SOVIET PHYSICS JETP VOLUME 35(8), NUMBER 3 MARCH, 1959

OPTICAL MODEL FOR THE INTERACTION OF NEUTRONS OF INTERMEDIATE ENERGY WITH NUCLEI

A. V. LUK'IANOV, Iu. V. ORLOV, and V. V. TUROVTSEV

Scientific Research Institute for Nuclear Physics, Moscow State University

Submitted to JETP editor April 11, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) 35, 750-756 (September, 1958)

An optical nuclear model is investigated in which the nucleus is described by a complex potential with a fall-off given by a third-degree polynomial. Model parameters have been found which yield the best agreement between the theoretical cross sections σ_t, σ_r, and σ_s and the corresponding experimental values for 14-Mev neutrons. For nuclei heavier than chromium the agreement is quite satisfactory. The angular distributions of elastically scattered neutrons computed with these parameters are also in satisfactory agreement with experiment. A preliminary study of the low-energy region indicates that the parameters depend weakly on the energy over a comparatively wide range.

At present, the most convenient model for the description of the interaction between nucleons and nuclei over a wide energy range is the optical model, in which the nucleus is described by a complex potential. The square-well potential, investigated by Feshbach, Porter, and Weisskopf[1] for the case of neutron scattering, leads at least to qualitative agreement between theory and experiment. However, it cannot reconcile simultaneously the total neutron cross sections, the reaction cross sections, and the elastic scattering cross sections (the reaction cross sections come out too low).[2,3] In this connection Nemirovskii, and thereafter several other authors, were led to use a model with diffuse boundary, which had some definite success. To this time, however, the choice of the form of the potential and the values of its parameters, as well as the limits of applicability of the optical model, cannot be regarded as definitely established. Nemirovskii[4] considers mainly the optical model with diffuse boundary in the region of small neutron energies. The most recent of the papers published on this model (Beyster, Walt, and Salmi[5]) treats various energies up to 14 Mev. However, these authors base their choice of parameters on insufficiently complete experimental data (only for four nuclei).

In this paper we use a very simple diffuse boundary potential different from all previous ones. It satisfies the condition of smoothness. The boundary region of the potential is described by a third-degree polynomial. Our aim is to find the parameters, independent of the mass number, for which the proposed model gives the best fit to the experimental data on the total cross sections σ_t, the reaction cross sections σ_r, and the ratios between the scattering cross sections σ_s and the reaction cross sections for neutrons of energy $E_n = 14$ Mev.[6-9]

1. SOLUTION OF THE SCATTERING PROBLEM

We investigate the nuclear potential of form:

$$U(r) = V(r) + iW(r) = -U_0(1 + i\zeta) f(r), \tag{1}$$

where

$$f(r) = \begin{cases} 1, & r \leqslant R - d & \text{(I)} \\ 1 + (r - R - 2d)(r - R + d)^2 / 4d^3, & R - d \leqslant r \leqslant R + d & \text{(II)} \\ 0, & r \geqslant R + d & \text{(III)} \end{cases}$$

(see Fig. 1).

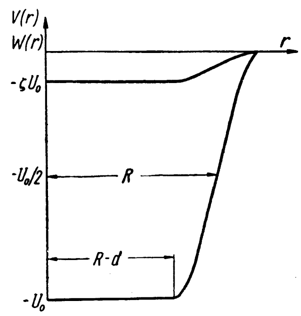

FIG. 1. Shape of the nuclear potential.

The equation for the radial part u(r) of the wave function is solved analytically in regions I and III and numerically in region II. In the computation it is expedient to introduce the dimensionless independent variable $x_0 = k_0 r$ $(k_0 = \sqrt{2MU_0/\hbar^2})$. We also lower the order of the equation by going over to the logarithmic derivative. We separate the real and imaginary parts of the equation. After these transformations we obtain the following system of first order equations with real coefficients:

$$v_1' = v_2^2 - v_1^2 - a^2 + l(l+1)x^{-2} - f(x), \quad v_2' = -2v_1v_2 - \zeta f(x), \tag{2}$$

