# on the size spectrum of extensive air showers 

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Experimental data are presented on the size spectrum of extensive air showers in the region $10^{5}$ to $2 \times 10^{6}$ particles. An analysis of these and other data available in the literature indicates that there is, very probably, an irregularity in the shower size distribution curve in the region between $10^{6}$ and $10^{7}$ particles. This is considered to constitute an argument in favor of the metagalactic origin of cosmic rays with energies above $10^{16} \mathrm{ev}$.

IN connection with the problem of a change in the character of the elementary interaction in the ultra high energy range, Vernov ${ }^{1}$ suggested the idea that such a change could reflect itself as an irregularity in the size spectrum of extensive air showers (EAS). On the other hand, the study of the size spectrum of EAS is of considerable interest for the investigation of the primary component of cosmic radiation.

In the majority of experiments carried out so far the density spectrum of EAS was actually measured, and the size spectrum was calculated on its basis. Such a method of obtaining the size spectrum of an EAS has, however, many disadvantages. In fact, it is known that the density spectrum is the result of an integration of the size spectrum over a wide range, in which the number of particles varies by 2 or 3 orders of magnitude. As the result, the density spectrum cannot reflect all details of the size spectrum.

In the present work the size spectrum of EAS has been studied directly. The measurements were carried at sea level in May 1954.

## EXPERIMENTAL SETUP AND METHOD

The measurements were carried out using the array described in reference 2, by the method of correlated hodoscopes ${ }^{3}$ which, in principle, permits it to study each shower individually.

The size and position of the hodoscope counter groups are shown in Fig. 1. The array permitted us to locate the shower axis and find the number of particles in showers in the range from $2 \times 10^{4}$ to $2 \times 10^{6}$ particles. The hodoscope arrangement displayed the number of discharged counters in various counter groups for each shower. Knowing the position of counters struck in the plane of observation, one can find the total number of par-
ticles N and the coordinates of the axis $\mathrm{X}_{0}, \mathrm{Y}_{0}$ of the recorded shower, under the assumption that (1) the shower is axially symmetric and (2) the trajectories of the shower particles are distributed according to the Poisson law and possible fluctuations of the lateral distribution function in a shower with given N are governed only by that law.

The probability that, when $m_{i}$ out of $n_{i}$ counters are struck at the i-th point with coordinates $X_{i}, Y_{i}$, the recorded shower consists of $N$ particles and its axis has coordinates $\mathrm{X}_{0}, \mathrm{Y}_{0}$, is proportional to

$$
C_{n_{i}}^{m_{i}}\left[1-\exp \left(.-\sigma_{i} \rho_{i}\right)\right]^{m_{i}} \exp \left[-\sigma_{i} \rho_{i}\left(n_{i}-m_{i}\right)\right],
$$

where $\sigma_{i}$ is the area of a single counter in the i-th group, and

$$
\begin{gathered}
\rho_{i}=k N f\left(\left|\mathbf{r}_{i}-\mathbf{r}_{0}\right|\right), k=2 \cdot 10^{-3} \mathrm{~m}^{-1}\left[^{2}\right], \\
f\left(\left|\mathbf{r}_{i}-\mathbf{r}_{0}\right|\right)=\frac{1}{\left|\mathbf{r}_{i}-\mathbf{r}_{0}\right|} \exp \left(-\frac{\left|\mathbf{r}_{i}-\mathbf{r}_{0}\right|}{58}\right), \\
\left|\mathbf{r}_{i}-\mathbf{r}_{0}\right|=\left[\left(X_{i}-X_{0}\right)^{2}+\left(Y_{i}-Y_{0}\right)^{2}\right]^{1 / 2} .
\end{gathered}
$$

The probability that, for an occurring combination $\left(m_{i}, n_{i}\right)$, the recorded shower will have individual characteristics $N, X_{0}, Y_{0}$, is then given (to a normalization constant) by the following expression:

$$
\begin{gather*}
W\left(X_{0}, Y_{0}, N\right) \\
\approx \prod_{i=1}^{56} C_{n_{i}}^{m_{i}}\left[1-\exp \left(-\sigma_{i} \rho_{i}\right)\right]^{m_{i}} \exp \left[-\sigma_{i} \rho_{i}\left(n_{i}-m_{i}\right)\right] . \tag{1}
\end{gather*}
$$

Determination of the shower size and its axis location is therefore reduced to the following extremum problem: find the maximum of Eq. (1) with respect to the variables $\mathrm{N}, \mathrm{X}_{0}$, and $\mathrm{Y}_{0}$, knowing the value of other parameters in the expression. The problem was solved with the electronic computer of the Computing Center of Moscow State University.


