

⁴Hermansfeldt, Maxson, Stähelin, and Allen
Phys. Rev. **107**, 641 (1957).

⁵Wu, Ambler, Hayward, Hoppes, and Hudson,
Phys. Rev. **105**, 1413 (1957).

⁶Ambler, Hayward, Hoppes, Hudson, and Wu,
Phys. Rev. **106**, 1361 (1957).

⁷Ambler, Hayward, Hoppes, and Hudson, Phys.
Rev. **108**, 503 (1957).

⁸Burgy, Epstein, Krohn, Novey, Raboy, Ringo,
and Telegdi, Phys. Rev. **107**, 1731 (1957).

⁹Goldhaber, Grodzins, and Sunyar, Phys. Rev.
109, 1015 (1958).

¹⁰L. Rosenson, Phys. Rev. **109**, 958 (1958).

Translated by W. H. Furry

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*POLARIZATION CORRELATION OF BETA PARTICLES AND GAMMA QUANTA IN
ALLOWED DECAY OF ORIENTED NUCLEI*

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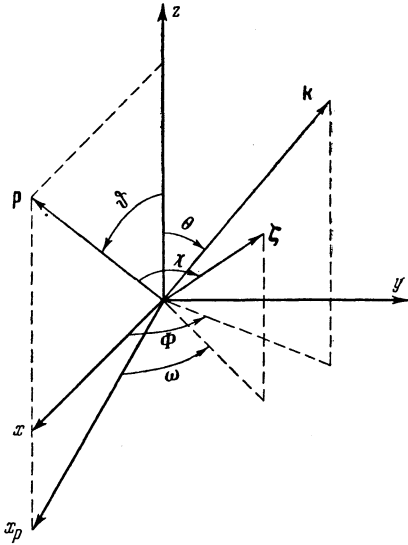
Formulas are obtained for the correlation between the polarization of particles and the circular polarization of the subsequent gamma quanta in allowed beta decay of oriented nuclei.

THE extensive study of angular and polarization correlation in beta transformations, undertaken recently, have made it possible to gather much important information on beta interactions. It was established that the assumed A-V interaction in parity nonconservation is apparently not in contradiction with existing experiments. However, there are clearly not enough data for an unambiguous statement. This is why the determination of the type of beta interaction remains the most important problem in the theory of beta decay. It is also desirable to know the values and the relative signs of the β -interaction constants. To explain these problems, it is desirable to study all aspects of the beta transformations and, in particular, to investigate the polarization correlation between the β -particles and subsequent gamma quanta in beta decay of oriented nuclei. The advantages of such experiments is that they can yield complete information on the β -interaction constants. The pseudoscalar interaction does not make a noticeable contribution in allowed beta transitions. This leaves therefore eight (generally speaking complex) constants c_S, c_T, c_V, c_A and $c'_S, c'_T, c'_V,$ and c'_A for the scalar, tensor, vector and axial-vector beta interactions, which must be determined

experimentally. The quantities c'_i , unlike c_i , enter into those interaction terms that vanish when parity is conserved (these terms contain an additional matrix $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$, where $\gamma_4 = -\beta$). A study of the correlation considered here, in accordance with (1), makes it possible to determine the real and imaginary parts of eight independent combinations of c_i and c'_i . The information obtained in this manner will actually be complete. Let us remark however that the quantity $\text{Im}(c_S c'_V^* + c'_S c_V^*)$ enters only with a small multiplier $\alpha Z/E$, where $\alpha = 1/137$, Z is the charge of the nucleus, and E the total energy of the beta electron (we use units in which $\hbar = m = c = 1$). Therefore, if the experimental accuracy is insufficient to discern quantities of order $\alpha Z/E$ from quantities of order $p/E \equiv v/c$, we obtain not 16 but only 15 independent relations for the 16 real quantities. In practice however, this causes no complications, for it is enough to take into account the results of any of the already-performed independent experiments to obtain more relations than necessary.

Since the procedural aspect of the calculation of the sought correlation has been treated in an earlier work by this author,¹ we merely cite the end results.

