matrix in terms of the functions ${}_{s_1 s_2}Y^{jM}_{\alpha_1\alpha_2}(\mathbf{n})$ describing the state of a system of two particles whose total angular momentum is j. The component of j and the components of the spins of the two particles along the direction given by $\mathbf{n}$ are M, α_1, and α_2, respectively. In terms of these functions, the scattering matrix is[2,3]

$$M(\mathbf{n}_f, \mathbf{n}) = \sum_{\alpha j M} Y^{jM}_{\alpha_1\alpha_2}(\mathbf{n}_f)\,[Y^{jM}_{\alpha'_1\alpha'_2}(\mathbf{n}_i)]^*\, A^j_{\alpha_1\alpha_2\alpha'_1\alpha'_2},$$

$${}_{s_1 s_2}Y^{jM}_{\alpha_1\alpha_2}(\mathbf{n}) = \sum_{s,l}\langle s_1\alpha_1 s_2\alpha_2 \,|\, s_1 s_2 s a\rangle \times \langle s a l 0 \,|\, s l j a\rangle \sqrt{\frac{2l+1}{2j+1}}\;{}_{s_1 s_2}Y^{jM}_{sl}(\mathbf{n})$$

Let $S(\mathbf{n})$ be a rotation that carries the vector $\mathbf{n}$ into the third axis, and consider the functions $\varphi_{\sigma_1\sigma_2}(\mathbf{n})$ whose components are

$$[\varphi_{\sigma_1\sigma_2}(\mathbf{n})]_{\alpha_1\alpha_2} = D^{s_1}_{\alpha_1\sigma_1}(S^{-1}(\mathbf{n}))\, D^{s_2}_{\alpha_2\sigma_2}(S^{-1}(\mathbf{n})),$$

where $D^j_{m_1 m_2}(S)$ are the matrix elements of an irreducible representation of the three-dimensional rotation group.[4] These functions describe a state in which the first and second particles have spins whose components are σ_1 and σ_2, respectively, along $\mathbf{n}$.

The functions $Y^{jM}_{\alpha_1\alpha_2}(\mathbf{n})$ satisfy the relation

$$Y^{jM}_{\alpha_1\alpha_2}(\mathbf{n}) = \varphi_{\alpha_1\alpha_2}(\mathbf{n})\sqrt{\frac{2j+1}{4\pi}}\, D^j_{\alpha_1+\alpha_2, M}(S(\mathbf{n})),$$

so that the matrix element for the transition from the state $\varphi_{\alpha'_1\alpha'_2}(\mathbf{n}_i)$ to the state $\varphi_{\alpha_1\alpha_2}(\mathbf{n}_f)$ is*

$$(\alpha_1\alpha_2 \,|\, M \,|\, \alpha'_1\alpha'_2) = \sum_j A^j_{\sigma_1\alpha_2\alpha'_1\alpha'_2} \times \sqrt{\frac{2j+1}{4\pi}}\, D^j_{\alpha_1+\alpha_2, \alpha'_1+\alpha'_2}(S(\mathbf{n}_f)\, S^{-1}(\mathbf{n}_i)). \tag{1}$$