INVESTIGATION OF THE ANGULAR DISTRIBUTION OF SECONDARY PARTICLES IN SHOWERS PRODUCED BY HIGH-ENERGY NUCLEONS

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The angular distribution of secondary shower particles produced in the stratosphere by nucleons with energies above 50 Bev is investigated. An analysis of the distribution is carried out on basis of the theory of multiple meson production in nucleon-nucleon collisions and collisions between several nucleons according to the tunnel effect model.

THE angular distribution of shower particles reflects the processes taking place in the target nucleus in interaction with the incident nucleon. In order to study such interactions, we examined showers produced by protons. The showers were selected from a large number of stars produced in emulsion stacks exposed in the atmosphere at the latitudes of Moscow and Italy in 1955. (Stacks I-f-55 and R).

The particle detection efficiency was determined from the $\pi \rightarrow \mu \rightarrow e$ decay sensitivity and the emulsion was calibrated by measuring the grain density of electron tracks.[1] The angular distribution of thin tracks (shower particles) was determined for all showers.

The space angle θ_i between the direction of the primary and the i-th shower particle was determined by the formula:

$$\cos\theta_i = \cos\varphi_i \cos\varphi_0 (\tan\varphi_i \tan\varphi_0 + \cos\lambda_i),$$

where φ_i is the angle between the direction of the i-th shower particle and the emulsion plane, φ_0 is the angle between the direction of the primary (or the shower axis) and the emulsion plane, and λ_i is the angle between the i-th shower particle and the primary (or the shower axis) in the plane of emulsion.

For showers produced by neutral particles we determined the mean direction (axis) of the shower, with respect to which the spatial angle θ_i was measured. For that case

$$\sin\varphi_0 = Z/\sqrt{X^2+Y^2+Z^2};$$

$$\sin\lambda_0 = X/\sqrt{X^2+Y^2+Z^2}\cos\varphi_0,$$

$$X = \sum_{i=1}^{n_s}\cos\varphi_i \sin\lambda_i'; \quad Y = \sum_{i=1}^{n_s}\cos\varphi_i\cos\lambda_i'; \quad Z = \sum_{i=1}^{n_s}\sin\varphi_i;$$

where λ_0 and λ_i', are the angles in the emulsion plane of the shower axis and the i-th particle, respectively, with the direction of any arbitrarily chosen track near the axis.

The energy of the primary particles was determined from the angular distribution of shower particles; it was assumed that the incident particle interacts either with a single nucleon or with a column of nuclear matter. For nucleon-nucleon interactions, the primary particle energy was determined in first approximation[2] from the formula

$$E = 2Mc^2/\tan^2\theta_{1/2},$$

where $\theta_{1/2}$ is the angle containing a half of the shower particles.

For interactions between a nucleon and nucleons of the tunnel l/d long, the energy was found from the equation

$$E = Mc^2 \times [l/d + \sqrt{(l/d)^2 + \tan^2\theta_{1/2}(\tan^2\theta_{1/2} + (l/d)^2 + 1)}]/\tan^2\theta_{1/2}. \quad (1)$$

Such an energy estimate corresponds to the assumption that the produced particles are monoenergetic in the center-of-mass system (c.m.s.). ($\beta_\pi^* = v_\pi^*/c = 1$, where v_π^* is the velocity of π mesons in the c.m.s.). The energy spectrum of shower particles was accounted for in the estimate of the primary energy according to reference 2, and the energy spectrum of the primary particles — according to Ref. 3. Both refinements strengthen our conclusions on the role of secondary interactions inside the nucleus.

1. INTEGRAL AND DIFFERENTIAL ANGULAR DISTRIBUTIONS OF SHOWER PARTICLES

In the study of the angular distribution of shower particles in the laboratory system of coordinates* (l.s.) it is assumed that the distribution in the c.m.s. is symmetric with respect to the plane perpendicular to the line joining the interacting nucleons. The relation between the angle θ^* in the c.m.s. and θ in the l.s. is given by the equation

$$\gamma_c \tan\theta = \sin\theta^* / (m + \cos\theta^*), \quad m = \beta_c/\beta_\pi^*; \quad (2)$$

where γ_c is the primary energy in the c.m.s. and β_c is the velocity of the center-of-mass system.