We introduce the following notation: $j_0, j_1,$ and j_2 are the angular moments of the nucleus for the β - γ transition $j_0(\beta)j_1(\gamma)j_2$; $\mu_0, \mu_1,$ and μ_2 are their projection on the z axis. $\mathbf{k}(k, \theta, \Phi)$ is the momentum of the gamma quantum, l is its multiplicity, $\sigma = 1$ or -1 determines the right-handed or left-handed circular polarization of the quantum; $\mathbf{p}(p, \vartheta, \varphi)$ is the electron momentum and $\boldsymbol{\xi}(l, \chi, \omega)$ is the pseudovector of its polarization in the rest system. The angles θ, Φ and ϑ, φ are given in a coordinate system in which the z axis is along the direction of the predominant orientation of the nuclear spins, while the angles χ and ω are in a coordinate system with the z axis along the direction of \mathbf{p} (see diagram).



The expression for the correlation between $j_0, \mathbf{p}, \mathbf{k},$ and $\boldsymbol{\xi}$ for $\sigma = \pm 1$ can be represented in the following form*

$$W(j_0, \mathbf{p}, \boldsymbol{\xi}, \mathbf{k}, \sigma) = \sum (-1)^\epsilon \sqrt{2S+1} h_g(j_0) B_{S\sigma} \times Z_{JgS}^{ba} F_{JgS}^{ba}(\mathbf{p}, \boldsymbol{\xi}, \mathbf{k}); \quad (1)$$

$$h_g(j_0) = \sum_{\mu_0} (-1)^{j_0 - \mu_0} C_{j_0 \mu_0 j_0 - \mu_0}^{g0} \omega(\mu_0); \quad (2)$$

$$B_{S\sigma} = U(j_2 l j_1 S; j_1 l) C_{JgS0}^{I\sigma}; \quad (3)$$

$$F_{JgS}^{ba}(\mathbf{p}, \boldsymbol{\xi}, \mathbf{k}) = 4\pi i^\lambda (-1)^{J-a} \sqrt{2a+1} \sum_{\sigma, \beta} C_{g0S\sigma}^{J\sigma} \times Y_{S\sigma}(\theta\Phi) D_{\sigma\beta}^{J*}(\varphi\theta) C_{a0b\beta}^{J\beta} Y_{b\beta}^*(\chi\omega). \quad (4)$$

The summation in (1) is over all possible values of the indices $a, d, J, g,$ and S within the following limits: $b + J \geq a \geq |b - J|$; $0 \leq S \leq 2I$; $J + S \geq g \geq |J - S|$; $b = 0, 1$; $J = 0, 1$; $2\lambda = 1 - (-1)^{a+b+S+g}$.

The quantities $C_{a\alpha b\beta}^{c\gamma}$ are the known Clebsch-

*The common factors that do not affect the correlation are omitted everywhere.

Gordan coefficients. $w(\mu_0)$ is the probability of a given value of the projection μ_0 of an oriented nucleus. $h_g(j_0)$ determines the degree of orientation of the initial nuclei; the values for $f_g(j_0) = h_g(j_0)/j_0^g$ for various particular cases are given in reference 2. For aligned nuclei, g can be only even. The quantity

$$U(j_2 l j_1 S; j_1 l) = \sqrt{(2j_1 + 1)(2l + 1)} W(j_2 l j_1 S; j_1 l), \quad (5)$$

where $W(j_2 l j_1 S; j_1 l)$ is the Racah function. In the particular case when $S = 0$ or 1 , we have

$$B_{00} = 1, \quad B_{1\sigma} = \sigma \frac{j_1(j_1 + 1) - j_2(j_2 + 1) + l(l + 1)}{2l(l + 1)j_1(j_1 + 1)} \quad (6)$$

If the γ -quanta polarization is not investigated, it is necessary to take $B_S = B_{S1} + B_{S-1}$ instead of B_S . $B_S \neq 0$ only for even S

$$B_S = \left[1 - \frac{S(S+1)}{2l(l+1)} \right] U(j_2 l j_1 S; j_1 l) C_{JgS0}^{I0}. \quad (7)$$

If the observed β - γ cascade is of the form $j_0(\beta)j_1(\gamma_1)j_2(\gamma_2) \dots j_{N-1}(\gamma)j_N$ and if the experiment is aimed at investigation of the gamma quanta of the $j_{N-1}(\gamma)j_N$ transition, then, denoting the multiplicities of the quanta by $I_1, I_2, \dots, I_{N-1}, I$ respectively, we obtain a formula for the correlation, provided we multiply the expression $B_{S\sigma}$ in (1) by the product

$$\prod_{k=2}^{N-1} U(j_k I_k S_j j_{k-1}; j_{k-1} j_k). \quad (8)$$

The quantity $F_{JgS}^{ba}(\mathbf{p}, \boldsymbol{\xi}, \mathbf{k})$ depends only on the angles. Its explicit form is given in the Appendix for several specific cases.

