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Letters to the Editor

WAVE-FRONT VELOCITY IN ELECTRODYNAMICS CONTAINING HIGHER DERIVATIVES

L. G. IAKOVLEV

Uzbek State University

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THE wave-front velocity in Maxwell-Lorentz electrodynamics and in nonlinear electrodynamics has been the subject of several studies.[1,2] Some of the references cited used the method of Levi-Civita, which is the simplest and clearest. We propose to use this method to analyze electrodynamics with higher derivatives. We shall restrict our considerations to fourth-order differential equations for the potential.

First we use the Lagrangian formalism to obtain the general form of the equations of electrodynamics with higher derivatives. Since the correspondence principle must always remain fulfilled, let us assume that the Lagrangian density L depends on the two invariants $I_1 = H_{ik}H_{ik}/2$ and $I_2 = H_{ik,l}H_{ik,l}$, where, as usual,

$$H_{ik} = A_{k,i} - A_{i,k}. \tag{1}$$

This last expression gives us the first group of equations, namely*

$$H_{ik,l} + H_{kl,i} + H_{li,k} = 0. \tag{2}$$

The second group is obtained from the variationally derived Euler-Lagrange equations

$$\frac{\partial}{\partial x_k}\frac{\partial L}{\partial A_{i,k}} - \frac{\partial^2}{\partial x_k \partial x_l}\frac{\partial L}{\partial A_{i,kl}} = 0. \tag{3}$$

Writing out Eq. (3) using the relations

$$\frac{\partial L}{\partial A_{i,k}} = \frac{\partial L}{\partial I_1}\frac{\partial I_1}{\partial A_{i,k}}$$

etc., we obtain the general form of the second group of equations, namely

$$\begin{aligned} &AH_{ik,k} + BH_{ik,kll} + CH_{ik}H_{lm}H_{lm,k} + D(H_{ik}H_{lm,n}H_{lm,nk} + 2H_{mn}H_{mn,l}H_{ik,kl} + 2H_{mn}H_{mn,k}H_{ik,ll} + 2H_{mn}H_{ik,l}H_{mn,kl} \\ &+ 2H_{ik,l}H_{mn,k}H_{mn,l}) + \mathcal{E}(H_{mn,p}H_{mn,lp}H_{ik,kl} + H_{mn,p}H_{mn,kp}H_{ik,ll} + H_{ik,l}H_{mn,pk}H_{m,npl} + H_{ik,l}H_{mn,p}H_{mn,klp}) \\ &+ GH_{mn}H_{pr}H_{mn,k}H_{pr,l}H_{ik,l} + K(H_{mn}H_{mn\,k}H_{ik,l}H_{ns,p}H_{ns,pl} + H_{mn}H_{mn,l}H_{ik,l}H_{rs,p}H_{rs,pk}) + MH_{ik,l}H_{mn,p}H_{rs,t}H_{mn,pl}H_{rs,tk} = 0 \end{aligned} \tag{4}$$

$$(A = \partial L/\partial I_1, \quad B = 2\partial L/\partial I_2, \quad C = \partial^2 L/\partial I_1^2, \quad D = \partial^2 L/\partial I_1 \partial I_2,$$

$$\mathcal{E} = 2\partial^2 L/\partial I_2^2, \quad G = \partial^3 L/\partial I_1^2 \partial I_2, \quad K = \partial^3 L/\partial I_1 \partial I_2^2, \quad M = 2\partial^3 L/\partial I_2^3).$$

The wave front is a surface of weak discontinuity. In the present case all the H_{ik} and all but their very highest derivatives are continuous on the wave front, that is, all but the $H_{ik,lmn}$. According to the method we are using, we must find the differences which occur in Eqs. (2) and (4) when passing through the wave front. Writing $h_{ik,lmn}$ for the nonvanishing differences of the $H_{ik,lmn}$, Eqs. (2) and (4) give

$$\begin{aligned} &h_{ik,lmm} + h_{kl,imm} + h_{li,kmm} = 0, \\ &Bh_{ik,kll} + \mathcal{E}H_{ik,l}H_{mn,p}h_{mn,pkl} = 0. \end{aligned} \tag{5}$$

Let us consider a plane front. Let $E = E_x(z, t)$ and $H = H_y(z, t)$, which means that $H_{14}(x_3, x_4)$ and $H_{13}(x_3, x_4)$ do not vanish. Writing the relations

$$H_{ik,lm}(x_3 + \Delta x_3, x_4 + \Delta x_4)$$
$$= H_{ik,lm}(x_3, x_4) + H_{ik,lm3}\Delta x_3 + H_{ik,lm4}\Delta x_4$$

for points in front of and behind the wave front and taking the differences, we find that the discontinuities in the second derivatives give

$$\begin{aligned} h_{ik,lm4} &= ivh_{ik,lm3} \\ v &= \lim(i\Delta x_3/\Delta x_4) \text{ as } \Delta x_4 \to 0. \end{aligned} \tag{6}$$

It can be shown, using (2), (5), and (6), that the index 4 in Eq. (5) can be replaced by 3 if the additional factor iv is added (for instance, $h_{14,444} = v^4 h_{13,333}$). Then dividing by $h_{13,333}$ and writing the result in three-dimensional vector form, Eq. (5) gives

$$\begin{aligned} &(1 + 2\alpha E_{x,t}^2)v^4 - 4\alpha E_{x,t}E_{x,z}v^3 - 2(1 + 2\alpha E_{x,t}H_{y,z} \\ &- 4\alpha E_{x,z}^2)v^2 - 8\alpha H_{x,z}H_{y,z}v + 1 + 2\alpha H_{y,z}^2 = 0, \end{aligned} \tag{7}$$

where $\alpha = \mathcal{E}/B$.

