ON THE SCATTERING OF "DRESSED" PARTICLES IN QUANTUM FIELD THEORY

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A scattering theory is considered in which the virtual "clouds" around the particles are always taken into account exactly. The theory is based on the representation of the basis functionals as products of single-particle functionals. In the absence of vacuum polarization the equations for the matrix elements are automatically renormalized, and the matrix elements can be expressed in terms of the single-nucleon matrix elements.

In scattering problems in quantum theory it is usually assumed that for $t = \pm\infty$ the particles do not interact with the vacuum. The adiabatic introduction of the interaction then includes both the interaction between the particles (by way of the field) and also the interaction with the vacuum. Since the particles always interact with the vacuum, it is interesting to consider the question of a scattering theory in which the interaction of the particles with the vacuum is at all times taken into account exactly (the scattering of "dressed" particles). This is also one of the possible approaches to a problem that has recently been much discussed, that of the construction of the S matrix on the basis of ideas about the physical particles, without the introduction of the concept of "bare" particles.[1]

As the first aspect of the problem of the scattering of "dressed" particles we can take the problem of constructing basis functionals Φ which asymptotically describe noninteracting dressed particles. In the present paper we consider from this point of view some problems of the scattering of dressed particles in the case in which the functionals Φ are represented in the form of products of single-particle functionals, which are solutions of the Schrödinger equation with interaction. Such functions have been discussed by Ekstein[2] and have been used by the present writer for the derivation of a two-nucleon potential depending on the phase shifts of the π-N scattering.

As soon as the system of basis functions Φ has been constructed, the apparatus of the formal scattering theory[2] at once leads to a system of nonlinear integral equations for the scattering amplitudes. A characteristic feature of this method is the presence in these equations of the matrix elements of the field operators between products of single-nucleon states. The method described below (in Sec. 2) for calculating such quantities is a generalization of the method we have proposed[3] earlier for the case of two bound nucleons.

The scattering scheme so developed takes proper-energy effects into account automatically; on the other hand it seems to be impossible to extend the scheme to the vacuum polarization without the explicit introduction of renormalization. This is a consequence of the assumption of the spatial localizability of the virtual field (only a "cloud" around the particles), which is the essential basis of the choice of the Φ. Therefore for practical calculations the method can be used for the treatment of the interaction of nucleons, mesons, and hyperons in the region of relatively low energies.

1. THE BASIS FUNCTIONS

Let us consider the case in which there is no vacuum polarization, and go over to the configuration space for the nucleons. The total Hamiltonian for n nucleons and antinucleons interacting with the meson field can be written in the form

$$H = H_\pi + \sum_i^n [H_N(i) + U_i], \tag{1}$$

where H_π is the Hamiltonian of the free meson field, $H_N(i)$ refers to the i-th "bare" nucleon, and U_i is the operator for the interaction of the i-th nucleon with the meson field, which we shall assume is linear in the meson creation operators a^+ and annihilation operators a:

$$U_i = \sum_q (V_{iq} a_q e^{i\mathbf{q}\cdot\mathbf{r}_i} + V_{iq}^+ a_q^+ e^{-i\mathbf{q}\cdot\mathbf{r}_i}). \tag{2}$$

Here q denotes all the quantum numbers of a meson. Let us consider the construction of the wave functions $\Phi(1, 2, \ldots n; \bar{a})$ of a system of n noninteracting dressed nucleons. Since the nucleons have meson clouds, Φ depends on the meson field variables $\bar{a}$, or

$$\Phi(1, 2 \ldots n; \bar{a}) = \Phi(1, 2 \ldots n; a^+)\Lambda_0,$$

where Λ_0 is the vacuum state vector. (Here the physical meson vacuum Λ_0 is the same as the mathematical state.) Strictly speaking, because of the identity of the mesons belonging to the meson clouds of different nucleons, one cannot define a state of noninteracting nucleons for arbitrary distances between them. Only for large distances R_{ij} between the nucleons does the effect of the identical nature of the mesons become negligible, so that the state of the noninteracting dressed nucleons can be defined only in the asymptotic sense for $R_{ij} \to \infty$.