If the particles produced are monoenergetic ($\beta_\pi^* = 1$) then, knowing the angular distribution function in the c.m.s. $N(\theta^*)$, it is easy to find the distribution function in the l.s. $N(\theta)$.

In fact, for an isotopic distribution of shower particles in the c.m.s. we have

$$N(\theta^*) = \text{const} \cdot d\Omega$$

the angular distribution function in the l.s. is of the form[4]

$$N(\theta) = \frac{N\gamma_c\alpha[m + \sqrt{1-(m^2-1)\alpha^2}]^2}{2\cos^2\theta\,(\alpha^2+1)^2\sqrt{1-(m^2-1)\alpha^2}}$$

for $m \le 1$, or

$$N(\theta) = \frac{N\gamma_c\alpha}{2\cos^2\theta\,(\alpha^2+1)^2}\left[\frac{m \pm \sqrt{1-(m^2-1)\alpha^2}}{\sqrt{1-(m^2-1)\alpha^2}}\right]$$

for $m \ge 1$, where $\alpha = \gamma_c \tan\theta$.

The fraction of shower particles f contained in the angle θ_f is defined as

$$f = \frac{1}{N}\int_0^{\theta_f} N(\theta)d\theta, \quad (3)$$

where N is the total number of particles. Substituting $N(\theta)$ and integrating, we find:

$$f = \frac{1}{2}\left[1 + m - \frac{m + \sqrt{1-(m^2-1)\alpha^2}}{\alpha^2+1}\right] \quad \text{for } m \leqslant 1, \quad (4)$$

$$f = 1 - \frac{\sqrt{1-(m^2-1)\alpha^2}}{\alpha^2+1} \quad \text{for } m \geqslant 1. \quad (5)$$

Accounting for the energy spectrum of shower particles in the c.m.s., we can find in an analogous way, for a given $N(\theta^*)$, the corresponding integral distribution in the l.s.

The differential angular distribution in the l.s., for an isotropic distribution in the c.m.s. and $m = 1$, is

$$\frac{df}{dy} = \frac{df}{d\alpha}\frac{d\alpha}{dy} = \frac{2\cdot 10^{2y}}{(10^{2y}+1)^2}\ln 10; \quad y = \log\alpha. \quad (6)$$

For an anisotropic distribution in c.m.s., e.g., for $N(\theta^*) \sim \cos^2\theta^*$, we have

$$df/dy = 4\cdot 10^{2y}(10^{2y}-1)(10^{2y}+1)^{-3}\ln 10.$$

The above expression, however, tends to zero for $y \to 0$, which is not observed experimentally. We shall consider therefore the following distribution function of shower particles:

$$N(\theta^*) = a^2\cos^2\theta^* + \sin^2\theta^*;$$

which is proportional to the radius vector of the ellipsoid of rotation with the major axis parallel to the line joining the colliding nucleons. The total number of particles is then

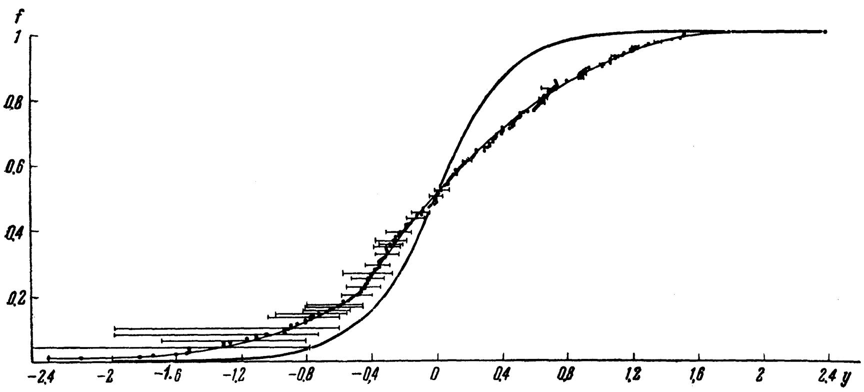

FIG. 1. Summary integral angular distribution of showers with $E_0 \geq 1000$ Bev. The solid curve shown for comparison represents an isotopic distribution of shower particles in the c.m.s. for $m = 1$. Experimental errors are shown in the figure. The abscissa represents $y = \log\gamma_c \tan\theta_f$.

