ON THE THEORY OF THE WEAK INTERACTIONS. II

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A scheme is constructed in which the non-Hermitian weak-interaction Hamiltonian H_W is a component part of the Hermitian total Hamiltonian operating in a "doubled" Hilbert space.

1. INTRODUCTION

IN the writer's previous paper[1] the question was left open as to how the non-Hermitian Hamiltonian H_W can be included in a consistent quantum-mechanical scheme. One of the possible solutions of this question is proposed in the present paper. The essentials of the scheme are as follows.

The weak-interaction Hamiltonian H_W appears in the total Hamiltonian not simply added to the strong-interaction Hamiltonian H_S, but as one of the "blocks" of a Hermitian total Hamiltonian;

$$\hat{H} = \begin{pmatrix} H_s H_w \\ H_w^+ H_s \end{pmatrix}, \tag{1.1}$$

which acts in a Hilbert space with a doubled number of dimensions. In order to give a physical meaning to the doubling of the Hilbert space, we postulate as the form of the inversion operator the matrix

$$\hat{P} = \begin{pmatrix} 0 & P \\ P & 0 \end{pmatrix}.$$

With such a definition of the inversion the "second" Hilbert space is the mirror image of the "first" space.

The class of permissible Hamiltonians of the form (1.1) is restricted by the requirement that the operators $\hat{H}$ and $\hat{P}$ commute. In other words, in the theory developed here the inversion $\hat{P}$ is a constant of the motion.

Finally, we introduce a certain "condition of observation" that enables us to interpret physically the results of the theory. The condition of observation is not invariant with respect to the inversion $\hat{P}$; this corresponds to the experimentally known violation of the conservation of parity in the weak interactions.

It is curious to note that within the framework of the proposed theory the non-Hermitian character of the Hamiltonian H_W is necessary. If we replace H_W by a Hermitian operator, all the effects associated with the non-conservation of parity vanish. At the same time, the Hermitian character of the Hamiltonian H_S and the conservation of parity in the strong interactions are necessary consequences of the scheme.

2. THE EXPLICIT EXPRESSION OF THE HAMILTONIAN H_W

The Hamiltonian H_W is expressed extremely simply in terms of certain four-component field spinors describing pairs of charged particles. With the pair of leptons (e, μ) we associate the field

$$\chi_L = \begin{pmatrix} \varphi_a(\mu^-) + \varphi_c(e^+) \\ \dot{\varphi}_a(e^-) - \dot{\varphi}_c(\mu^+) \end{pmatrix}, \tag{2.1}$$

and with the pair of baryons (p, Σ^+) we associate the field

$$\chi_B = \begin{pmatrix} \varphi_a(\tilde{p}) + \varphi_c(\Sigma^+) \\ \dot{\varphi}_a(\tilde{\Sigma}^+) - \dot{\varphi}_c(p) \end{pmatrix} \tag{2.2}$$

(the notation is the same as in reference 1). We shall not include the other charged particles in the scheme of the direct Fermi interaction.

It is convenient to combine the fields χ_L and χ_B into a single χ-field. For this purpose we introduce explicitly the dependence of the field on the variable β canonically conjugate to the nucleonic charge N, and define the single field by the equation

$$\chi(x, t, \beta) = \chi_B(x, t)e^{i\beta} + \chi_L(x, t). \tag{2.3}$$

We introduce into the scheme the direct Fermi interaction of the following neutral particles: the neutrino, the Λ^0 particle, and their antiparticles. The neutral particles come into the theory in an essentially different way from the charged particles (the rule of polarization of lines[1] fixes the type of the φ-fields only for charged particles). Because of this the neutral particles have to be described by ordinary ψ-fields. We define the single ψ-field of the neutral particles by the equation

$$\psi(\mathbf{x}, t, \beta) = \{\psi_n(\mathbf{x}, t) + \psi_{\Lambda^0}(\mathbf{x}, t)\} e^{-i\beta} + \psi_\nu(\mathbf{x}, t). \quad (2.4)$$

We define the operation R as the interchange of the particles in the pairs (e, μ) and (p, Σ^+). The result of applying R will be written with an upper index. For the neutral particles we have by definition that R is the identical transformation: $\psi^R \equiv \psi$.

We can now write the weak-interaction Hamiltonian density $\mathcal{H}_W(\mathbf{x}, t)$, satisfying all the rules formulated in reference 1, in the form

$$\mathcal{H}_w(\mathbf{x}, t) = (g/4\pi) \int_{-\pi}^{\pi} d\beta \{(\bar{\chi}\psi)^R (\bar{\psi}\chi) + (\bar{\psi}\chi)(\bar{\chi}\psi)^R\}. \quad (2.5)$$

In the expression (2.5) the integration with respect to β selects the terms corresponding to conservation of the nucleonic charge.

