A	136	180	184	212	264	276	312	372	392
I/I_{hydr}	10.5	8.8	7.9	10.4	9.4	5.0	6.5	6	15.6

where $X \equiv k_0R_0$, and $X^2 \approx 100$. Therefore the range of applicability of perturbation theory for the moment of inertia is determined, strictly speaking, by $X^2\alpha_2$ since the states with $l = 0; 1; \ldots$ give a large contribution. However, one can expect that the averaged value of the moment will be correct over a substantially larger range of α_2. This can be explained as follows. We divide all states into two groups, states with large l and states with small l, where the latter contain the cases $l = 0; 1; \ldots$. Both groups will give approximately equal but opposite contributions to the moment of inertia, with an appreciable mutual cancellation of terms containing the parameter $(\alpha_2X^2)^2$ taking place. This cancellation, incidentally, turns out to be incomplete. This explains the obtained oscillations of the moment of inertia as a function of A.

[1] D. Inglis, Phys. Rev. **96**, 1059 (1954).
[2] D. Inglis, Phys. Rev. **103**, 1786 (1956).
[3] A. Bohr and B. Mottelson, Kgl. Dansk. Vidensk. Selsk. Mat.-Fys. Medd, **30**, No. 1 (1955).

Translated by M. Danos
32

SOVIET PHYSICS JETP VOLUME 35 (8), NUMBER 1 JANUARY, 1959

ON THE POLARIZATION OF RECOIL NUCLEONS IN THE PHOTOPRODUCTION OF PIONS

N. F. NELIPA and V. A. FEOKTISTOV

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

Submitted to JETP editor February 24, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **35**, 244-248 (July, 1958)

A general expression for the polarization of recoil nucleons appearing during the production of pions by photons has been obtained on the basis of the momentum and parity conservation laws. As an example, an expression is derived for the polarization in pion production in s, p, and d states.

1. Measurement of the polarization of the recoil nucleons that appear in photoproduction of mesons would help clarify, in principle, many important problems connected with the difference between the Fermi and Yang solutions, with the determination of small phase shifts in meson-nucleon scattering, with the elimination of the Minami ambiguity, etc.

A theoretical study of the problems of polarization can be made, roughly speaking, in two ways. The first,[1] based on the use of the density matrix, leads to the most general expressions for polarization, a particular case of interest to us being the expression for the polarization P of recoil nuclei in meson photoproduction. The second method, which employs the phenomenological scattering S matrix,[2] is simpler, albeit more limited. An expression for P, was obtained by the last method in reference 3.*

In the present work we should like to call attention to still another possibility of obtaining a general expression for P within the framework of the S matrix. Unlike the authors of reference 3, we obtained a more general expression for P, in which summation over the spin projections of the initial particles leads to the Racah coefficient.† The use of such a formula facilitates the calcula-

*An expression for P, in the particular case when the mesons are produced only in the s and p states, is given by Fel'd[4] without proof.

†Naturally, the expression we obtained for P is the same as obtained with the aid of the density matrix.

tions considerably, for tables are available for the coefficients that enter into the formula. By way of an example, we have calculated the expression for P in that case when the mesons are produced in the s, p, and d states. If d-state meson production is disregarded, our expression for P goes into the corresponding expression of Fel'd.[4]

2. We shall consider the reaction

$$\gamma + N \rightarrow N' + \pi. \tag{1}$$

To simplify the calculations we assume that the incident photon moves along the z axis and that the nucleon is emitted in the xz plane. Then the non-vanishing nucleon polarization will be that along the y axis, i.e., we must find an expression for the following quantity

$$P = (d\sigma_+ - d\sigma_-)/(d\sigma_+ + d\sigma_-), \tag{2}$$

where $d\sigma_+$ and $d\sigma_-$ are the differential cross sections for meson photoproduction, corresponding to the production of recoil nucleons with spins along and against the y axis. We calculate $d\sigma_+$ and $d\sigma_-$ by the same method as used to derive an expression for the differential cross section for photoproduction (see for example reference 5 and the bibliography contained therein). The only difference is that we do not sum over the finite nucleon spin projections, but calculate the cross sections for the two spin projections separately, thus obtaining $d\sigma_+$ and $d\sigma_-$.