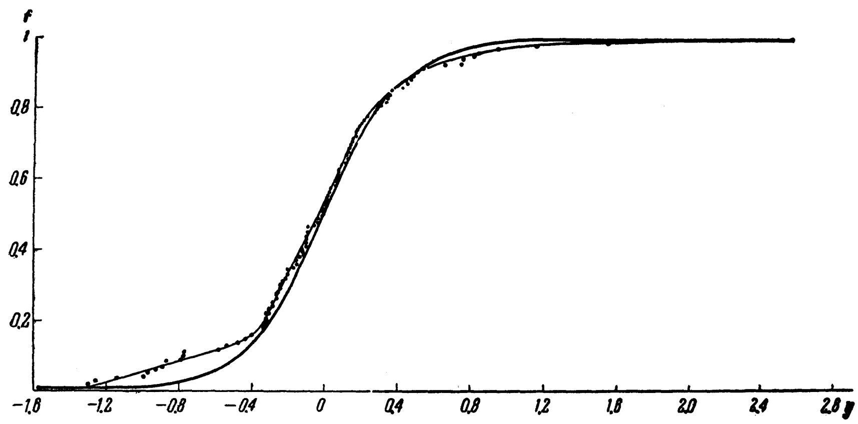

FIG. 2. Same as Fig. 1, only for $E_0 \leq 100$ Bev.

$$N = \int_0^\pi C\,(a^2\cos^2\theta^* + \sin^2\theta^*)^{1/2}\sin\theta^* d\theta^*,$$

$$C = \frac{N}{a + \ln(a+\sqrt{a^2+1})/\sqrt{a^2-1}},$$

where a is the ratio of the axes of the ellipsoid of rotation.

The differential angular distribution in the l.s. is given by the following expression:

$$\frac{\partial f}{\partial y} = \frac{4C}{N}\left\{\left(\frac{1-10^{2y}}{1+10^{2y}}\right)^2(a^2-1)+1\right\}^{1/2}\frac{10^{2y}\ln 10}{(10^{2y}+1)^2}. \quad (7)$$

In the analysis of showers we constructed the curves of the integral angular distribution, which should be symmetric about $f = 1/2$ for $m = 1$. The majority of experimental curves are asymmetric. This means either that $m \neq 1$ or that the assumption about a symmetric emission is incorrect. It is possible that the deviations from symmetry are a consequence of multiple collisions of the primary nuclear inside the nucleus.

2. COMPARISON WITH EXPERIMENTAL DATA

Showers initiated by primaries with energy $E_0 \leq 10^{11}$ ev and $E_0 \geq 10^{12}$ ev were selected from a large number of showers (over 200) for a study of the differences in the shower production mechanism at different energies.

The summary experimental integral angular distributions are shown in Figs. 1 and 2; curves corresponding to a symmetric distribution of shower particles in the c.m.s. are also included. Comparison shows that the angular distribution in the region $E_0 \leq 10^{11}$ ev is isotropic. Showers from references 5 and 6 have been included in the group $E_0 \geq 10^{12}$ ev together with showers detected and reduced in our laboratory.

It can be seen from Fig. 1 that, for $E_0 \geq 10^{12}$ ev, the distribution of shower particles in the c.m.s. is anisotropic.