where

$$v_1 = \mathrm{Re}(u'/u), \quad v_2 = \mathrm{Im}(u'/u), \quad a^2 = E_n/U_0.$$

Knowing, at the point $x_2 = k_0(R+d)$, the solution of the system (2) satisfying the boundary conditions at the point $x_1 = k_0(R-d)$ obtained by the formulas of reference 1, we can calculate the amplitudes of the scattered waves η_l.* Using the recurrence relations for the Hankel functions of half odd-integer order, we obtain the following working formulas for the amplitudes η_l:

$$\mathrm{Re}\,\eta_l = \frac{D^2 - A^2 + B^2 - C^2}{(A+D)^2 + (B-C)^2}, \quad \mathrm{Im}\,\eta_l = \frac{2(AB+CD)}{(A+D)^2+(B-C)^2},$$
$$A = a\alpha_l - v_1(x_2)\gamma_l, \quad B = a\beta_l - v_1(x_2)\delta_l, \quad C = v_2(x_2)\gamma_l, \quad D = v_2(x_2)\delta_l. \tag{3}$$

Here the quantities α_l, β_l, γ_l, δ_l are computed by the recurrence formulas

$$\gamma_{-1} = \cos x, \quad \gamma_0 = \sin y, \quad \gamma_i = \frac{2i-1}{y}\gamma_{i-1} - \gamma_{i-2} \quad (i = 1, 2, \ldots l),$$
$$\delta_{-1} = \sin y, \quad \delta_0 = -\cos y, \quad \delta_i = \frac{2i-1}{y}\delta_{i-1} - \delta_{i-2} \quad (i = 1, 2, \ldots l), \tag{4}$$
$$\alpha_l = \gamma_{l-1} - l\gamma_l/y, \quad \beta_l = \delta_{l-1} - l\delta_l/y,$$

where $y = ax_2$.

The solution of the system (2) for many values of the parameters is quite laborious. These calculations can successfully be carried out only with the help of a high-speed computing machine. We used the "Strela" computer of Moscow State University. The whole computational procedure to obtain η_l as well as the cross sections was completely automatic. The numerical solution of system (2) was obtained by the Runge-Kutta method with automatic selection of the step. The constants determining the accuracy were chosen such that the amplitudes η_l were guaranteed to be correct within four places after the decimal. In the cross section formulas l was summed from 0 to l_0, where l_0 was automatically chosen by the machine from the condition

$$\max\{(1 - \mathrm{Re}\,\eta_{l_0}), |\mathrm{Im}\,\eta_{l_0}|\} \leqslant 10^{-4}.$$

It appeared that l_0 varies from 4 or 5 to 12 or 13, depending on the nuclear radius and on the neutron energy. As should be expected, l_0 increases with the nuclear radius and with the neutron energy. As a check, all results were computed twice.

2. METHOD OF SELECTION OF THE PARAMETERS

Our model contains four independent parameters U_0, ζ, d, and R. The simultaneous variation of all these parameters is a very complicated task. Therefore, in the first step, we fixed the parameter of the depth of the potential at $U_0 = 42$ Mev. The chief justification for this is the U_0-R ambiguity,[1,5] which, apparently, is independent of the form of the fall-off of the potential (the change in the quantity U_0 within reasonable limits can be compensated for by a corresponding change in R, such as to leave the cross sections essentially the same).

The nuclear radius is given by the formula $R = (r_0A^{1/3} + \delta) \times 10^{-13}$ cm.[5] The theoretical cross sections were computed for eight values of R ($10^{13}R$ = 3.75, 4.05, 4.99, 5.39, 6.14, 6.63, 7.42, 8.01 cm) and various d and ζ. Thus it was possible to establish the dependence of $\sigma_t^{\mathrm{theoret}}$ and $\sigma_r^{\mathrm{theoret}}$ on the parameter d with various values ζ for each R. The cross section depends strongly on d: $\sigma_t^{\mathrm{theoret}}$ and $\sigma_r^{\mathrm{theoret}}$ increase with increasing d, where the rate of increase becomes faster as we go to heavier nuclei. In an analogous way the dependence of the cross sections on ζ for fixed values of the diffuseness parameter d was found. The reaction cross section increases, of course, with ζ. It is zero for $\zeta = 0$ and reaches an upper limit for sufficiently large ζ. A characteristic feature is the weak dependence of $\sigma_t^{\mathrm{theoret}}$ on ζ for fixed d: the total cross sections increase slowly with increasing ζ for some values R, and decrease slightly for others.