We give here the values of the quantities Z_{JgS}^{ba} that enter into (1):

$$\begin{aligned} Z_{0gS}^{00} &= [M_0 - \gamma E^{-1} N_0 + U(g j_1 j_0 1; j_1 j_0) (M_1 - \gamma E^{-1} N_1)] \delta_{Sg}; \\ Z_{1gS}^{01} &= (2/3) U(j_0 S j_0 1; j_0 g) E^{-1} [-(p \operatorname{Re} Q_m + \alpha Z \operatorname{Im} Q_n) \delta_{gS \pm 1} + (p \operatorname{Im} Q_m - \alpha Z \operatorname{Re} Q_n) \delta_{gS}] \\ &\quad - \sqrt{2(2g+1)(2j_0+1)(2j_1+1)/3} X(j_1 j_0 1, j_1 j_0 1, Sg1) \\ &\quad \times E^{-1} (p \operatorname{Re} Q_1 + \alpha Z \operatorname{Im} Q_1); \\ Z_{1gS}^{10} &= (2/3) U(j_0 S j_0 1; j_0 g) [\operatorname{Re} D \delta_{Sg \pm 1} - \operatorname{Im} D \delta_{Sg}] \\ &\quad + (1/3) \sqrt{2(2g+1)(2j_0+1)(2j_1+1)/3} X(j_1 j_0 1, j_1 j_0 1, Sg1) G; \\ Z_{0gS}^{11} &= \sqrt{1/3} E^{-1} [p \operatorname{Re} Q_0 + \alpha Z \operatorname{Im} Q_0 + U(g j_1 j_0 1; j_1 j_0) (p \operatorname{Re} Q_1 + \alpha Z \operatorname{Im} Q_1)] \delta_{gS}; \\ Z_{1gS}^{11} &= (2/3) \sqrt{2/3} U(j_0 S j_0 1, j_0 g) E^{-1} [(p \operatorname{Re} Q_n + \alpha Z \operatorname{Im} Q_m) \delta_{gS} - (p \operatorname{Im} Q_n - \alpha Z \operatorname{Re} Q_m) \delta_{Sg \pm 1}] \\ &\quad - 2 \sqrt{2(2g+1)(2j_0+1)(2j_1+1)} X(j_1 j_0 1, j_1 j_0 1, Sg1) E^{-1} \\ &\quad \times (p \operatorname{Im} Q_1 - \alpha Z \operatorname{Re} Q_1) \delta_{Sg \pm 1}; \\ Z_{1gS}^{12} &= (2/3) (1 - \gamma E^{-1}) \{ (\sqrt{2/3}) U(j_0 S j_0 1, j_0 g) [\operatorname{Im} (D_0 + D_1)] \delta_{Sg} \\ &\quad - \operatorname{Re} (D_0 + D_1) \delta_{Sg \pm 1} - \sqrt{2(2g+1)(2j_0+1)(2j_1+1)/3} \\ &\quad \times X(j_1 j_0 1, j_1 j_0 1, Sg1) (M_1' + N_1) \}; \\ \gamma &= \sqrt{1 - (\alpha Z)^2}. \end{aligned}$$

The quantities $X(abc, def, ghi)$ are the Fano functions. Their explicit form, many of their properties, and particular values, are given in references 3 and 4. The number triplets abc , def , and ghi can be transposed cyclically without changing the function X . Non-cyclic transposition of the numbers changes the function by a factor $(-1)^\nu$, where $\nu = a + b + c + d + e + f + g + h + i$. $X(abc, def, ghi) = X(adg, beh, cfi)$ and $X(abc, def, gh0) = U(gdbc; ae) \delta_{cf} \delta_{gh} / \sqrt{(2g+1)(2c+1)(2a+1)(2e+1)'}.$