For the energy region 5×10^{10} to 10^{11} ev we also studied the differential angular distribution of a summary shower with $n_s = 161$ particles. The curves, shown in Fig. 3, were calculated according to Eq. (7) for $a = 1$ and $a = 1.4$. The fit between experimental and theoretical curves was examined by the χ^2 test. An analysis of the differential angular distribution shows that it is almost totally isotropic in c.m.s.

3. ANALYSIS OF SHOWERS FROM THE POINT OF VIEW OF THE MULTIPLE MESON PRODUCTION THEORY

The theory of a pure multiple production of mesons assumes that all mesons are produced in a single interaction between the incident nucleon and a nucleon or a group of nucleons of the target nucleus (tunnel effect). It is assumed that the nucleon group forms a continuous amorphous mass of nuclear matter, since the collisions between the incident nucleon and the nucleons of the nucleus are not sharply divided in time. In the instant of collision, the nucleons of the nucleus are not in the same state with respect to the incident nucleon; the shape of the tunnel may, therefore, not be cylindrical. For small energies (< 50 Bev) the deviation from such a shape may be large, but for the region > 50 Bev we assume that the tunnel shape is almost cylindrical. In the following we assume that a nucleon incident upon the nucleus interacts with a column of nuclear matter in a tunnel whose length depends on the impact parameter. If M (column)/M (nucleon) $= l/d$ nucleons of the nucleus take part in the collision, one can estimate[7] the energy of the primary nucleon $\gamma = E/Mc^2$ by means of Eq. (1). In the

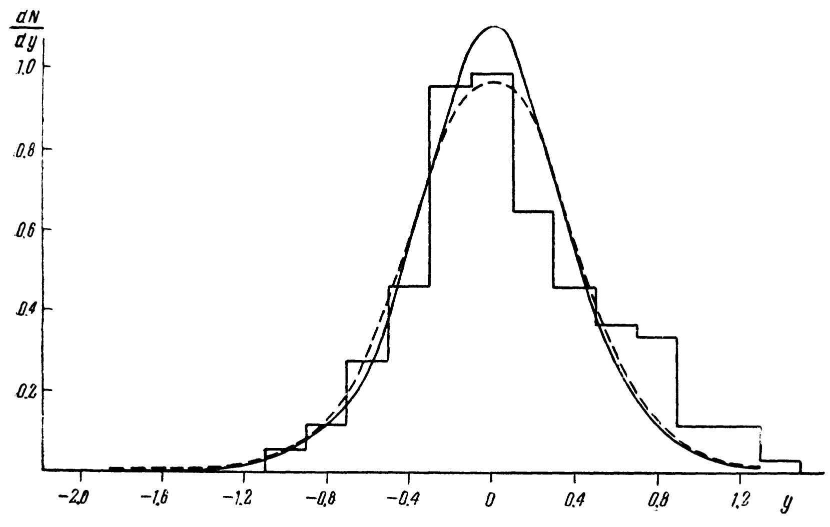

FIG. 3. Differential angular distribution for 161 shower particles. The solid and dashed curves correspond to Eq. (7) for a = 1 and a = 1.4 respectively.