It is easy to see that we cannot separate off from the Hamiltonian (1) any part corresponding to noninteracting dressed nucleons. Therefore $\Phi(1, 2, \ldots; \bar{a})$ must be a solution of the Schrödinger equation with the total Hamiltonian H for large relative distances,

$$\Big\{H - \sum_i E(i)\Big\}\Phi(1, 2, \ldots; a^+)\Lambda_0 \to 0, \quad R_{ij} \to \infty, \tag{3}$$

where $E(i)$ is the energy of the i-th free nucleon. Moreover, asymptotically Φ must also be an eigenfunction of the total momentum. The wave function Φ is constructed from products of single-nucleon state vectors $F_\alpha(1, \bar{a})$ (α denotes all the quantum numbers of a nucleon), which are eigenvectors of the energy and momentum operators:

$$H_i F_\alpha(i, \bar{a}) = \{H_N(i) + H_\pi + U_i\} F_\alpha(i, \bar{a}) = E_\alpha(i) F_\alpha(i, \bar{a}), \tag{4}$$

$$\{\hat{\mathbf{p}}_N + \mathbf{u}\} F_\alpha(i, \bar{a}) = \mathbf{p}_i^\alpha F_\alpha(i, \bar{a}), \tag{5}$$

where $\mathbf{u}$ is the momentum of the meson field and $\hat{\mathbf{p}}_N$ is the momentum operator of a nucleon. Furthermore, $F_\alpha(i, \bar{a})$ is an eigenfunction of the total spin operator. The wave function of the system of noninteracting dressed nucleons is then

$$\Phi_a(1, 2 \ldots; a^+)\Lambda_0 = \sum_{\alpha\beta\ldots} C^a_{\alpha\beta\ldots} F_\alpha(1, a^+) F_\beta(2, a^+) \ldots \Lambda_0, \tag{6}$$

where the coefficients $C^a_{\alpha\beta\ldots}$ are such that asymptotically Φ_a has the required symmetry properties.

Such a method for constructing Φ has much in common with the construction of functions in the theory of molecules. The role of the electrons, in whose coordinates one has to antisymmetrize, is now played by the mesons, in whose coordinates Φ is symmetric. The antisymmetrization with respect to the electrons belonging to the various nuclei destroys the orthogonality of the basis functions. Here, in just the same way, because of the identity of the mesons in all the one-nucleon states F_α, the functions Φ_a and Φ_b are not orthogonal, and so on.

Let us verify that the conditions (3) are satisfied. We introduce the notation

$$U_i = U_i^{(+)} + U_i^{(-)}, \tag{7}$$

where $U^{(+)}$ contains the absorption operators and $U^{(-)}$ the emission operators. Since the operator $h = H_\pi + \sum U_i^{(+)}$ is linear in the absorption operators, the quantity $[h, F(i, a^+)]$ does not depend on the operators a and commutes with any other $F(j, a^+)$. Using the equations (4), we have

$$\begin{aligned}\Big\{H - \sum_i E(i)\Big\} F(1, a^+) F(2, a^+) \ldots \Lambda_0 \\ = \sum_{i \neq j} F(1, a^+) F(2, a^+) \ldots [U_i^{(+)}, F(j, a^+)]\Lambda_0.\end{aligned} \tag{8}$$

According to Eq. (5),

$$F(j, a^+)\Lambda_0 = \exp\{i(\mathbf{p}_j - \mathbf{u})\cdot\mathbf{r}_j\} F^0(j, a^+)\Lambda_0, \tag{9}$$

where F^0 does not depend on the coordinates. Therefore it follows from Eq. (4) that

$$\begin{aligned} &U_i^{(+)} F_\alpha(j, a^+)\Lambda_0 \\ &= -\sum_q \sum_n \exp\{i\mathbf{p}_\alpha\cdot\mathbf{r}_j + i\mathbf{q}\cdot(\mathbf{r}_i - \mathbf{r}_j)\} \\ &\times \frac{V_{iq} F^{(n)}(j)\,(F^{(n)}(j), V^+_{jq} F_\alpha(j, \bar{a}))}{E_n + \omega_q - E_\alpha},\end{aligned} \tag{10}$$

where $F^{(n)}(j)$ is the eigenstate of the Hamiltonian H_j with the momentum $\mathbf{p}_n = \mathbf{p}_\alpha - \mathbf{q}$, and the energy $E_n = (M_n^2 + p_n^2)^{1/2}$, $M_n > M_N$; and $\omega_q = (1 + q^2)^{1/2}$. The denominator in Eq. (10) cannot vanish. For $|\mathbf{r}_i - \mathbf{r}_j| \to \infty$ the quantity (10) decreases exponentially. In a similar way one can verify that the function Φ is asymptotically an eigenfunction of the total momentum or angular momentum.