The method used for the selection of the parameters was the following. For a given pair of values r_0 and δ ($1 \le r_0 \le 1.5$) each of the eight values of R was set in correspondence, by formula (5), to a definite value A, and hence, to a definite pair

*The quantities η_l are connected with cross sections by the well-known relations:

$$\sigma_t = 2\pi\lambda\!\!\!^{-2} \sum_{l=0}^{\infty}(2l+1)(1 - \mathrm{Re}\,\eta_l), \quad \sigma_r = \pi\lambda\!\!\!^{-2} \sum_{l=0}^{\infty}(2l+1)(1 - |\eta_l|^2).$$

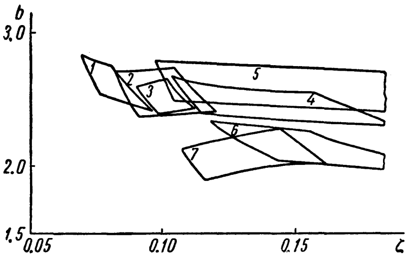

FIG. 2. Determination of the parameters $b = k_0 d$ and ζ for the given values $U_0 = 42$ Mev; $r_0 = 1.25$. $\delta = 0.42$ (the labels 1, 2, ..., 7 refer, respectively, to $A = 220, 173, 121, 62, 19, 47, 24$).

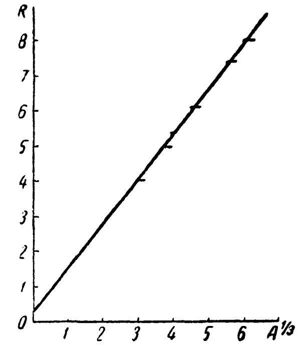

FIG. 3. Determination of the parameters r_0 and δ for fixed values $U_0 = 42$ Mev, $b = 2.5$, $\zeta = 0.12$.

of experimental cross sections $\sigma_t \pm \Delta\sigma_t$ and $\sigma_r \pm \Delta\sigma_r$ ($\Delta\sigma$ is the experimental error). In cases where there were no measurements, the cross sections were found by interpolation. This is possible on account of the sufficiently smooth change of the cross sections when going from one nucleus to the other, and the great number of nuclei investigated experimentally, particularly with respect to the total cross sections (54 nuclei in the region $2.08 \leq A^{1/3} \leq 6.2$). With the help of the curves representing the dependence of the cross sections on the parameters, figures (we call them "quadrangles") are then constructed in the ζd plane which define the region of values of the parameters d and ζ for which $\sigma_t^{\text{theoret}}$ and $\sigma_r^{\text{theoret}}$ agree with the experimental data within the limits of error. In order to fit the experimental data for the greatest possible number of nuclei by one and the same parameters, r_0 and δ have to be varied in this procedure until maximal closeness of the "quadrangles" is reached (see Fig. 2). In this case the common "center of gravity" of the "quadrangles" determines the required values of the parameters. During this operation a certain ambiguity was noted: several different sets of parameters r_0, δ, d, and ζ (d and ζ are restricted to a certain definite region) gave satisfactory agreement between the computed cross sections and the experimental data. A different method was used to choose the best among these sets of parameters. For given values of d and ζ in the above-mentioned region the dependence of R on $A^{1/3}$ was established. d and ζ were now fixed already in each single case, while r_0 and δ were to be determined. The given values R, d, and ζ determine the values of $\sigma_t^{\text{theoret}}$ and $\sigma_r^{\text{theoret}}$. Using, for example, the experimental curves for the total cross section, we determined the region of values of $A^{1/3}$ which correspond to a theoretical cross section such that $\sigma_t^{\text{theoret}} = \sigma_t^{\text{exptl}}$ within the experimental errors. The corresponding curve is represented in Fig. 3. Taking into account the weak dependence of $\sigma_t^{\text{theoret}}$ on ζ, we first chose the best d, based on the requirement that there be agreement between theoretical and experimental cross sections within the limits of error for the greatest possible number of values of R considered, i.e., that the short lines in the diagram $R = R(A^{1/3})$ lie on a straight line. The parameters of this straight line, determine, of course, the parameters r_0 and δ. The final choice of the parameter ζ was made by comparing the theoretical results using the obtained values of the parametrs d, r_0, and δ and various different values of ζ, with the experimental values of σ_r and $\sigma_0 = \sigma_t/\sigma_r - 1 = \sigma_s/\sigma_r$.