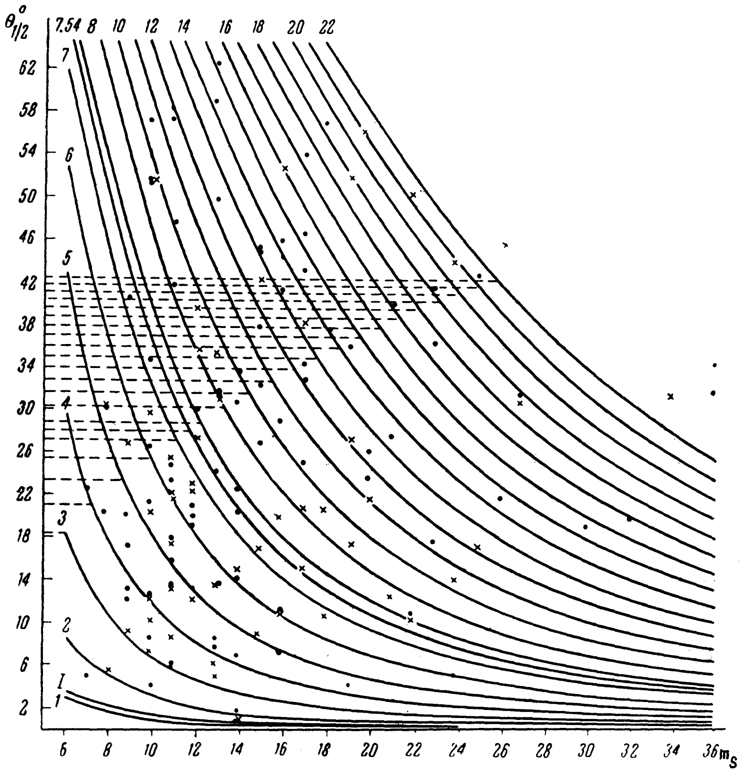

FIG. 4. Dependence of $\theta_{1/2}$ on n_s for different l/d. Experimental points for 138 showers (● – showers of the stack I – f; × – showers of the stack R). The region below dashed lines corresponds to $E_0 > 50$ Bev for each l/d.

derivation of (1) it has been assumed that (a) the number of particles is large (b) the emission of particles in c.m.s. is symmetric, and (c) all particles are relativistic in c.m.s., i.e. $\beta_\pi^* = 1$, $m = \beta_c$.

The angle $\theta_{1/2}$ is found from Eq. (1) as a function of γ and l/d:

$$\tan^2\theta_{1/2} = \frac{1 + 2\gamma l/d + (l/d)^2}{\gamma^2 - 1}. \qquad (8)$$

The total number of particles N taking part in the interaction and produced in it is determined from the hydrodynamical theory:[8]

$$N = 0.84\,(l/d + 1)\,\gamma^{1/4} \text{ for } l/d \leqslant 3.7, \qquad (9)$$

$$N = 1.55\,(l/d - 0.25)^{3/4}\gamma^{1/4} \text{ for } l/d > 3.7. \qquad (10)$$

On the other hand, depending on the nature of the primary particle (p, n), we have

$$(p)\ N = 3n_s/2 + l/4d - 1/2;$$
$$(n)\ N = 3n_s/2 + l/4d + 1;$$
$$(p, n)\ N = 3n_s/2 + l/4d + 1/2.$$

The value of γ was found by substituting N into Eqs. (9) and (10), and $\tan\theta_{1/2}$ was then obtained from Eq. (8).

The analysis of all reduced showers (138 in all) was carried out from the point of view of the interaction with a column of nucleons in the tunnel. The dependence of $\theta_{1/2}$ on n_s was plotted for different tunnel lengths l/d (Fig. 4). Curves for proton-produced stars were used (for stars with primary neutrons the curves are flatter and lie lower) and experimental points were marked in coordinates $\theta_{1/2}$, n_s for showers produced by protons and neutrons. The diameters of emulsion nuclei range from 1 to 7.54 in units of d. The parameter l/d can, therefore, vary for emulsion from 0 to 7.54.

If the tunnel-effect model is correct, the experimental points should all lie below the curve corresponding to $l/d = 7.54$. In reality it was found that a considerable part of the points fell above the curve (cf. Fig. 4).

The large scatter of experimental points cannot be explained by fluctuations of n_s and $\theta_{1/2}$. In fact, allowing for fluctuations, the number of particles in the forward and backward cones in the c.m.s. is

$$n_s/2 \pm \sqrt{n_s/2}.$$

The angle $\theta_{1/2}$ also fluctuates from $\theta_{1/2\min}$ to $\theta_{1/2\max}$, representing the angles that contain $n_s/2 \mp \sqrt{n_s/2}$ particles (in the c.m.s.).