FIG. 1. Diagram of the hodoscope array used for the study of the size spectrum of EAS. $\quad$ - group of 24 hodoscope counters, $330 \mathrm{~cm}^{2}$ in area each: $\Delta$ - group of 48 hodoscope counters, 24 of area of $100 \mathrm{~cm}^{2}$ each, and 24 of area of $24 \mathrm{~cm}^{2}$ each,

- master groups.

N was found with an accuracy of $10 \%$, and $\mathrm{X}_{0}$ and $Y_{0}$ with an accuracy of 0.5 m for shower axes falling within the area $S_{1}$. The errors were $15 \%$ and 1.0 m respectively when the axis fell within the area $\mathrm{S}_{3}$.

The axis location and the number of particles for each individual shower being known, it is possible to find the absolute rate of showers of a given size. The dimensions of the collecting area were different for showers of different size, being determined not only by the position of hodoscope points but also by the probability of shower detection using the chosen six-fold counter coincidence arrangement.

## RESULTS AND DISCUSSION

The experimental results which are made use of in the present article were gathered during 285 hours of operation of the array. The number of showers of a size above given value, the axes of which fell in the area $\mathrm{S}_{1}=78 \mathrm{~m}^{2}$ for $\mathrm{N}>8 \times 10^{4}$ $\mathrm{S}_{2}=400 \mathrm{~m}^{2}$, for $\mathrm{N}>1.6 \times 10^{5}$, and $\mathrm{S}_{3}=576 \mathrm{~m}^{2}$, for still larger N (the relative position of these areas is shown in Fig. 1), used for constructing the spectrum, are given in the table. The probability that a shower incident upon the above areas was recorded by the array was greater than $95 \%$.

The integral size spectrum constructed from using the data is shown in Fig. 2. The ordinate represents the number of showers above certain size in $\mathrm{cm}^{-2} \mathrm{sec}^{-1}$ sterad $^{-1}$. It was shown ${ }^{4}$ that the rate of showers arriving vertically per unit solid angle $F(N, 0)$ is related to the total $F(N)$ for the given array by

$$
F(N, 0)=F(N)(\nu+1) / 2 \pi,
$$

when the zenith angle distribution of shower axes incident upon the array is proportional to $\cos ^{2} \theta$, with $\nu=8 .{ }^{5}$


FIG. 2. Integral size spectrum of EAS. - - measurements of the present experiment, 0 - measurements of reference 7.

The data indicate with a high probability that a change in the character of the spectrum may occur in the investigated region $\mathrm{N}=8 \times 10^{4}$ to $1.5 \times 10^{6}$. In the region $\mathrm{N}=8 \times 10^{4}$ to $8 \times 10^{5}$ the integral spectrum can be approximated by a power law with an exponent $\kappa=1.5 \pm 0.1$. For $\mathrm{N}>8 \times 10^{5}$ the spectrum is steeper. For a quantitative determination of the exponent $\kappa$ in that region we used the data of reference 6 , in which each shower was measured individually.* These data are also shown in Fig. 2. For the region $N=8 \times 10^{5}$ to $3 \times 10^{6}$ we then obtain $\kappa=2.2 \pm 0.3$. Using the results of reference 6 , for the region $\mathrm{N}=10^{7}$ to $10^{8}$ we have $\kappa=1.5 \pm 0.2 . \dagger$

The exponent of the size spectrum of EAS was
*It should be mentioned that the lateral distribution function used in reference 6 does not differ from that used by us by more than $25 \%$.
$\dagger$ In reference 6 the size-spectrum exponent was determined for the whole range of measurements $\mathrm{N}=8 \times 10^{5}$ to $10^{8}$ and was found to be $x=1.84 \pm 0.15$. A $\chi^{2}$ test shows, however, that the probability that the spectrum has two exponents $x=2.2$ $\pm 0.3$ and $x=1.5 \pm 0.2$ is three times higher then the probability of a single exponent $x=1.84 \pm 0.15$. The probabilities are 75 and $25 \%$ respectively.

| Number of <br> particles in <br> shower, N | $0.8 \cdot 10^{5}$ | $1.6 \cdot 10^{5}$ | $3.2 \cdot 10^{5}$ | $6.4 \cdot 10^{6}$ | $8.0 \cdot 10^{5}$ | $12.8 \cdot 10^{5}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> showers with <br> number of <br> particles > N | 157 | 276 | 1.38 | 46 | 24 | 6 |

measured earlier using arrays with widely spaced counters connected in coincidence. ${ }^{7,8}$ It is well known that such arrays select effectively showers with size varying by two orders of magnitude. The values of the exponent $\kappa \approx 1.9$ to 2.0 obtained in these experiments for the range $10^{6}<\mathrm{N}<10^{8}$ do not contradict the above data on variation of the exponent in that range.