The quantities M_0, N_0, Q_m , etc., which enter into Z_{JgS}^{ba} have the following explicit form:

$$\begin{aligned} M_0 &= (|c_S|^2 + |c'_S|^2) |K_S|^2 + (|c_V|^2 + |c'_V|^2) |K_V|^2; \\ M_1 &= (|c_T|^2 + |c'_T|^2) |K_T|^2 + (|c_A|^2 + |c'_A|^2) |K_A|^2; \\ N_0 &= 2\text{Re}(c_S c'_V + c'_S c_V) K_S K_V^*; \\ N_1 &= 2\text{Re}(c_T c'_A + c'_T c_A) K_T K_A^*; \\ \text{Re } Q_0 &= \text{Re}(c_S c'_S |K_S|^2 - c_V c'_V |K_V|^2); \\ \text{Im } Q_0 &= \text{Im}(c_V c'_S + c'_V c_S) K_V K_S^*; \\ \text{Re } Q_1 &= \text{Re}(c_T c'_T |K_T|^2 - c_A c'_A |K_A|^2); \\ \text{Im } Q_1 &= \text{Im}(c_A c'_T + c'_A c_T) K_A K_T^*; \\ Q_m &= (c'_S c'_T + c_S c_T) K_S K_T^* - (c_V c'_A + c'_V c_A) K_V K_A^*; \\ Q_n &= (c_V c'_T + c'_V c_T) K_V K_T^* - (c_S c'_A + c'_S c_A) K_S K_A^*; \\ D_0 &= (c_S c'_T + c'_S c_T) K_S K_T^* + (c_V c'_A + c'_V c_A) K_V K_A^*; \\ D_1 &= (c_V c'_T + c'_V c_T) K_V K_T^* + (c_S c'_A + c'_S c_A) K_S K_A^*; \\ D &= (D_0 - \gamma E^{-1} D_1) - 2(D_1 - \gamma E^{-1} D_0); \\ G &= (M_1 - \gamma E^{-1} N_1) - 2(N_1 - \gamma E^{-1} M_1). \end{aligned}$$

The quantities K_S, K_V, K_T , and K_E are the nuclear matrix elements for the S, V, T, and A interactions

$$K_S = \int \psi_{i,\mu_1} \beta \psi_{j,\mu_2} d\tau \equiv \int \beta; \quad (9)$$

$$K_T = -[C_{10j_i\mu_1}^{j_0\mu_2}]^{-1} \int \psi_{i,\mu_1}^* \beta \psi_{j,\mu_2} d\tau \equiv -\int \beta \sigma; \quad (10)$$

K_V and K_A differ from K_S and K_T in the absence of the matrix β under the integral sign. In the non-relativistic approximation for the nucleons we have

$$K_V \equiv \int 1 = -K_S, \quad K_A \equiv \int \sigma = -K_T. \quad (11)$$

Since the strong interactions are apparently invariant under time inversion, the phase shift between K_S, K_T , etc. is zero or π .

If the beta interaction is invariant under time inversion, the constants c and c' should be real. In references 5 to 7 it was proposed to study the β - γ correlation in oriented nuclei to determine $\text{Im } Q_m$. The first experimental results⁸ do not lead to definite conclusions. Assuming the AV

or TS interaction to take place, proof of the absence of $\text{Im } Q_m$ would be sufficient to establish the invariance of the beta interaction under time inversion. However, if some other combination of the β -interaction variants takes place, say TV or AVS etc, it becomes necessary to study the phenomena that are determined by $\text{Im } Q_n, \text{Im } Q_1$, etc. To clarify the problem of the invariance under time inversion, it is not essential to investigate the correlation (1) completely. It is enough to study the polarization of the electrons emitted by the polarized nuclei. The corresponding formulas are given in our earlier work.¹

Let us note that it is not essential to have oriented nuclei in order to investigate this correlation. The same results are obtained by studying the correlation between the electron polarization and the circular polarization of the subsequent gamma quanta. For a given circular polarization σ , the probability of definite $\mathbf{p}, \boldsymbol{\xi}$, and \mathbf{k} is of the form