We shall perform the calculations in the center-of-mass system. The initial photon-nucleon system can be described in terms of the total angular momentum L of the incident photon, for the electric and magnetic multiplicities $p = 0$ or 1 respectively, and a nucleon spin I. The final meson-nucleon system will be described by a relative orbital meson momentum l' and a total meson and nucleon spin s', so that $s' = 1/2$. The probability of the transition from a photon-nucleon with total momentum $J = L + I$ into a nucleon-meson state having the same total momentum will be characterized by the matrix $S^J_{l's',LI}$. Then the amplitude of process (1), corresponding to the production of particles with definite values of each spin projection, is written as follows:[5]

$$f = \sum_{JLpl'} i^{L-l'+p} \sqrt{\frac{\pi}{2}} \times (2L+1)^{1/2} m^p C^{JM}_{Lm;\, Im_I} C^{JM}_{l'\mu';\, s'm_{s'}} S^J_{l's';\, LI} Y^{\mu'}_{l'}(\vartheta). \tag{3}$$

Here $m = \pm 1$ (two transverse photon polarizations), $C^{JM}_{j_1m_1j_2m_2}$ are the Clebsch-Gordan coefficients, $Y^{\mu'}_{l'}(\vartheta)$ spherical functions, ϑ the angle between the directions of motion of the photon and meson, and m_I, $m_{s'}$, μ' are the projections of I, s' and l'. Here $m_{s'} = 1/2$ corresponds to a recoil-nucleon spin directed along the z axis (let this state be described by the function α), and $m_{s'} = -1/2$ is directed along the negative z axis (this state is described by the function β). We are interested in the polarization of the recoil nucleons along the y axis, i.e., the relation between the state of a nucleon with spins along the positive y axis (this state will be described by the function γ) and the state of one with spins along the negative y axis (function δ). The functions α and β are expressed in terms of the functions γ and δ as follows:

$$\alpha = (\gamma + \delta)/\sqrt{2}, \quad \beta = -i(\gamma - \delta)/\sqrt{2}. \tag{4}$$

Taking these relations into account, the expressions for the amplitudes f_+ and f_-, for meson photoproduction accompanied by production of recoil nucleons with spins along the positive and negative y axis, are written

$$f_\pm = \sum_{JLpl'} i^{L-l'+p} \frac{\sqrt{\pi}}{2} (2L+1)^{1/2} m^p C^{JM}_{LmIm_I} S^J_{l's',LI} Y^{\mu'}_{l'}(\vartheta) \times [C^{JM}_{l'\mu'^1/_2{}^1/_2} \mp C^{JM}_{l'\mu'^1/_2-^1/_2}]. \tag{5}$$

The differential cross section averaged over the projections of the initial spins (m and m_I) will be of the form

$$d\sigma_\pm = \frac{1}{4k^2} \sum_{mm_I} |f_\pm|^2 d\Omega, \tag{6}$$

where k is the wave number of the photon.

The expressions for $d\sigma_+$ and $d\sigma_-$ contain terms with products of the Clebsch-Gordan coefficients, both with equal signs of the projections $m_{s'}$ (denoted by a_+ and a_-) and with different signs of $m_{s'}$ (i.e., b_+ and b_-). It turns out that $a_+ = a_- = a$ and $b_+ = -b_-$, i.e.,

$$d\sigma_+ = a\,d\Omega + d\sigma_n, \quad d\sigma_- = a\,d\Omega - d\sigma_n, \tag{6'}$$

where $d\sigma_n = b_+ d\Omega$.

Using the relation

$$Y^{\mu*}_l(\vartheta)\, Y^{\mu'}_{l'}(\vartheta) = (-1)^\mu \sum_{n=|l-l'|}^{l+l'} \sqrt{\frac{(2l+1)(2l'+1)}{4\pi(2n+1)}}\, C^{n0}_{l'0l0} C^{nM}_{l-\mu l'\mu'} Y^M_n(\vartheta), \tag{7}$$

we obtain

$$d\sigma_n = \frac{1}{16k^2} \sum_{\substack{J_1L_1p_1l'_1\\ J_2L_2p_2l'_2 n}} i^{L_1-L_2-l'_1+l'_2+p_1-p_2} \sqrt{\pi} \Big[(2L_1+1)(2L_2+1)$$