The observed variation of the size spectrum is evidently related to a corresponding variation of the spectrum of the primary ultra-high-energy cosmic radiation that initiates the EAS.* We shall make the simplest assumption that the total number of particles $N$ in EAS is connected with the primary energy $E$ by the following relation:

$$
E=k_{0} N
$$

where $\mathrm{k}_{0}$ is almost independent of N and, at sea-level, equals about $10^{10} \mathrm{ev} .{ }^{9}$ To the size range $\mathrm{N}=10^{5}$ to $10^{8}$ corresponds, therefore, the primary energy range $\mathrm{E}=10^{15}$ to $10^{18} \mathrm{ev}$.

The observed variation of the primary cosmicray spectra can be explained by the following model of propagation from their point of origin to the point of observation: we shall assume that sources of primary cosmic radiation exist in the galaxy, producing particles with energy $E$, up to the highest experimentally observed ${ }^{10}$ ones, integral spectrum of this particle being $\sim \mathrm{E}^{-\kappa}$. In view of the presence of regions with a higher magnetic field (magnetic clouds) in the galaxy, the particles are diffused when their energy is $\mathrm{E} \leq 300 \mathrm{Hl} \approx$ $10^{16} \mathrm{ev}$, where $l$ is the size of magnetic clouds and H is the magnetic field intensity ( $\mathrm{H} \sim 10^{-5} \mathrm{Oe}$ and $l$ is on the order of several light-years). As the result of the diffusion, the particles are accumulated in the galaxy, the storage factor being on the order of $\sim 10^{3}$ to $10^{4}$. ${ }^{11}$ Particles with energy $E>10^{16} \mathrm{ev}$ are diffused to a lesser extent, so that their concentration is strongly dependent on the energy. One can expect therefore a rather sharp variation of the primary-energy spectrum, namely that it becomes steeper for $E>10^{16} \mathrm{ev}$. Particles of $>10^{16} \mathrm{ev}$, propagating practically in straight lines, leave the galaxy. If we assume that a similar process takes place in other galaxies as well, it is evident that the particles with

[^0]$\mathrm{E} \geq 10^{16} \mathrm{ev}$ may have a metagalactic origin.
We shall estimate now the contribution of other galaxies to the flux of particles with $\mathrm{E}>10^{16} \mathrm{ev}$. We shall asume that the concentration of galaxies in the universe is $n$. If $S$ is the emission intensity per unit volume of each galaxy then the total flux of particles I through surface $\omega$ will be given by the expression
\[

$$
\begin{equation*}
I=S V 2 \pi \int_{0}^{R} r^{2} d r n 2 \int_{0}^{\pi / 2} \frac{\omega \cos \theta}{r^{2}} \sin \theta d \theta \tag{2}
\end{equation*}
$$

\]

where V is the galactic volume and R is the maximum distance traversed by the particles without interaction.* The flux of particles of similar energies through surface $\omega$ produced by our galaxy $\mathrm{I}_{\mathrm{G}}$ is

$$
I_{\mathrm{G}}=S 2 \pi \int_{0}^{R_{0}} r^{2} d r 2 \int_{0}^{\pi / 2} \frac{\omega \cos \theta}{r^{2}} \sin \theta d \theta
$$

where $R_{0}$ is the radius of galaxy (assuming that the galaxy is spherical). The ratio of these fluxes is $\mathrm{I} / \mathrm{I}_{\mathrm{G}}=\mathrm{nVR} / \mathrm{R}_{0}$. Assuming that $\mathrm{n}=10^{-18}$ cubic light years, $V \sim 10^{14}$ per cubic light year, $R=10^{12}$ light years (reference 12), and $\mathrm{R}_{0}=10^{5}$ light years, we obtain $I / I_{G}=10^{3}$.

The ratio of the particle flux produced by other galaxies to that produced by our galaxy is, in its order of magnitude, equal to the "storage factor" due to diffusion.

It is natural to assume that the energy spectrum of particles of metagalactic origin will be characterized by the same exponent к. Since the intensity of the galactic and metagalactic radiation is of the same order of magnitude, the observed spectrum is a superposition of the spectra of particles of galactic and metagalactic origin.

Finally, we should like to mention that the conclusions of the present work with respect to the irregularity in the size spectrum of EAS cannot be regarded as final, since the experimental data on which they are based do not possess a sufficient statistical accuracy. The results, however, stress the great urgency of further experiments on the size spectrum of EAS.

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[^0]:    *With regard to another possible interpretation, cf. reference 1.

[^1]:    *In connection with the presence of magnetic fields of $\sim 10^{-7} \mathrm{Oe}$ in the metagalactic space, ${ }^{11}$ the upper limit of integration in Eq. (2) will be limited by smaller values of R; it is then necessary, however, to account for the storage factor of the metagalactic space.

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