3. RESULTS AND DISCUSSION

The following parameter values were adopted as the best values as a result of this analysis:

$$U_0 = 42 \text{ Mev}, \; b = k_0 d = 2.5; \; \zeta = 0.12; \; r_0 = 1.27; \; \delta = 0.3 \qquad \text{(A)}$$

However, after the cross sections for a great number of nuclei were calculated with these parameters, it appeared that better agreement with experiment could be achieved with

$$r_0 = 1.25, \; \delta = 0.4 \qquad \text{(B)}$$

The comparison of the theoretical cross sections σ_t, σ_r, and σ_0 with the experimental results is shown in Figs. 4 and 5. The agreement is fully satisfactory for all intermediate and heavy nuclei, starting from chromium. The curve for $\sigma_0(A^{1/3})$ lies above the experimental values in the region of light nuclei. It could be lowered by increasing ζ, but this destroys the agreement for heavy nuclei. This discrepancy between theory and experiment for nuclei lighter than chromium may be connected with the dependence of the parameters on A in the region of light nuclei, or with the principal inapplicability of the proposed model in this region. This point requires special study.

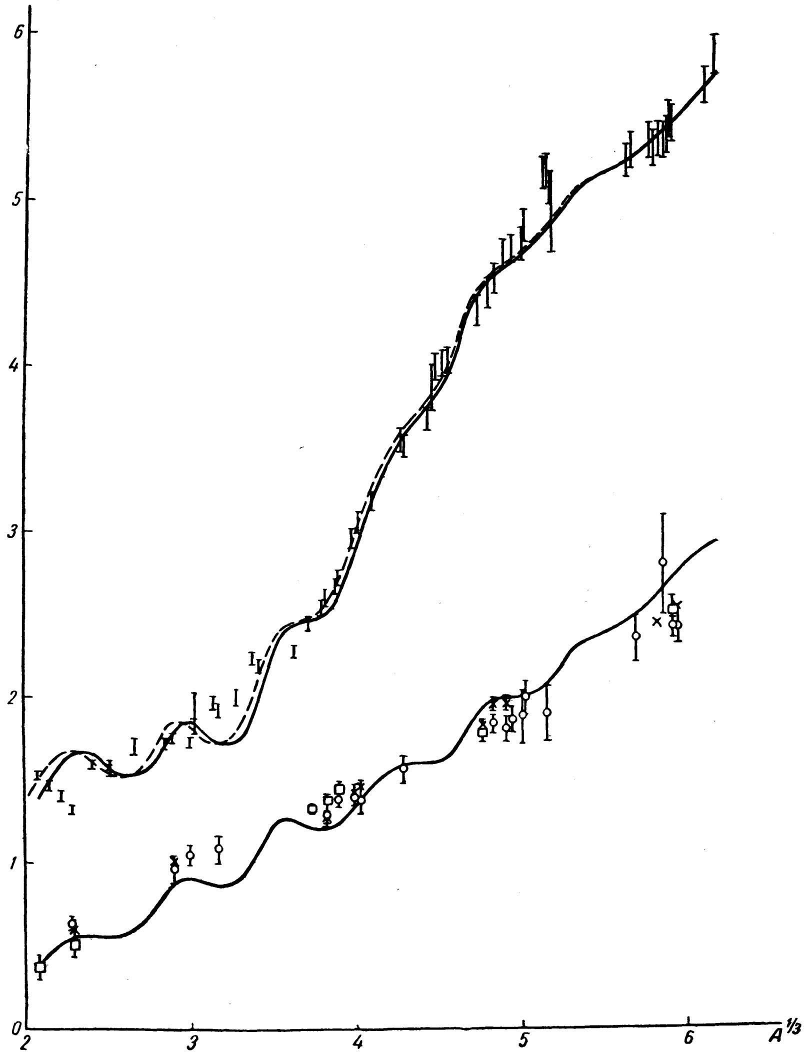

FIG. 4. Comparison of the theoretical and the experimental dependence of the cross section σ_t (upper curves) and σ_r (lower curve) on the mass number. Solid curve: theoretical cross sections computed with the parameter values (A); dotted curve: theoretical cross sections computed with the parameter values (B). Short lines: experimental data of reference 6; circles: reference 7; squares: reference 8; crosses: reference 9.

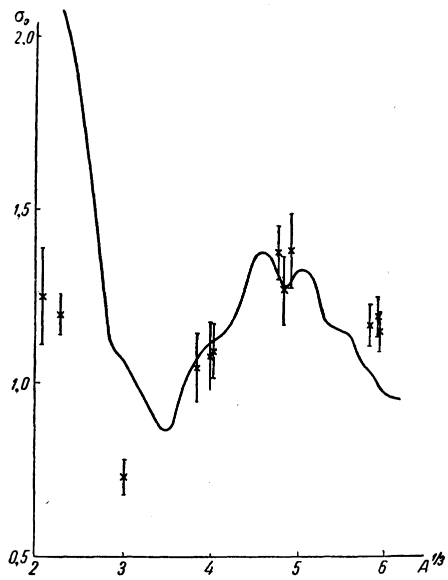

FIG. 5. Comparison of the theoretical and experimental[9] ratios between the scattering and reaction cross sections as a function of mass number (parameter A).

With the set of parameters (A) we calculated the differential cross sections for neutrons scattered elastically from the nuclei Al, S, Fe, Cu, In, Sn, Ta, Pb, and Bi (E_n = 14.1 Mev), and Mg, Ca, Cd, and Bi (E_n = 14.6 Mev) (see Fig. 6), for which the corresponding experimental data exist.[10,11] Although the parameters were chosen by studying only the values of σ_t, σ_r, and σ_0, the agreement between the angular distributions of the elastically scattered neutrons with the experimental data is not bad on the whole. The first maximum and the position of the maxima and minima are fitted rather well. For not too large angles the height of the maxima agrees well with experiment. The greatest discrepancey between theory and experiment shows up in the size of the minima, which are much deeper than the experimental data. This is independent of the character of the fall-off of the potential, since this occurs also for potentials of different type (see, for example, reference 5).