For an isotropic distribution of particles in the c.m.s. we know, on one hand, that the fraction of particles f contained within the angle $\theta_f^* = \pi/2 \mp \Delta\theta$ is $f = (1 - \cos\theta_f^*)/2$. On the other hand,

$$f = (n_s/2 \pm \sqrt{n_s/2})/n_s.$$

It follows hence that $\cos\theta_f^* = \mp\sqrt{2/n_s}$. Substituting this values into Eq. (2) we obtain

$$\tan\theta_{1/2}\left(\begin{smallmatrix}\min\\ \max\end{smallmatrix}\right) = \frac{\sqrt{n_s-2}}{\sqrt{n_s}\pm\sqrt{2}}\,\frac{\tan\theta_{1/2}}{\sqrt{1+\tan^2\theta_{1/2}}}\ . \qquad (11)$$

It can be seen from Fig. 5 that the width of fluctuation strips does not include the expected amount of experimental points and, consequently, fluctuations of the multiplicity and of the angle $\theta_{1/2}$ cannot explain the observed relatively large number of showers corresponding to tunnel lengths > 7.54. We therefore plotted curves for larger l/d (up to $l/d = 22$), in order to encompass all showers studied. The points corresponding to these showers lie below the curve $l/d = 7.54$; in extensive showers one must assume, for agreement with the theory, that $\theta_{1/2}$ may in some cases be 3 to 5 times larger. Such an increase may be due to secondary interactions in the target nucleus.

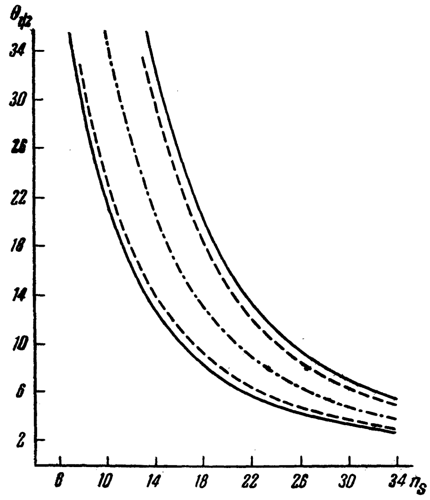

FIG. 5. Dependence of $\theta_{1/2}$ on n_s for $l/d = 7.54$ and fluctuation curves: dashed – for fluctuations of $\theta_{1/2}$, solid – for fluctuation of n_s.

The experimental points in Fig. 4 for showers with $E_0 > 50$ Bev were divided with respect to tunnel length (points lying above dotted curves were not taken into account). A histogram was then constructed of the distribution of showers with respect to l/d and compared with the computed histogram of the tunnel-length distribution of the relative number of interactions with emulsion nuclei.

If the interaction cross section is equal to the geometrical one $\sigma = \sigma_0 A^{2/3}$, then the number of interactions with a given nucleus is $n = \sigma q S t$, where q is the number of nuclei per unit volume, S is the flux of primary nucleons with a given primary energy, and t is the time. The total number of interactions can be obtained by integrating over the nuclei of all elements of the emulsion. The relative number of interactions with each nucleus is as follows:

I	Ag	Br	S	O	N	C	H
0.0082	0.3068	0.3815	0.0032	0.1037	0.0273	0.1154	0.0542

If we represent the nucleus by a spherical continuous body of radius $r_0 A^{1/3}$, we can relatively easily calculate the relative number of interactions in the emulsion for different tunnel lengths. The results of the calculation for Ilford G-5 emulsion are:

$l/d =$	0–1	1–2	2–3	3–4	4–5	5–6	6–7	7–7.54
	0.089	0.104	0.173	0.182	0.139	0.168	0.132	0.013

These data served as the basis for the histogram and were compared with the experimental histograms (Figs. 6 and 7).

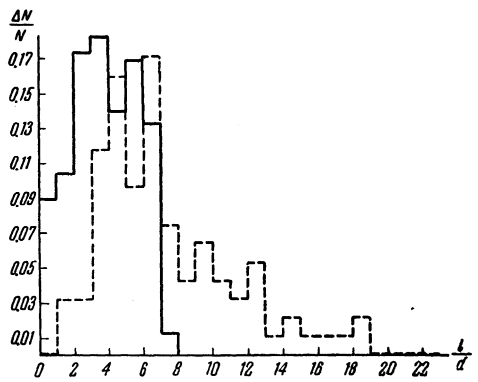

FIG. 6. Histograms of the tunnel-length distribution ($0 \leq l/d \leq 7.54$) of interactions. (Solid line – theoretical). Experimental histogram (dashed) includes errors. $E_0 > 50$ Bev. (94 showers).

