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CERENKOV RADIATION IN THE PASSAGE OF A CHARGED PARTICLE THROUGH A FERRITE

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The Cerenkov radiation produced by a charged particle which moves through a medium with gyrotropic magnetic permeability is considered. Losses due to spin-wave excitation by the particle are computed.

1. The electromagnetic radiation produced by the motion of a charged particle in a ferrite (ferrodielectric) is distinguished by a number of features which derive from the dispersion properties of the ferrite. In the gyrotropic magnetic permeability tensor $\hat{\mu}$, which characterizes the ferrite

$$\hat{\mu} = \begin{pmatrix} \mu_1, & i\mu_2 & 0 \\ -i\mu_2, & \mu_1 & 0 \\ 0 & 0 & \mu_3 \end{pmatrix}, \tag{1}$$

the components μ_1 and μ_3 approach unity as the frequency increases, whereas μ_2 approaches zero at relatively low frequencies.[1] For this reason the above-mentioned features can be observed only in the low-frequency region ($\omega \sim 10^{10}$ to 10^{11} sec^{-1}). Under these conditions the gyrotropic effects and the anisotropic effects[2] can lead to Cerenkov radiation at low particle velocities.

The tensor $\hat{\mu}$ does not give a complete description of the dispersion properties of a ferrite medium, because it does not take account of the possible propagation of spin waves. Spin-wave excitation can occur at almost any particle velocity. The energy losses associated with this excitation are of the same nature as polarization losses but are considerably smaller in magnitude.

It is the purpose of this note to investigate the effect of the gyrotropic properties of the magnetic permeability tensor on energy loss in the motion of a charged particle through a ferrite. For simplicity we assume that the motion takes place along the gyrotropic axis (z axis).

The radiation of a charged particle in a gyrotropic medium has been considered by Kolomenskii and Sitenko[3] who treated the gyrotropic electrical effects rather than the magnetic effects. Magnetic effects in the Cerenkov radiation have been examined in references 4 and 5. In reference 4 the magnetic premeability was assumed isotropic; in reference 5 it was characterized by an anisotropic symmetric tensor.

2. We shall assume that the ferrite is electrically isotropic. In point of fact the gyrotropic electrical effects generally found in ferrites should be manifest at frequencies associated with the electronic dispersion of the dielectric permittivity $\hat{\epsilon}$. As a rule, the electronic dispersion in $\hat{\epsilon}$ and the gyrotropic effect in $\hat{\mu}$ occur in different frequency regions so that in the present work we shall consider the gyrotropic magnetic effect only. These results, however, do not apply in the optical region. On the other hand, $\mu_{ik} \to \delta_{ik}$ in the optical region and the results obtained by Kolomenskii and Sitenko[3] can be used to calculate losses at these frequencies.

We calculate the intensity of the Cerenkov radiation in the usual way (cf. reference 1):

$$\begin{aligned} \frac{dW}{dz} = &-\frac{e^2}{c^2}\int\limits_{\mathrm{I}} \mu_1\left(1-\frac{1}{\beta^2\varepsilon\mu_1}-\frac{\mu_2^2}{\mu_1^2}\right)\omega d\omega - \frac{e^2}{2c^2}\int\limits_{\mathrm{II}}\Big\{\mu_1\left(1-\frac{1}{\beta^2\varepsilon\mu_1}-\frac{\mu_2^2}{\mu_1^2}\right) \\ &+\frac{1}{\beta^2\varepsilon|\mu_1|}\{(\beta^2\varepsilon\mu_1-1)^2(\mu_1-\mu_3)^2+2\beta^2\varepsilon\mu_2^2[(1+\beta^2\varepsilon\mu_1)\mu_3-\mu_1(\beta^2\varepsilon\mu_1-1)]+\beta^4\varepsilon^2\mu_2^4\}^{1/2} \\ &+\frac{\mu_3\{(\beta^2\varepsilon\mu_1-1)^2(\mu_1-\mu_3)-\beta^2\varepsilon\mu_2^2(1+\beta^2\varepsilon\mu_1)\}}{\beta^2\varepsilon|\mu_1|\{(\beta^2\varepsilon\mu_1-1)^2(\mu_1-\mu_3)^2+2\beta^2\varepsilon\mu_2^2[(1+\beta^2\varepsilon\mu_1)\mu_3-\mu_1(\beta^2\varepsilon\mu_1-1)]+\beta^4\varepsilon^2\mu_2^4\}^{1/2}}\Big\}\omega d\omega, \end{aligned} \tag{2}$$

where the region of integration is given in the table for $\mu_3 > 0$.