The basis functions Φ_A for states in which there are real mesons besides the nucleons are obtained from the expression (6) by multiplying by a suitable number of meson creation operators:

$$\Phi_A = a^+_{q_1} a^+_{q_2} \ldots \Phi_a; \quad A \equiv (q_1, q_2 \ldots; a). \tag{11}$$

To describe a meson spatially remote from the other particles one must take a wave packet, i.e., multiply Φ_a not by a^+_q, but by

$$f(r) = \sum_q c_q e^{-i\mathbf{q}\cdot\mathbf{r}} a^+_q.$$

Then asymptotically Φ_A will be a solution of the

Schrödinger equation. In fact, multiplying $\{H - \Sigma E(i) - \omega_q\} a_q^+ \Phi_a$ on the left by $\Sigma e^{-iqr} c_q$, we get on the right, in addition to terms of the type shown in Eq. (8), also terms that contain factors

$$\sum_i [U_i^{(+)}, f(r)] = \sum_i \sum_q e^{iq(r_i - r)} V_{iq} c_q,$$

that vanish for $|\mathbf{r} - \mathbf{r}_i| \to \infty$ in virtue of the properties of the coefficients c_q of the wave packet. Because of the asymptotic character of the conditions imposed on Φ, there still remains some arbitrariness in the expression for Φ. The expression (6) can be multiplied by a function of the relative distances R_{ij} that goes to unity for $R_{ij} \to \infty$. This arbitrariness is of no importance when we deal with scattering by the methods of quantum field theory, and becomes important when we go over to the phenomenological treatment (see Sec. 3).

2. THE EQUATIONS FOR THE TRANSITION AMPLITUDES

In the case of scattering of "dressed" particles we cannot define[2] the scattering matrix S in terms of an operator acting on the asymptotic states of the "dressed" particles. In this case the S matrix is defined by the relation

$$S_{BA} = (\Psi_B^{(-)}, \Psi_A^{(+)}) = \delta_{BA} - 2\pi i \delta (E_B - E_A) R_{BA}^{(\pm)}, \quad (12)$$

where $\Psi^{(\pm)}$ are the eigenfunctions of the Hamiltonian H with the respective boundary conditions:

$$\Psi_A^{(\pm)} = \Phi_A - \frac{1}{H - E_A \mp i\varepsilon} [(H - E_A) \Phi_A]. \quad (13)$$

According to the formal scattering theory[2] the scattering amplitudes

$$R_{BA}^{(-)} \equiv R_{BA} = (\Psi_B^{(-)}, (H - E_A) \Phi_A) \quad (14)$$

satisfy a system of nonlinear integral equations. These equations have the form:

$$\begin{aligned} R_{BA} = (\Phi_B, (H - E_A)\Phi_A) - \sum_n \frac{R_{nB}^* R_{nA}}{E_n - E_B - i\varepsilon} \\ - \sum_d \frac{(\chi_d, (H - E_B)\Phi_B)^* (\chi_d, (H - E_a)\Phi_a)}{E_d - E_B}, \end{aligned} \quad (15)$$

where the χ_d refer to the bound states, and the first sum is taken over all states n of the continuous spectrum of H. In the case of a single nucleon Eq. (15) is nothing other than the Low equation. For a larger number of nucleons the formalism with the equations (15) departs from the usual formalism of the S matrix; this can be seen in particular from the fact that matrix elements with the wave functions Φ in the form of products of single-nucleon functions are not encountered in the usual theory of the S matrix.

Let us examine the inhomogeneous terms in the equations (15). According to Eq. (11)

$$\begin{aligned} R_{BA}^0 &= (\Phi_B, (H - E_A) \Phi_A) \\ &= (\Phi_b, a_{q_1'} a_{q_2'} \ldots (H - E_A) a_{q_1}^+ a_{q_2}^+ \ldots \Phi_a). \end{aligned} \quad (16)$$