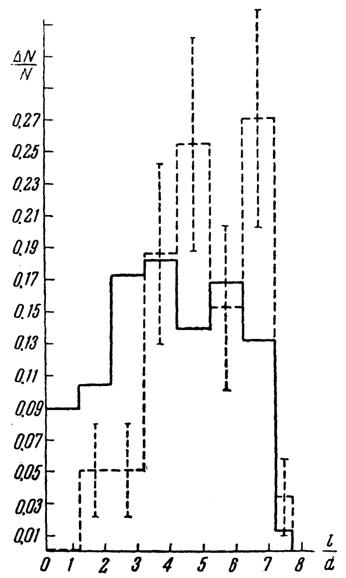

FIG. 7. Histograms of the tunnel-length distribution ($0 \leq l/d \leq 22$) of showers with $E_0 > 50$ Bev. Solid line – theoretical, dashed – experimental (59 events).

It can be seen that the experimental histogram is shifted towards longer tunnels. The same result follows from an estimate of the ratio of the number of interactions N_1 for longer tunnels

($7.54 \geq l/d > 3.6$) to the number of interactions N_2 involving shorter tunnels ($3.6 \geq l/d \geq 0$): the calculated ratio in emulsion is $N_1/N_2 = 1.45$, while the observed is $N_1/N_2 = 5.39 \pm 0.30$ for $E_0 > 50$ Bev and $N_1/N_2 = 2.92 \pm 0.31$ for $E_0 > 100$ Bev.

We have studied the variation of Eq. (1) and the form of dependence of $\theta_{1/2}$ on n_s when conditions (b) and (c) are not satisfied.

Let the condition (a) be satisfied, i.e., let the showers have a large multiplicity. We shall find γ for the case when the emission of particles in the c.m.s. is asymmetric with respect to the plane passing through the center of mass and perpendicular to the direction of the nucleon, but is symmetric in the system of equal velocities. In such a system the velocity of the incident nucleon (v_1'), of a tunnel nucleon (v_2'), and of the system (v_p') are equal.

The energy of a tunnel nucleon γ' is given, for $\beta_\pi' = 1$, by the expression:

$$\gamma' = 1/\sqrt{1-\beta_p'^2},\ \beta_p' = v_p'/C = 1/\sqrt{1+\tan^2\theta_{1/2}}.$$

The nucleon velocity v in the laboratory system and v_1' in the equal-velocity system are connected by the relation:

$$\beta = 2\beta_p'/(1+\beta_p'^2).$$

The primary particle energy in l.s. is then

$$\gamma = 1 + 2/\tan^2\theta_{1/2}.$$

If $\beta_\pi' \neq 1$ and $m = \beta_p'/\beta_\pi'$, we have

$$\gamma = 2/m^2\tan^2\theta_{1/2} - 1.$$

and the relation between $\theta_{1/2}$ and n_s is:

$$N^4/[0.84(l/d+1)]^4 = 2/m^2\tan^2\theta_{1/2} - 1 \quad \text{for } l/d \leqslant 3.7,$$
$$N^4/[1.55\,(l/d-0.25)^{3/4}]^4 = 2/m^2\tan^2\theta_{1/2} - 1 \quad \text{for } l/d > 3.7. \tag{12}$$

The dependence of $\theta_{1/2}$ on n_s for different m, calculated according to Eq. (12) for $l/d = 7.54$, is shown in Fig. 8. For comparison, analogous curves for the c.m.s. are shown in the figure as well, the energy of the primary for a symmetric distribution in the c.m.s. being defined by the formula

$$\gamma = \frac{(l/d)\,(1 - m^2\tan^2\theta_{1/2}) + \sqrt{(l/d)^2 + m^2\tan^2\theta_{1/2}\,(1-l/d)^2}}{m^2\tan^2\theta_{1/2}}. \tag{13}$$