$$\begin{aligned} W(\mathbf{p}, \boldsymbol{\xi}, \mathbf{k}, \sigma) &= \sum_{L=0}^1 [M_L - \gamma E^{-1} N_L] + E^{-1} [\rho \text{Re}(Q_0 + Q_1) \\ &+ \alpha Z \text{Im}(Q_0 + Q_1)] \cos \chi - B_{1\sigma} \{ [-2\rho \text{Re } Q_m - 2\alpha Z \text{Im } Q_n \\ &+ \lambda_{j_1 j_0} (\rho \text{Re } Q_1 + \alpha Z \text{Im } Q_1)] E^{-1} \cos \theta + (1/3) [2\text{Re } D \\ &+ \lambda_{j_1 j_0} G] [\cos \theta \cos \chi + \sin \theta \sin \chi \cos \omega] + [2\rho \text{Im } Q_n \\ &- 2\alpha Z \text{Re } Q_m - \lambda_{j_1 j_0} (\rho \text{Im } Q_1 - \alpha Z \text{Re } Q_1)] E^{-1} \\ &\times \sin \theta \sin \chi \sin \omega + (1/3) (1 - \gamma E^{-1}) [2\text{Re}(D_0 + D_1) \\ &- \lambda_{j_1 j_0} (M_1 + N_1)] [2\cos \theta \cos \chi - \sin \theta \sin \chi \cos \omega]; \end{aligned} \quad (12)$$

$$\lambda_{j_1 j_0} = [j_1(j_1 + 1) - j_0(j_0 + 1) + 2] [2\sqrt{j_1(j_1 + 1)}]^{-1}. \quad (13)$$

Since the nuclei are not oriented in the given case, the angles θ and χ are measured from the direction of \mathbf{p} , which was taken to be the z axis. The plane (\mathbf{pk}) is chosen the same as the (zx) plane and the angle ω is measured from the x axis in a right-handed system of coordinates.

If there exists a $t \rightarrow -t$ invariance then, according to (12), the probability of observing an electron polarized with or against the y axis ($\chi = \pi/2, \omega = t\pi/2$) is very small. In this case $\text{Im } Q_n = \text{Im } Q_1 = 0$, and the projection of the polarization vector (14) on the y axis is proportional to a small quantity, namely $2\alpha Z E^{-1} \times \text{Re}(Q_m - \lambda_{j_1 j_0} Q_1)$. Observation of a noticeable electron polarization ($\sim p/A$ and not $\sim \alpha Z/E$) along this axis would be evidence of the existence of a contribution from the VT or AT interaction and of violation of the $t \rightarrow -t$ invariance.

Since the observation of the circular polarization of gamma quanta yields information on the orientation of the nuclear spin after the beta decay,

a study of the correlation (12) is equivalent to a study of the polarization of the electrons emitted by oriented nuclei. To proceed to this case and to obtain $W(j_0, \mathbf{p}, \xi)$, it is sufficient to replace

$B_{1\sigma}$ in formula (12) by $-h_1(j_0) = -\sum_{\mu_0} \mu_0 w(\mu_0) / \sqrt{j_0(j_0+1)}$, to replace $\lambda_{j_1 j_0}$ by $\lambda_{j_0 j_1}$, and to replace θ by ϑ .

If we know the probability (1), then the polarization of the electrons emitted in the beta decay is determined by the vector $\bar{\xi}$, where $|\bar{\xi}| \leq 1$ gives the degree of polarization. Let $w(\chi, \omega)$ denote, in abbreviated form, the probability $W(j_0, \mathbf{p}, \xi, \mathbf{k}, \sigma)$ as a function of the angles χ and ω . Then the expression for the projection of $\bar{\xi}$ on the coordinate axes (in a system with z parallel to \mathbf{p}) can be represented in the form

$$\begin{aligned} \bar{\xi}_k &= [w_k - w_{-k}] / [w_k + w_{-k}], \quad k \rightarrow x, y, z, \\ w_x &= w(0, 0) \quad w_{-x} = w(\pi, 0) \quad w_y = w\left(\frac{\pi}{2}, 0\right), \\ w_{-y} &= w\left(\frac{\pi}{2}, \pi\right) \quad w_z = w\left(\frac{\pi}{2}, \frac{\pi}{2}\right) \quad w_{-z} = w\left(\frac{\pi}{2}, -\frac{\pi}{2}\right). \end{aligned} \quad (14)$$

All the above formulas pertain to a β^- decay. For β^+ decay it is necessary to make the substitutions

$$\begin{aligned} c_S &\rightarrow -c_S^*, \quad c_V \rightarrow c_V^*, \quad c_T \rightarrow c_T^*, \quad c_A \rightarrow -c_A^*, \\ c'_S &\rightarrow c'_S, \quad c'_V \rightarrow -c'_V, \quad c'_T \rightarrow -c'_T, \quad c'_A \rightarrow c'_A. \end{aligned} \quad (15)$$