$$\times (2l'_1+1)(2l'_2+1)\frac{1}{(2n+1)}\Big]^{1/2} \mathrm{Im}\{S^*(1)\,S(2)\} \sum_{mm_I} (-1)^{\mu'_2} m^{p_1+p_2} C^{J_1M_1}_{L_1mIm_I} \tag{8}$$

$$\times C^{J_2M_2}_{L_2mIm_I} C^{n0}_{l'_10l'_20} C^{n\mu'_1-\mu'_2}_{l'_1\mu'_1 l'_2-\mu'_2} \Big(C^{J_1M_1}_{l'_1\mu'_1 1/2\, 1/2} C^{J_2M_2}_{l'_2\mu'_2 1/2\, -1/2} - C^{J_1M_1}_{l'_1\mu'_1 1/2\, -1/2} C^{J_2M_2}_{l'_2\mu'_2 1/2\, 1/2} \Big) Y^{\mu'_1-\mu'_2}_{n}(\vartheta)\, d\Omega.$$

Inserting (6′) into (2) and bearing in mind that $d\sigma_+ + d\sigma_- = d\sigma$, we find

$$P = 2d\sigma_n/d\sigma. \tag{9}$$

The quantity P can be sought with the aid of (8), but this leads to rather cumbersome computations. The calculations become much simpler if the summation in expression (8) is over the projections of the spins m_I and m. To perform this summation, we use the following relations:[6]

$$C^{e\alpha+\beta}_{a\alpha b\beta} C^{c\alpha+\beta+\delta}_{e\alpha+\beta d\delta} = \sum_f \sqrt{(2e+1)(2f+1)}\, C^{c\alpha+\beta+\delta}_{a\alpha f\beta+\delta} C^{f\beta+\delta}_{b\beta d\delta}\, W(abcd;ef), \tag{10}$$

$$\sum_\beta C^{e\alpha+\beta}_{a\alpha b\beta} C^{c\gamma}_{c\alpha+\beta d\gamma-\alpha-\beta} C^{f\gamma-\alpha}_{b\beta d\gamma-\alpha-\beta} = \sqrt{(2e+1)(2f+1)}\, C^{c\gamma}_{a\alpha f\gamma-\alpha}\, W(abcd;ef), \tag{11}$$

where W(abcd; ef) are the Racah coefficients (the Latin and Greek letters denote momenta and projections of momenta respectively).

Using the symmetry properties of the Clebsch-Gordan coefficients together with relation (10) and (11) we get

$$d\sigma_n = \frac{1}{4k^2} \sum i^{L_1-l'_1+p_1-L_2+l'_2-p_2} \sqrt{\frac{\pi}{2}} [(2L_1+1)(2L_2+1)(2l'_1+1)(2l'_2+1)]^{1/2}$$

$$\times (2J_1+1)(2J_2+1)\left[\frac{2n(n+1)}{2n+1}\right]^{1/2} C^{n0}_{l'_10l'_20} C^{n0}_{L_11L_2-1}\, W\left(L_1J_1L_2J_2;\ \tfrac{1}{2}\, n\right) \tag{12}$$

$$\times \Big[W\left(\tfrac{1}{2}\, l'_1 n - \tfrac{1}{2}\, l'_2;\ J_1\, n\right) W\left(l'_2 J_1\ \tfrac{1}{2} n;\ n - \tfrac{1}{2} J_2\right) - W\left(\tfrac{1}{2}\, l'_1 n + \tfrac{1}{2}\, l'_2;\ J_1\, n\right) W\left(l'_2 J_1 \tfrac{1}{2} n;\ n + \tfrac{1}{2} J_2\right)\Big]$$

$$\times \mathrm{Im}\,\{S^*(1)\,S(2)\}\, Y_n^{-1} d\Omega.$$

The summation is over $J_1J_2L_1L_2p_1p_2l'_1l'_2$ and n. Using (12), and bearing (9) in mind, an expression can be found for P. Here it is necessary to employ the tabulated numerical values of the Racah coefficients[7] and of the Clebsch-Gordan coefficients.[8]

3. To illustrate the use of the formula obtained, let us find an expression for P for the case when the mesons are formed in the s, p and d states. The transitions possible in this case are listed in the table.