Thus in electrodynamics with higher derivatives, as in nonlinear electrodynamics, a wave front has in general four velocities of propagation different from the velocity of light in vacuo.

A special case is $\mathcal{E} = 0$ (or $\alpha = 0$), i.e., $L = f(I_1) + bI_2/2$ (the electrodynamics of Bopp and Podolsky is of this kind). It is easily seen that in this case (7) yields $v = 1$, which is the velocity of light in vacuo.

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*Here A_k is the four-vector whose components are A_x, A_y, A_z, and $i\varphi$, and $A_{k,l} = \partial A_k/\partial x_l$; we set $c = 1$.

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SELECTION RULES IN REACTIONS INVOLVING POLARIZED PARTICLES

CHOU KUANG-CHAO

Joint Institute for Nuclear Research

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SIMON and Welton[1] and Shirokov[2] have obtained the selection rules for a reaction of the type $a + b \rightarrow c + d$ in the form of relations between the polarization vectors and tensors. They assume that the initial state is not polarized. The present communication gives a derivation of the selection rules for any arbitrarily-polarized initial state. We shall use Shirokov's notation[2] and assume that all the particles have nonvanishing rest mass.

Consider the statistical tensors of the final state in the $a + b \rightarrow c + d$ reaction,

$$\rho'(q_c, \tau_c, q_d, \tau_d;\ g_c g_a^{-1}), \tag{1}$$

which depend essentially on the parameters of the rotation $g_c g_a^{-1}$ which carries the z_a, y_a, x_a coordinate system into the z_c, y_c, x_c coordinate system. The first of these systems is associated with the initial state, the z_a axis being parallel to $\mathbf{n}_a$, and the y_a axis being perpendicular to the production plane of particle a. The second of these systems has z_c parallel to $\mathbf{n}_c$, and y_c in the direction of the cross product $\mathbf{n}_a \times \mathbf{n}_c$. Here $\mathbf{n}_i$ is the unit vector along the direction of motion of particle i, and q is the rank of the statistical tensors. The spin indices τ are defined in terms of these particular coordinate systems. With this choice of coordinate systems, the Euler angles for the rotation $g_c g_a^{-1}$ are $\{-\pi, \theta_c, \pi - \varphi_c\}$, where θ_c and φ_c are the spherical angles of the unit vector $\mathbf{n}_c$ in the z_a, y_a, x_a coordinate system (see Shirokov[2]).

Let the state obtained from the initial one by space reflection be characterized by the statistical tensor ρ'_I. Under the reflection the z_a and z_c axes, chosen along the momenta of particles a and c, change direction, while the y_a and y_c axes remain invariant. The spherical angles θ_{cI} and φ_{cI} of the reflected $-\mathbf{n}_c$ vector in the reflected $\{z_a, y_a, x_a\}_I$ coordinate system are

$$\vartheta_{cI} = \vartheta_c,\ \ \varphi_{cI} = -\varphi_c. \tag{2}$$

The spin operators remain invariant under reflection. If θ_c and φ_c are replaced by θ_{cI} and φ_{cI} in Eq. (1), we obtain the ρ'_I statistical tensors from ρ'; the spin indices τ of the new ρ'_I tensors must be quantized with respect to the old nonreflected z_c, y_c, x_c system. Since the reflected $\{z_c, y_c, x_c\}_I$ coordinate system differs from the initial one only by rotation through an angle π about the y_c axis, the transformation properties of the statistical tensors[2] lead to the equations

$$\rho'_I(q_c, \tau_c, q_d, \tau_d; \vartheta_c, -\varphi_c) = \sum_{\tau'_c \tau'_d} D^{q_c}_{\tau_c \tau'_c}(0, \pi, 0)\, D^{q_d}_{\tau_d \tau'_d}(0, \pi, 0) \times \rho'(q_c, \tau'_c, q_d; \tau'_d; \vartheta_c, \varphi_c). \tag{3}$$

Here the spin indices τ are quantized with respect to their own proper coordinate systems. Since $D^q_{\tau\tau'}(0, \pi, 0) = (-1)^{q+\tau}\delta_{\tau,-\tau'}$ (see Shirokov[2] and Gel'fand and Shapiro[3]), Eq. (3) leads to

$$\rho'_I(q_c, \tau_c, q_d, \tau_d; \vartheta_c, -\varphi_c) = (-1)^{q_c+\tau_c+q_d+\tau_d}\rho'(q_c, -\tau_c, q_d, -\tau_d; \vartheta_c, \varphi_c). \tag{4}$$

The law of parity conservation may be stated in the following way: if the initial statistical tensors ρ are replaced by the reflected tensors ρ_I, the tensors ρ'' of the products of the reaction are the tensors ρ'_I which are obtained from ρ' by Eq. (4). In other words, if the statistical tensors ρ' are written $\rho' = F(\rho)$, then

$$\rho'_I = F(\rho_I). \tag{5}$$