In Eq. (16) let us displace all the creation operators to the left to Φ_b and all the annihilation operators to the right to Φ_a, and write R_{BA}^0 in the form

$$R_{BA}^0 = (\Phi_b, L(a, a^+) \Phi_a), \quad (17)$$

where the operator L contains only normal products of the creation and annihilation operators. Here the wave functions Φ_b and Φ_a, which describe noninteracting dressed nucleons (Eq. (6)) depend on the meson variables only in a small volume around a nucleon (the "meson cloud"). Therefore in calculating the expression (17) we may use the method of introducing individual coordinates for the mesons in the cloud of each nucleon.[3] We introduce a_{jq} and a_{jq}^+, the creation and annihilation operators for a meson in the cloud of the j-th nucleon,

$$[a_{jq}, a_{ik}^+] = \delta_{ij} \delta_{qk}; \; [a_{jq}, a_{ik}] = 0. \quad (18)$$

The state vector of a single nucleon is then written as $F(i, a_i^+) \Lambda_0$, and the state vector for n free nucleons (with neglect of the identical nature of the mesons in the clouds of different nucleons) has the form

$$\Phi_a^0 = \sum_{\alpha\beta\ldots} C_{\alpha\beta\ldots}^a F_\alpha (1, a_1^+) F_\beta (2, a_2^+) \ldots \Lambda_0. \quad (19)$$

The vectors Φ^0 are orthogonal. Asymptotically Φ_a coincides with Φ_a^0. Our problem is to reduce the matrix element (17) between the states Φ_b and Φ_a (i.e., with account taken of the identical nature of the mesons in the clouds of different nucleons) to matrix elements between states Φ_b^0 and Φ_a^0 (i.e., with only mesons belonging to the same nucleon treated as identical particles). This problem is solved in general form by Eq. (A.1) of the Appendix. We have

$$(\Phi_b, L(a, a^+) \Phi_a) = (\Phi_b^0, :[1 + \hat{N}] L(A, A^+): \Phi_a^0), \quad (20)$$

where :PT: denotes the normal product of the operators P and T, and A^+ and A are sums of creation and annihilation operators:

$$A_q = \sum_i a_{iq}; \; A_q^+ = \sum_i a_{iq}^+. \quad (21)$$

The weight operator $\hat{N}$ has the form

$$1 + \hat{N} = :\exp \sum_{i \neq j} \sum_q a_{iq}^+ a_{jq}:. \quad (22)$$

In Eq. (20) each of the meson operators occurring in $:L(1+\hat{N}):$ acts only on the state vector of a single nucleon. Therefore the matrix element (20) is expressed in terms of products of one-nucleon matrix elements of normal products of meson operators a, a^+ and vertex operators V:

$$(F_\alpha, a_q^+ \ldots a_k \ldots V_s \ldots F_\beta). \tag{23}$$

We obtain the simplest matrix elements by omitting $\hat{N}$. Each successive term in the expansion of $1+\hat{N}$ adds one meson operator in the one-nucleon matrix elements (23), so that the inhomogeneous term (17) is represented as an infinite sum of products of one-nucleon matrix elements of the type (23). The operator $:(1+\hat{N})L:$ relates only to the interaction of the particles; all self-energy effects contained in L are eliminated automatically, since we are using functionals with the eigenfields F. The operator N itself describes the exchange of virtual mesons between the various nucleons ($i \neq j$ in Eq. (22)).

The rule (20) for calculating matrix elements of field operators between products of one-nucleon functionals completes the specification of the system of equations (15) for the transition amplitudes R_{BA}, in the sense that it expresses the inhomogeneous terms in the system in terms of quantities well known in the quantum field theory, the single-nucleon matrix elements. If the solution of the system (15) exists, we accordingly require, in order to find it, a knowledge of the one-nucleon problem, i.e., of the quantities (23). The charge renormalization also relates to the one-nucleon problem, since a knowledge of $(F_\alpha, V_s F_\beta)$ is required for the determination of the renormalized charge. All the one-nucleon matrix elements are already renormalized. In the static theory the quantities (23) can be related to the scattering amplitudes on the energy surface, and the charge renormalization is trivial.[4]