3. THE SYMMETRY OF THE THEORY WITH RESPECT TO INVERSIONS

Under proper Lorentz transformations the fields χ and ψ transform as bispinors. The behavior of the χ-fields under inversions is not obvious a priori and requires special consideration. We shall examine inversions of the following three types: spatial inversion P, charge conjugation C, and the transformation S, consisting of reflection of all four space-time axes and replacement of particles by antiparticles. All other inversions are expressible in terms of products of these "elementary" inversions.

We shall introduce the inversion operators acting in the state space in a way like that used in reference 2 for Lorentz transformations. We associate with the inversions P and C the unitary transformations U_P and U_C. To the inversion S, which involves time reversal, we assign an antiunitary transformation $U_S J$, where U_S is a unitary operator and J is the operation of complex conjugation. The forms of the operators U are given in the Appendix. The χ-field (2.3) transforms in the following ways under the elementary inversions:

$$\chi(\mathbf{x}, t, \beta) \rightarrow \begin{cases} U_P \chi U_P^+ = \gamma_4 \chi^R(-\mathbf{x}, t, \beta) & (P) \\ U_C \chi U_C^+ = C\chi^+(\mathbf{x}, t, -\beta). & (C). \\ U_S J\chi J U_S^+ = C\gamma_5 \chi^R(-\mathbf{x}, -t, -\beta) & (S) \end{cases} \quad (3.1)$$

In the right members of Eq. (3.1) C denotes the spinor conjugation matrix, which has the properties:

$$C\gamma_\mu C^{-1} = -\gamma_\mu^*, \quad C^* = C^T = C^+ = C; \quad C^2 = 1.$$

The appearance of the matrix C in the third of the equations (3.1) is due to the antilinear character of the transformation S. The ψ-field (2.4) also transforms by the formulas (3.1), the only difference being that for it R is the identical transformation

It is now easy to find the transformation law of the weak-interaction Hamiltonian. We give the result for the whole Hamiltonian H_W in the Schrödinger representation:

$$U_p H_w U_p^+ = U_c H_w U_c^+ = U_s J H_w J U_s^+ = H_w^+. \quad (3.2)$$

Under each of the elementary inversions H_W goes over into the Hermitian adjoint operator H_W^+. Under any combined inversion equal to the product of two elementary inversions H_W remains unchanged. This result has been obtained in a different way in the preceding paper.[1]

4. THE TOTAL HAMILTONIAN $\hat{H}$

Our further considerations are based on the hypothesis that the complete physical definition of the inversions must include, in addition to the transformations of the space of the state vectors Ψ discussed in the preceding section, also a transformation of a certain internal variable characterizing the properties of the space. We obtain a realization of this idea in the following scheme.

We shall regard the total state vector Ψ as a two-component quantity

$$\hat{\Psi} = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}, \quad (4.1)$$

where Ψ_1 and Ψ_2 are two Hilbert vectors for the state of the system. We define the scalar product of two vectors of the type (4.1) by

$$(\hat{\Psi}, \hat{\Phi}) = (\Psi_1, \Phi_1) + (\Psi_2, \Phi_2). \quad (4.2)$$

It is convenient to introduce matrices $\hat{\Lambda}_i$ acting on the vectors $\hat{\Psi}$, in analogy with the Pauli spin matrices:

$$\Lambda_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \Lambda_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \Lambda_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Let I be any one of the elementary inversions. We define the complete inversion $\hat{I}$ by the equation

$$\hat{I} = I\Lambda_1. \quad (4.3)$$

Finally, let us define the total Hamiltonian H, containing both the strong and the weak interactions:

$$\hat{H} = H_s + \frac{1}{2}(H_w + H_w^+)\Lambda_1 + \frac{i}{2}(H_w - H_w^+)\Lambda_2. \quad (4.4)$$

The Hamiltonian $\hat{H}$ is Hermitian in the metric given by the scalar product (4.2). In virtue of the relation (3.2), the Hamiltonian H satisfies the relation

$$[\hat{H}, \hat{I}] = 0. \tag{4.5}$$

In the derivation of Eq. (4.5) it has been assumed that there is conservation of all the inversion parities for the strong interactions, i.e., that we have the relations

$$[H_s, I] = 0. \tag{4.6}$$

The converse proposition is also true: the relations (4.6) and (3.2) are consequences of the relation (4.5) and the definitions (4.3) and (4.4).