Region	$\mu_1<0$	$\mu_1>0$	Number of cones
I	$\beta^2\varepsilon\lvert\mu_1\rvert+1<\beta^2\varepsilon\lvert\mu_2\rvert$	$\beta^2\varepsilon\mu_1-1>\beta^2\varepsilon\lvert\mu_2\rvert$	2
II	$\beta^2\varepsilon\lvert\mu_1\rvert+1>\beta^2\varepsilon\lvert\mu_2\rvert$	$-\beta^2\varepsilon\lvert\mu_2\rvert<\beta^2\varepsilon\mu_1-1<\beta^2\varepsilon\lvert\mu_2\rvert$	1

The conditions given in the table do not take account of the fact that macroscopically we are not actually considering short waves.[2] This limitation must be taken into account close to points at which the integrand diverges. By introducing a finite attenuation we avoid the divergence and obtain a determinate result.

When $\mu_2 = 0$ the integral vanishes in region II and we obtain the result given by Sitenko:[4]

$$\frac{dW}{dz} = -\frac{e^2}{c^2}\int\limits_{\beta^2\varepsilon\mu_1>1} \mu_1\left(1-\frac{1}{\beta^2\varepsilon\mu_1}\right)\omega d\omega. \tag{3}$$

3. In considering Cerenkov radiation in the radio-frequency region (wavelengths greater than 0.1 to 1 mm) we use a simplified model in which the ferrite is characterized by a spontaneous magnetization M. Then, following reference 7, the quantities μ_1 and μ_2 are given by

$$\mu_1 = \frac{\omega_1^2-\omega^2+2i\omega\lambda(H_e+2\pi M)/M}{\omega_0^2-\omega^2+2i\omega\lambda H_e/M};$$

$$\mu_2 = \frac{\omega}{\omega_0}\,\frac{\omega_1^2-\omega_0^2}{\omega_0^2-\omega^2+2i\omega\lambda H_e/M}; \quad \mu_3 = 1; \tag{4}$$

$$\omega_0 = gH_e, \quad \omega_1 = g\sqrt{H_eB}; \quad B = H_e+4\pi M, \ H_e = H+\beta M$$

where H is the magnetic field applied along the axis of easiest magnetization, β is the anisotropy constant and λ is the relaxation frequency in the Landau-Lifshitz equation.

Substitution of Eq. (4) in Eq. (2) (with $\lambda \ll \omega_0$) yields

$$\frac{dW}{dz} = -\frac{e^2}{c^2}\int\limits_{\mathrm{I}}\left(1-\frac{1}{\beta^2\varepsilon}+\omega_1^2\frac{\mu_0-1}{\omega_1^2-\omega^2}\right)\omega d\omega$$

$$-\frac{e^2}{2c^2}\int\limits_{\mathrm{II}}\left\{1-\frac{1}{\beta^2\varepsilon}+\omega_1^2\frac{\mu_0-1}{\omega_1^2-\omega^2}\right. \tag{5}$$

$$\left.+\frac{\omega_1^2(\beta^2\varepsilon\mu_0-1)^2-\omega^2[(\beta^2\varepsilon-1)^2\mu_0-(\mu_0-1)(1+\beta^2\varepsilon)]}{2\beta^2\varepsilon\left[\beta^2\varepsilon\mu_0\frac{\omega^2}{\omega_1^2}+\frac{1}{4}(\beta^2\varepsilon\mu_0-1)^2\right]^{1/2}\lvert\omega_1^2-\omega^2\rvert}\right\}\omega d\omega,$$

where μ_0 is the value of $\mu_1(\omega)$ at $\omega = 0$.