The curves, obtained under the assumption of a symmetric emission in the equal-velocity system, lie below the corresponding curves in the c.m.s. since $\beta_p' \geq \beta_c'$. If we drop conditions (b) and (c), a still larger number of experimental points will fall above the curve for $l/d = 7.54$ and, consequently, will not be explained by the multiple meson production theory.

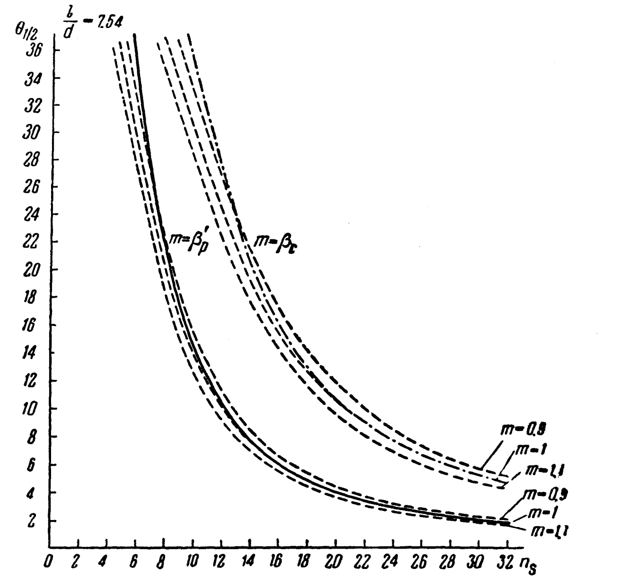

FIG. 8. Dependence of $\theta_{1/2}$ on n_s for different m. Top curves calculated assuming symmetry in c.m.s., bottom – assuming symmetry in the equal velocity system.

In the above calculations the energies were estimated neglecting the energy spectrum of shower particles. Such a procedure decreases the primary energy. If we correct the value of γ_c according to reference 2, the calculated curves fall lower, as can be seen in Fig. 9.

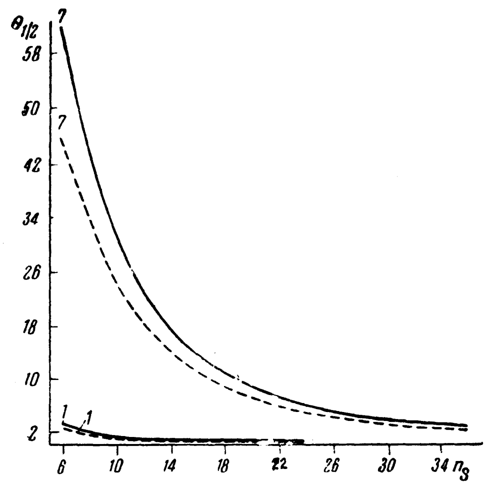

FIG. 9. Curves for $l/d = 1$ and 7, calculated assuming a power energy spectrum of shower particles (dashed), and assuming that the particles are monoenergetic ($\beta_\pi^* = 1$) (solid).

CONCLUSION

The integral angular distribution of shower particles in stars produced by 5×10^{10} to 10^{12} ev primaries is asymmetric about $f = 1/2$. An analysis of the symmetry of the integral-distribution curve shows that the angular distribution of shower particles in the c.m.s. is nearly isotropic. The experimentally observed relation between n_s and $\theta_{1/2}$ is greatly different from that calculated on basis of the multiple meson production theory. The discrepancy cannot be explained by fluctuations of multiplicity and angle $\theta_{1/2}$.

The observed values of n_s and $\theta_{1/2}$ can be explained if one assumes the production of mesons in secondary interactions in about 40% of showers produced in the stratosphere at 30 to 33 km altitude.

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