A possible method for the solution of the system (15) with the inhomogeneous terms in the form (20) is an expansion in terms of the number of mesons (if we disregard perturbation theory). We may assume that the larger the number of mesons by which a state n differs from the initial and final states A and B, the smaller is the part played by this state, and on this basis we can throw out some of the terms of the sum in Eq. (15). We must then break off the expansion of $1+\hat{N}$ in a similar way. When this is done, however, if we express the quantities (23) in terms of matrix elements $(F_\alpha, V_s F_\beta)$, $(F_\alpha^q, V_s F_\beta)$ (F_α^q is a state with one nucleon and one meson) and the other simplest one-nucleon quantities that can be taken as a basis, then in general each of the quantities (23) is a series involving summation over the eigenstates of the single-nucleon Hamiltonian. From this series also one must take only a suitable number of terms. The range of usefulness of such a method can be judged only from the results. We note that the series (22) that defines the operator $1+\hat{N}$ arose as a result of the expansion of the one-nucleon state vector $F(i, A^+)$ in a series in $A^+ - a_i^+$. The successive terms of the expansion will fall off rapidly in cases in which the meson clouds of the various nucleons "overlap" only slightly, so that the mesons in a cloud interact much more strongly with "their own" nucleon than with "a different" nucleon. Therefore it is to be expected that the higher the energy of the colliding nucleons (i.e., the greater the "overlapping" of the clouds) the more terms in the series (22) will have to be taken into account.

If the number and types of the particles do not change in the collision some simplification is possible. In this case one can use a treatment in which the methods of quantum field theory are used for the calculation of an effective potential, and the scattering amplitude is then determined by solving the Schrödinger equation with this potential. Such an approach is possible only for the region of moderate energies, in which the concept of a potential can be used and the polarization of the vacuum can be neglected.

3. DETERMINATION OF THE EFFECTIVE POTENTIAL

Let us denote by K + W the total Hamiltonian in the phenomenological treatment, where W is the potential. For the eigenfunctions $\psi_a^{(\pm)}$ of the operator K + W that belong to the continuous spectrum, the formal scattering theory gives

$$\psi_a^{(\pm)} = \psi_a^0 - \frac{1}{K - E_a \mp i\varepsilon} W\psi_a^{(\pm)} = \psi_a^0 - \frac{1}{K + W - E_a \mp i\varepsilon} W\psi_a^0,$$

where ψ_a^0 is an eigenfunction of the free-particle Hamiltonian K. For the scattering amplitude $R_{ba} = (\psi_b^{(-)}, W\psi_a^0)$ we can then write either the linear integral equation

$$R_{ba} = W_{ba} - \sum_c \frac{R_{bc} W_{ca}}{E_c - E_b - i\varepsilon}, \tag{24}$$

or the nonlinear equation

$$R_{ba} = W_{ba} - \sum_c \frac{R_{cb}^* R_{ca}}{E_c - E_b - i\varepsilon} - \sum_d \frac{(\psi_d, W\psi_b^0)^* (\psi_d, W\psi_a^0)}{E_d - E_b}, \tag{25}$$

where the summation goes over both the continuous spectrum c and the discrete spectrum d.

Here $W_{ba} = (\psi_b^0, W\psi_a^0)$ is the Born approximation to the scattering amplitude. Usually the potential is determined by solution of the linear equation (24). In the treatment we are considering it is more convenient to use the nonlinear equation (25). In addition one can generally avoid solving the equation connecting R_{ba} with W_{ba}.

Let us separate out from the whole sum over n in Eq. (15) the sum over the states c (of the continuous spectrum) with the same particles as there are in the states a and b (it is assumed that the collision does not change the types of the particles):

$$R_{ba} = (\Phi_b, (H - E_a)\Phi_a) - \sum_{n \neq c,d} \frac{R^*_{nb} R_{na}}{E_n - E_b - i\varepsilon} - \sum_c \frac{R^*_{cb} R_{ca}}{E_c - E_a - i\varepsilon} - \sum_d \frac{(\chi_f, (H - E_b)\Phi_b)^* (\chi_f, (H - E_a)\Phi_a)}{E_d - E_b}. \quad (26)$$

The last two terms in Eq. (26) are analogous in form to the last two terms in Eq. (25). Let us now determine the potential W from the requirement that the transition amplitudes between the two-nucleon states be equal to the corresponding amplitudes in the phenomenological treatment:

$$(\Psi_b^{(-)}, (H - E_a)\Phi_a) = (\psi_b^{(-)}, W\psi_a^0), \quad (\chi_d, (H - E_a)\Phi_a) = (\psi_d, W\psi_a^0). \quad (27)$$

Then comparison of Eqs. (26) and (25) shows that

$$W_{ba} = (\Phi_b, (H - E_a)\Phi_a) - \sum_{n \neq c,d} \frac{R^*_{nb} R_{na}}{E_n - E_a - i\varepsilon}, \quad (28)$$

where the sum is over all states except the states with the original particles. This determination of the potential is valid also in the case in which nucleon-antinucleon pairs are taken into account.