It is possible that there exist parameter values which yield an even better agreement between the differential cross sections and the experimental data without worsening the agreement for σ_t, σ_r, and σ_0 appreciably. The answer to this question requires a further careful analysis of the dependence of the theoretical differential cross sections on the model parameters. However, radical improvements can hardly be expected. In particular, the appearance of very deep minima in the theoretical angular distributions can apparently not be avoided. The same picture emerges for the most distinct values of d and ζ. Changing r_0 and δ also makes little difference. The dependence of the differential cross sections on $A^{1/3}$ was computed from 0° to 130° in steps of 10°, with b = 2.3 and ζ = 0.125. The corresponding curves have, starting from 30°, deep and wide minima, while changes in r_0 and δ reduce the depth of the minimum for one nucleus, and increase it for another.

4. OTHER ENERGIES

The reasons for choosing the experimental cross sections for neutrons with energy $E_n = 14$ Mev as the basic data for the determination of the model parameters are, firstly, the great amount of published experimental material, and, secondly, the circumstance that, at this energy, one can neglect the compound elastic scattering cross section[1] and compare the experimental cross sections for elastic scattering and reactions with the theoretical values immediately. Taking d, r_0, and δ to be energy-independent parameters, we made several calculations for smaller energies. The comparison of the calculated cross sections with the experimental data[5,6] gives the possibility of determining the dependence of the parameters U_0 and ζ on the energy. Preliminary results indicate that the parameters U_0 and ζ stay practically the same when the neutron energy is lowered to ~ 7 Mev. The parameter ζ becomes smaller, while the parameter U_0 increases slowly. For 3.5 Mev, ζ decreases by $\sim 30\%$, and U_0 increases by $\sim 5\%$, as compared to their values at the energy 14 Mev. As in reference 5, the model parameters U_0 and ζ depend weakly on energy over a rather wide energy range.