If the polarization of the beta particles is not being investigated, it is necessary to put $b = 0$ in formula (1). Therefore only Z_{0gS}^{00} and Z_{1gS}^{01} will enter into $W(j_0, \mathbf{p}, \mathbf{k}, \sigma)$. When $S = g$, the quantity Z_{1gS}^{01} contains the term $(p/E) \text{Im } Q_m$. This makes it possible, by investigating $W(j_0, \mathbf{p}, \mathbf{k})$ or $W(j_0, \mathbf{p}, \sigma)$ to obtain information on the $t \rightarrow -t$ invariance for the TS or AV interaction.

APPENDIX

The expression (4) contains the quantity $D_{\alpha\beta}^j(\varphi \neq 0)$. This equals

$$\begin{aligned} D_{\alpha\beta}^0 &= \delta_{\alpha 0} \delta_{\beta 0}, \quad D_{00}^1 = \cos \vartheta, \\ 2D_{11}^1 &= 2D_{-1-1}^{1*} = (1 + \cos \vartheta) e^{i\varphi}, \\ 2D_{1-1}^1 &= 2D_{-11}^{1*} = (1 - \cos \vartheta) e^{i\varphi}; \\ \sqrt{2}D_{0-1}^1 &= -\sqrt{2}D_{01}^1 = \sin \vartheta, \\ \sqrt{2}D_{10}^1 &= -\sqrt{2}D_{-10}^1 = \sin \vartheta e^{i\varphi}. \end{aligned}$$

References 1 and 5 give particular values of the quantity $F_{SgJ}(\vartheta \varphi \theta \Phi)$, which in our notation is $i^{-\lambda} F_{JgS}^{0J}(\mathbf{p}, \xi, \mathbf{k})$. The following are a few particular values of $F_{JgS}^{ba}(\mathbf{p}, \xi, \mathbf{k})$ for $b \neq 0$:

$$\begin{aligned} F_{000}^{11} &= \sqrt{3} \cos \chi; \quad F_{110}^{11} = \frac{3}{\sqrt{2}} \sin \vartheta \sin \chi \sin \omega; \\ F_{101}^{11} &= 3\sqrt{3/2} \{ \cos \theta \sin \vartheta \sin \chi \sin \omega \\ &\quad + \sin \theta \sin \chi \cos \omega \sin(\Phi - \varphi) \\ &\quad - \cos \vartheta \sin \chi \sin \omega \sin \theta \cos(\Phi - \varphi) \}; \\ F_{110}^{10} &= -\sqrt{3} [\cos \vartheta \cos \chi - \sin \vartheta \sin \chi \cos \omega]; \\ F_{101}^{12} &= (3/\sqrt{2}) \{ \cos \theta [2 \cos \vartheta \cos \chi + \sin \chi \sin \vartheta \cos \omega] \\ &\quad + [2 \sin \vartheta \cos \chi - \sin \chi \cos \vartheta \cos \omega] \sin \theta \cos(\Phi - \varphi) \\ &\quad - \sin \chi \sin \omega \sin \theta \sin(\Phi - \varphi) \}; \\ F_{110}^{12} &= \sqrt{3/2} [2 \cos \vartheta \cos \chi + \sin \vartheta \sin \chi \cos \omega]. \end{aligned}$$

¹A. Z. Dolginov, Nucl. Phys. **5**, 512 (1958).

²I. A. M. Cox and H. A. Tolhoek, Physica **19**, 101 (1953).

³H. Matsunobo and H. Takebe, Progr. Theoret. Phys. **14**, 589 (1955).

⁴Kennedy, Sears, and Sharp, Chalk River Report CRT, 569, 1954.

⁵A. Z. Dolginov, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 1363 (1957), Soviet Phys. JETP **6**, 1047 (1958).

⁶M. Morita and R. S. Morita, Phys. Rev. **107**, 1316 (1957).

⁷R. B. Curtis and R. R. Lewis, Phys. Rev. **107**, 1381 (1957).

⁸Ambler, Hayward, Hoppes, and Hudson, Phys. Rev. **108**, 503 (1957).

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