Transition	L	J	l'
E_{11}	1	$1/2\,-$	0
E_{13}	1	$3/2\,-$	2
M_{11}	1	$1/2\,+$	1
M_{13}	1	$3/2\,+$	1
E_{23}	2	$3/2\,+$	1
M_{23}	2	$3/2\,-$	2
M_{25}	2	$5/2\,-$	2
E_{35}	3	$5/2\,-$	2

The letters in the first column denote the matrix elements of the corresponding transitions; E and M represent electric and magnetic transitions respectively (the first index is determined by the quantity L, and the second equals 2J).

Taking these transitions into account, the expression for P becomes

$$P = \frac{\sin\vartheta\, d\Omega}{4k^2 d\sigma}\, \mathrm{Im}\Big\{E^*_{11}\Big[-M_{11} - \frac{1}{2} M_{13} - \frac{\sqrt{3}}{2} E_{23}\Big) + \Big(\frac{3}{2} E_{13} - \frac{3\sqrt{3}}{2} M_{23}$$

$$- \sqrt{3} M_{25} - \sqrt{6} E_{35}\Big)\cos\vartheta\Big] + M^*_{11}\Big[\Big(\frac{3}{2} M_{13} + \frac{3\sqrt{3}}{2} E_{23}\Big)\cos\vartheta$$

$$+\left(-\frac{1}{2}E_{13}+\frac{\sqrt{3}}{2}M_{23}+\frac{\sqrt{3}}{2}XM_{25}+\sqrt{\frac{3}{2}}\,XE_{35}\right)\Big]+M_{13}^{*}\Big[2E_{13}$$
$$+\sqrt{3}(1-3\cos^2\vartheta)M_{23}+\sqrt{3}\left(-1+\frac{1}{2}\cos^2\vartheta\right)M_{25}-\frac{\sqrt{6}}{8}XE_{35}\Big]$$
$$+E_{23}^{*}\Big[-\sqrt{3}(1-3\cos^2\vartheta)E_{13}-6\cos^2\vartheta M_{23}-\frac{3}{2}\cos^2\vartheta M_{25} \tag{13}$$
$$+\frac{3}{8}\sqrt{2}(-3-\cos^2\vartheta)E_{35}\Big]+E_{13}^{*}\Big[-\frac{5\sqrt{3}}{2}M_{25}-\frac{5\sqrt{6}}{56}(19+9\cos^2\vartheta)E_{35}\Big]\cos\vartheta$$
$$+M_{23}^{*}\Big[\frac{5}{14}(15+6\cos^2\vartheta)M_{25}+\frac{5\sqrt{2}}{56}(69+15\cos^2\vartheta)E_{35}\Big]\cos\vartheta\Big\},$$

where $X = (5\cos^2\vartheta - 1)$.

If meson production in the d state is disregarded (i.e., putting $E_{13} = M_{23} = M_{25} = E_{35} = 0$), Eq. (13) goes into the corresponding expression obtained by Fel'd (reference 4).*

[1] A. Simon, Phys. Rev. **92**, 1050 (1953).

[2] J. Blatt and V. E. Weiskopf, Theoretical Nuclear Physics, N. Y., 1952.

[3] Hayakaw, Kawaguchi, and Minami, Progr. Theoret. Phsy. **12**, 355 (1953).

[4] B. T. Fel'd, Nuovo cimento **12**, 425 (1954).

[5] N. F. Nelipa, Usp. Fiz. Nauk (in press).

[6] G. Ia. Liubarskii, Теория групп и ее применения в физике (Group Theory and its Applications in Physics), GITTL, M. 1957.

[7] Simon, VanderSluis, and Biedenharn, Oak Ridge National Laboratory report ORNL-1679.

[8] A. Simon, Oak Ridge National Laboratory Report ORNL-1718 (1954).

[9] Keck, Tollestrup, and Walker, Phys. Rev. **101**, 1159 (1956).

*It must be noted that the expression given in reference 9 for P, follows from our expression if we make, in addition, the following substitutions:

$$E_{11}\to E_{11},\ M_{11}\to -M_{11},\ M_{13}\to -M_{13},\ E_{23}\to E_{23}/2\sqrt{3}.$$

Translated by J. G. Adashko
33