The limits of the regions of integration which are determined by the radiation conditions (cf. table), can be obtained by using the explicit forms of $\mu_1(\omega)$ and $\mu_2(\omega)$. A characteristic feature of Cerenkov radiation in a ferrite is the existence of a low-frequency spectrum at small particle velocities. In terms of the velocity of the charged particle, $\beta = v/c$, the limits of the low-frequency radiation region are given by

$$\text{for } 0<\beta n\leqslant 1: \quad \Omega<\omega<\omega_1$$

$$\text{for } 1\leqslant\beta n\leqslant\mu_0^{1/4}: \quad 0<\omega<\omega_1$$

$$\text{for } \mu_0^{1/4}\leqslant\beta n\leqslant\mu_0^{1/2}: \quad 0<\omega<\Omega.$$

Here

$$n=\sqrt{\mu_0\varepsilon_0}, \quad \varepsilon_0=\varepsilon\rvert_{\omega=0}, \quad \Omega=\omega_0\lvert\beta^2n^2-1\rvert/\lvert\beta^2\varepsilon_0-1\rvert.$$

The integral in Eq. (5) must be computed before we can obtain the total intensity of the low-frequency radiation. Computing this integral we obtain the following results:

for $0 < \beta n < 1$:

$$\frac{dW}{dz} = -\frac{e^2}{2c^2}\omega_1^2(\mu_0-1)\times\left[\ln\frac{\omega_1}{2\lambda}\frac{(\mu_0-1)(1+\beta^2\varepsilon_0\mu_0)}{\mu_0(1-\beta^2\varepsilon_0)}-1-\frac{\mu_0-1}{2\mu_0(1-\beta^2\varepsilon_0)}\right], \tag{6}$$

for $1 < \beta n < \mu_0^{1/4}$:

$$\frac{dW}{dz} = -\frac{e^2}{2c^2}\omega_1^2(\mu_0-1)\ln\frac{\omega_1}{2\lambda}\frac{(1-\beta^2\varepsilon_0)(1+\beta^2\varepsilon_0\mu_0)}{\beta^2\varepsilon_0(\mu_0-1)}$$

$$-\frac{e^2}{4c^2}\omega_1^2\frac{2(\mu_0-1)(\beta^6\varepsilon_0^3\mu_0-1)+2\mu_0(\beta^2\varepsilon_0-1)^2-(\beta^4\varepsilon_0^2\mu_0-1)^2}{\beta^4\varepsilon_0^2\mu_0(1-\beta^2\varepsilon_0)}, \tag{7}$$

for $\mu_0^{1/4} < \beta n < \mu_0^{1/2}$:

$$\frac{dW}{dz} = -\frac{e^2}{2c^2}\omega_1^2(\mu_0-1)\ln\frac{\omega_1}{2\lambda}\frac{(1-\beta^2\varepsilon_0)(1+\beta^2\varepsilon_0\mu_0)}{\beta^2\varepsilon_0(\mu_0-1)}$$

$$-\frac{e^2}{4c^2}\omega_1^2\frac{4\beta^2\varepsilon_0(\beta^2\varepsilon_0-1)+2\beta^6\varepsilon_0^3(\mu_0-1)+\frac{1}{\mu_0}(1-\beta^8\varepsilon_0^4\mu_0^4)}{\beta^4\varepsilon_0^2(1-\beta^2\varepsilon_0)}. \tag{8}$$

We have not considered the Cerenkov-radiation intensity for particle velocities which satisfy the condition $\beta n > \mu_0^{1/2}$ because in this case there is no separate low-frequency region (isolated in terms of frequency) and it is necessary to introduce the actual frequency dependence of ϵ to obtain the integrated intensity.

4. In deriving Eqs. (5) to (8) we used Eq. (4) which was derived assuming a simplified model for the ferrite. Actually, a ferrite must be considered as a system of magnetized sub-lattices. Analyzing the motion of the magnetic moments of each of these sub-lattices in a high-frequency magnetic field, we find the following expressions for μ_1 and μ_2:

$$\mu_1 = (\omega^2 - \omega_{1a}^2)(\omega^2 - \omega_{2a}^2)/(\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2),\ \mu_2 = -4\pi\omega(g_1M_1 + g_2M_2)(\omega^2 - \omega_{3a}^2)/(\omega^2 - \omega_1^2)(\omega^2 - \omega_2^2),\ \mu_3 = 1;$$
$$\omega_1 = g_1g_2(M_2H_{A1} + M_1H_{A2})/(g_1M_1 + g_2M_2),\ \omega_{1a}^2 = \omega_1[\omega_1 + 4\pi(g_1M_1 + g_2M_2)],$$
$$\omega_2 = -\alpha(g_1M_2 + g_2M_1) + \frac{g_1^2M_2H_{A1} + g_2^2M_1H_{A2}}{g_1M_2 + g_2M_1} - 2\frac{g_1g_2M_1M_2(g_1H_{A1} - g_2H_{A2})^2}{\alpha(g_1M_2 + g_2M_1)^3}, \tag{9}$$
$$\omega_{2a}^2 = \omega_2^2 + 4\pi(g_1 - g_2)^2|\alpha M_1M_2, \qquad \omega_{3a}^2 = \omega_2^2 + (g_1 - g_2)^2|\alpha M_1M_2|(\omega_1 + \omega_2)/(g_1M_1 + g_2M_2),$$

where $H_{Aj} = \beta_j M_j + H$ and M_j, β_j and g_j are respectively the magnetic moment per unit volume, the anisotropy constant and the gyromagnetic ratio for the j-th sub-lattice. The dimensionless parameter α characterizes the exchange reaction between the sub-lattices. In Eq. (9) it is assumed that the axes of easiest magnetization of the sub-lattices are parallel to each other and that the magnetic field H is along this direction.

It is apparent from Eq. (9) that "magnetic" Cerenkov radiation should be observed in the region of the high-frequency resonance ($\omega \sim \omega_2$)* as well as in the region of the usual ferromagnetic resonance ($\omega \sim \omega_1$). This result is of interest because ω_2 is a sensitive function of temperature; thus it is possible to obtain Cerenkov radiation in different regions of the spectrum (from radio frequencies to the infrared) at low particle velocities.

5. Up to this point we have not considered the possibility that spin waves can be excited by the particle. In other words, we have not taken account of the spatial dispersion in the tensor $\hat{\mu}$; this dispersion is a result of the exchange interaction between the spins.

Inasmuch as the coupling between the electromagnetic waves and the spin waves is weak,[9] we can compute the intensity of the spin-wave excitation by successive approximations, assuming zero electric field in the first approximation.

The following expressions, which give the loss due to spin-wave excitation, can be obtained by some rather laborious computations:

$$\frac{dW}{dz} = -\frac{1}{4\pi}\frac{e^2}{c^2}gM\frac{\Theta_c}{\hbar}\left(\frac{\Theta_c}{\mu M}\right)^2\left(\frac{v}{v_s}\right)^4 \times\left[\ln\left(\frac{\Theta_c}{\mu M}\frac{v^2}{v_s^2}\right) - 1\right], \quad \sqrt{\frac{\mu M}{\Theta_c}}v_s \gg v; \tag{10}$$

$$\frac{dW}{dz} = -\pi\frac{e^2}{c^2}gM\frac{\Theta_c}{\hbar}\left(\frac{v}{vs}\right)^2, \quad v_s \gg v \gg \sqrt{\frac{\mu M}{\Theta_c}}v_s; \tag{11}$$

$$\frac{dW}{dz} = -\pi^5\frac{e^2}{c^2}gM\frac{\Theta_c}{\hbar}\left(\frac{v_s}{v}\right)^2, \quad v \gg v_s, \tag{12}$$

where Θ_c is on the order of the Curie temperature and $v_s = \Theta_c a/h$ is the maximum ($k = 1/a$) phase velocity for the spin waves. Equation (10) to (12) are obtained under the assumption that $H \ll M$.

It is interesting to note that conservation of energy and conservation of momentum (the radiation conditions) permit excitation of spin waves at almost any particle velocity.

Comparing Eqs. (10) to (12) with the polarization losses in a dielectric, one is easily convinced that the excitation of spin waves by the charged particle is responsible for only a small part of the total energy loss. The polarization losses are of order $dW/dz \approx (e^2/c^2)\omega_p^2$ where $\omega_p^2 = 4\pi Ne^2/m$ ($\omega_p \sim 10^{15}\ \mathrm{sec}^{-1}$). Hence the ratio of spin-wave loss to polarization loss is of order $(gM/\omega_p) \times (\Theta_c/\hbar\omega_p) \ll 1$.

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*The high-frequency resonance in ferrites will be considered in detail in a separate paper.

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