As is well known,[5-7] the introduction of an effective potential by different methods can lead to different results. This ambiguity is due to the arbitrariness noted above in the choice of the basis functions Φ and arises from the fact that in quantum field theory the concept of noninteracting dressed nucleons is defined only asymptotically. In the phenomenological treatment, on the other hand, the wave function of the noninteracting nucleons is defined for all values of R_{ij}. Therefore the unique determination of the potential requires a knowledge of the state vector of the noninteracting dressed nucleons for arbitrary R_{ij}.

The formulas (27) and (28) for the potential W do not leave any ambiguity, but in these formulas it is assumed implicitly that one has chosen for Φ the wave function that "correctly" describes the noninteracting dressed nucleons for finite R_{ij} also. This assumption is included in the equations (27) outside the energy surface, since the replacement of the basis functions Φ by other functions Φ' (with the same asymptotic behavior) leaves Eq. (27) unchanged only on the energy surface.

In the case of two nucleons the adiabatic potential derived by Klein and McCormick[5] corresponds to the representation (6) for Φ without additional factors, and the potential obtained by Miyazawa[8] and the writer[3] corresponds to the choice of the function Φ with normalized amplitude

$$(\Phi_a | \Phi_a) = 1,$$

where $(|)$ means that in the scalar product the integration over the space coordinates is not carried out.

APPENDIX

We shall derive the formula (20). We shift the operators a^+ to the left to Λ_0 and the absorption operators a to the right to Λ_0; then according to Eq. (6) a typical one of the quantities making up the left member of Eq. (20) will be

$$M = (F_1'(1, a^+) F_2'(2, a^+) \ldots \Lambda_0, F_1(1, a^+) F_2(2, a^+) \ldots \Lambda_0),$$

where F_i' and F_i can differ from F by the absence of one or more virtual mesons. We have

$$M = F_1'^*\left(1, \frac{\partial}{\partial \bar{a}}\right) F_2'^*\left(2, \frac{\partial}{\partial \bar{a}}\right) \ldots F_1(1, \bar{a}) F_2(2, \bar{a}) \ldots \Big|_{\bar{a}=0}$$

$$= F_1'^*\left(1, \frac{\partial}{\partial \bar{a}_1} + \frac{\partial}{\partial \bar{a}_2} + \ldots\right) F_2'^*\left(2, \frac{\partial}{\partial \bar{a}_1} + \frac{\partial}{\partial \bar{a}_2} + \ldots\right) \times \ldots F(1, \bar{a}_1) F(2, \bar{a}_2) \ldots \Big|_{\bar{a}_i = 0}$$

Since $[\bar{\lambda}_i, \bar{a}_i] = 0$

$$F(i, \bar{\lambda}_i + \bar{a}_i) = \exp\left[\sum_q \bar{\lambda}_{iq} \frac{\partial}{\partial \bar{a}_{iq}}\right] F(i, \bar{a}_i).$$

From this, writing $\lambda_i = \sum_j \partial/\partial \bar{a}_j$ for $i \neq j$, we have

$$M = F_1'^{*}\left(1, \frac{\partial}{\partial \bar{a}_1}\right) F_2'^{*}\left(2, \frac{\partial}{\partial \bar{a}_2}\right) \ldots : \prod_i \exp\left[\sum_q \lambda_{iq} \bar{a}_{iq}\right] : F_1(1, \bar{a}_1) F_2(2, \bar{a}_2) \ldots \Big|_{\bar{a}_i = 0}$$

$$= \left(F_1'(1, a_1^+) F_2'(2, a_2^+) \ldots \Lambda_0, \; : \prod_i^n \exp\left[\sum_q \hat{\lambda}_{iq} a_{iq}^+\right] : F_1(1, a_1^+) F_2(2, a_2^+) \ldots \Lambda_0 \right), \quad \hat{\lambda}_{iq} = \sum_{i \neq j} a_{jq} = A_q - a_{iq}. \tag{A.1}$$

Changing back from the quantities F_i and F_i' to the one-nucleon functions F, we get from this the formula (20).

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