5. THE PHYSICAL INTERPRETATION OF THE THEORY

Let us construct in our scheme the theory of the S matrix that gives the probabilities of transitions between unperturbed states. We subject the unperturbed states to the symmetry condition $\hat{\Psi} = \Lambda_1\hat{\Psi}$, or in terms of the components

$$\hat{\Psi} = \begin{pmatrix}\Psi\\ \Psi\end{pmatrix}. \tag{5.1}$$

Vectors of the form (5.1) can be regarded as eigenstates of either the strong-interaction Hamiltonian H_S or the free-particle Hamiltonian H_0. Let $\hat{S}$ be the S matrix related in the usual way to the Hamiltonian $\hat{H}$. We define the probability of transition from the symmetric state $\hat{\Psi}_0$ to the symmetric state $\hat{\Psi}_f$ by the equation

$$W(0\rightarrow f) = \left|\left(\hat{\Psi}_f, \frac{1+\Lambda_3}{2}\hat{S}\hat{\Psi}_0\right)\right|^2. \tag{5.2}$$

It is easy to see that the definition (5.2) of the probability goes over into the usual definition if we neglect the weak interactions.

On the basis of Eq. (5.2) the total transition probability is given by

$$\sum_f W(0\rightarrow f) = \left(\hat{\Psi}, \frac{1+\Lambda_3}{2}\hat{\Psi}\right), \quad \hat{\Psi} = \hat{S}\hat{\Psi}_0. \tag{5.3}$$

In the derivation of Eq. (5.3) we have used only the completeness of the system of final states Ψ_f. In order for the definition (5.2) of the transition probability to be free from contradiction, the condition of conservation of the total probability must hold, that is,

$$\left(\hat{\Psi}, \frac{1+\Lambda_3}{2}\hat{\Psi}\right) = \left(\hat{\Psi}_0, \frac{1+\Lambda_3}{2}\hat{\Psi}_0\right). \tag{5.4}$$

It can be shown that Eq. (5.4) is valid if the initial state $\hat{\Psi}_0$ has a definite intrinsic parity, i.e., satisfies the condition

$$P_0\Psi_0 = \lambda\Psi_0, \quad (\lambda = \pm 1).$$

By intrinsic parity we mean parity in the center-of-mass system. We define the intrinsic parity operator P_0 by the equation

$$P_0 = U_L U_P U_L^+,$$

where L is the Lorentz transformation from the laboratory coordinate system to the center-of-mass system, and U_L is the operator of the unitary representation of the Lorentz group discussed in reference 2.

if we apply1 the definition (5.2) to the calculation in first-order perturbation theory of the transition probability occasioned by the weak interaction, we get the relation

$$W(0\rightarrow f) = \left|\left(\Psi_f, \int \mathcal{H}_w(x)\, d^4x \Psi_0\right)\right|^2,$$

from which it can be seen that in this case the transition probability is calculated by the usual rules, on the basis of the Hamiltonian $\mathcal{H}_W(x)$.

APPENDIX

It is easy to find the explicit forms of the operators U by the method discussed in reference 2:

$$U_p = \exp\left\{-i\frac{\pi}{2}\sum a^+(\mathbf{p})\,[a(\mathbf{p}) - a(-\mathbf{p})] + b^+(\mathbf{p})\,[b(\mathbf{p}) + b(-\mathbf{p})]\right\};$$

$$U_c = \exp\left\{-i\frac{\pi}{2}\sum [a^+(\mathbf{p}) - b^+(\mathbf{p})]\,[a(\mathbf{p}) - b(\mathbf{p})]\right\};$$

$$U_s = \exp\left\{\frac{\pi}{2}\sum a^+(\mathbf{p})\,\varepsilon a(\mathbf{p}) - b^+(\mathbf{p})\,\varepsilon b(\mathbf{p})\right\}.$$

All the operators (A.1) are unitary. The antiunitary transformation is $S = U_S J$, where J denotes complex conjugation. By definition the operator J does not change the vacuum state: $J\Phi_0 = \Phi_0$. Furthermore, J commutes with the operators for creation and annihilation of particles (since these operators are represented by real matrices): $JaJ = a$, $Ja^+J = a^+$.

Thus under the transformation S the creation operators do not go over into annihilation operators, and conversely. Our definition of S agrees with the definition of time reversal in nonrelativistic quantum mechanics (cf., e.g., reference 3).

[1] Iu. A. Gol'fand, J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 170 (1958), Soviet Phys. JETP **8**, 118 (1959).

[2] Iu. A. Gol'fand, J. Exptl. Theoret. Phys. (U.S.S.R.) **31**, 535 (1956), Soviet Phys. JETP **4**, 446 (1957).

[3] L. D. Landau and E. M. Lifshitz, Квантовая механика (Quantum Mechanics), Sect. 16. Gostekhizdat, Moscow-Leningrad 1948. [Engl. Transl. Addison-Wesley, 1958].

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