The present investigation of the optical model shows that a great amount of experimental data on the scattering of neutrons from nuclei can be described by the model of a complex potential with a fall-off given by a very simple function.

In conclusion we take occasion to express our deep gratitude to Prof. I. S. Shapiro for suggesting the topic and interest in this work, and to Prof. A. N. Tikhonov, under whose guidance the numerical part of this work was carried out. The authors also thank G. A. Samoilova for her great help and initiative in the programming and in the treatment of the results of the computations.

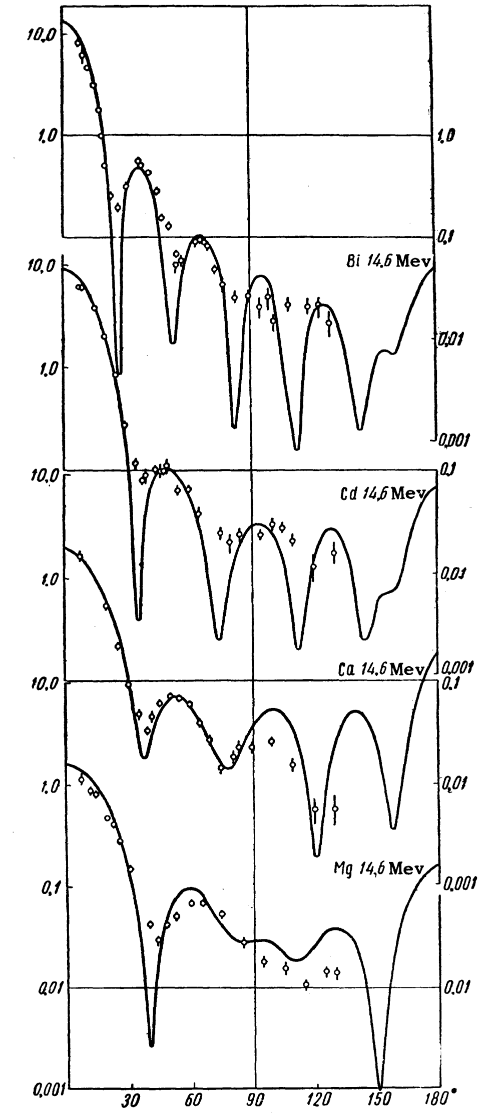

FIG. 6. Comparison of the theoretical angular distribution of the elastically scattered neutrons with the experimental data.[11] Abscissa: angle in dregees; ordinate: differential cross section in barns/sterad in logarithmic scale.

[1] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).

[2] Kawai, Ui, and Nagasaki, Progr. Theoret. Phys. **14**, 263 (1955).

[3] M. Walt and J. R. Beyster, Phys. Rev. **98**, 677 (1955).

[4] P. E. Nemirovskii, Reports of the Soviet Delegation to the International Conference on the Peaceful Use of Atomic Energy, Geneva, 1955, M., 1955, p. 287. J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 1143 (1957); Soviet Phys. JETP **5**, 932 (1957).

[5] Beyster, Walt, and Salmi, Phys. Rev. **104**, 1319 (1956).

[6] Coon, Graves, and Barschall, Phys. Rev. **88**, 562 (1952).

[7] V. I. Strizhak, Атомная энергия (Atomic Energy) **2**, 68 (1957)

[8] Taylor, Loensjoe, and Bonner, Phys. Rev. **100**, 174 (1955).

[9] E. R. Graves and R. W. Davis, Phys. Rev. **97**, 1205 (1955).

[10] J. O. Elliot, Phys. Rev. **101**, 684 (1956).

[11] Culler, Fernbach, and Sherman, Phys. Rev. **101**, 1047 (1956).

Translated by R. Lipperheide
147