**Reference 3 considers negative ion concentrations which fulfill the condition $b_p \varkappa / b_e \ll 1$. Then, as is easily seen from the expression for the flow given in reference 2, there must be a source of negative ions at the wall. We therefore consider the boundary condition $N_n(R) = 0$ not to be stringent enough.

††The inclusion of surface sources of negative ions will only shift the boundary between the regions toward the wall.

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SURFACE WAVES ON THE BOUNDARY OF A GYROTROPIC MEDIUM

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Submitted to JETP editor January 31, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 1635-1637 (June, 1958)

1. We consider the surface waves (s.w.) $\exp[i(hz - \omega t) + \gamma x]$ propagating along the interface $x = 0$ of two semi-infinite media. Medium 1 $(x > 0)$ is isotropic $(\epsilon = \epsilon_0,\ \mu = \mu_0)$. Medium 2 $(x < 0)$ is gyrotropic with a dielectric constant ϵ and magnetic permeability μ_{ik}:

$$\mu_{xx} = \mu_{zz} = \mu_1; \quad \mu_{yy} = \mu_3; \quad \mu_{xz} = -\mu_{zx} = i\mu_2, \tag{1}$$

that is, the direction of the external magnetizing field is such that $H_0 \parallel OY$. We shall consider H-mode surface waves $(H_z \neq 0)$ in a medium with tensor μ_{ik} (ferrites). All of our results will also be valid for media with tensor ϵ_{ik} (plasma, Hall effect etc.) when E, H, ϵ, μ_{ik}, ω are replaced by H, E, μ, ϵ_{ik}, $-\omega$.

2. From the continuity of E_y and H_z at $x = 0$ (H_z is given in terms of E_y in reference 2) we obtain an equation for $u = -hc/\omega$, the retardation coefficient of the wave ($u = c/v_\phi$, where v_ϕ is the phase velocity):

$$\mu_0 (u^2 - \varepsilon\mu_\perp)^{1/2} + \mu_\perp (u^2 - \varepsilon_0\mu_0)^{1/2} = \mu_0 \Gamma u, \tag{2}$$

$$\Gamma = \mu_2 / \mu_1.$$

Equation 2 was analyzed graphically. Some of the results follow. For $\Gamma > 0$ and $\mu_\perp > 0$, Eq. (2) has a real root if

$$\text{for } \varepsilon\mu_\perp > \varepsilon_0\mu_0:\ \mu_\perp + \mu_0 > \mu_0\Gamma > \mu_0 (1 - \varepsilon_0\mu_0 / \varepsilon\mu_\perp)^{1/2},$$
$$\text{for } \varepsilon\mu_\perp < \varepsilon_0\mu_0:\ \mu_\perp + \mu_0 > \mu_0\Gamma > \mu_0 (1 - \varepsilon_0\mu_\perp / \varepsilon\mu_0)^{1/2}. \tag{3}$$

This is the only root; the wave thus propagates in only one direction $(h < 0)$. For $\epsilon_0\mu_0 \neq \epsilon\mu_\perp$ slight gyrotropy $(\Gamma \ll 1)$ cannot invalidate the law for an isotropic boundary. Just as in the isotropic case,[1] a s.w. does not propagate for $\epsilon > 0$, $\mu > 0$. But with $\epsilon\mu_\perp$ close to $\epsilon_0\mu_0$ a unidirectional wave (only in the z direction) is possible even for slight gyrotropy (theoretically also for paramagnetics).

For $\mu_\perp < 0$, $\Gamma > 0$, and $|\mu_\perp| > \mu_0$ the condition for propagation of the direct wave $(h > 0)$ is $\mu_0\Gamma < |\mu_0 + \mu_\perp|$, while for the reverse wave $(h < 0)$, $\Gamma < (1 + |\epsilon\mu_\perp|/\epsilon_0\mu_0)^{1/2}$. Thus, depending on the values of ϵ and μ_{ik}, both waves are propagated or one alone, or, finally, s.w. are impossible.

3. We now consider the more complicated case of s.w. in a gyrotropic plate 3 $(0 < x < d,\ \mu = \mu_{ik})$ between isotropic media 1 $(\epsilon = \epsilon_0;\ \mu = \mu_0)$ and 2 $(\epsilon = \tilde{\epsilon};\ \mu = \tilde{\mu})$. Let d be large; for the boundary $x = d$ we set up an equation similar to (2), different from (2) only in the sign of the right-hand side, i.e., the boundary $x = d$ guides s.w. in a direction opposite to that on the boundary $x = 0$. Accordingly the field of the direct wave is concentrated at one boundary and the field of the reverse wave at the opposite boundary: for $\epsilon_0 = \tilde{\epsilon}$ and $\mu_0 = \tilde{\mu}$:

$$E_y = A_1 e^{\gamma_3 x} + A_2 e^{-\gamma_3 x}, \quad A_{1,2} = \gamma_3\mu_0 \pm \gamma_1\mu_\perp \mp \mu_0\Gamma h.$$

When the boundary which conducts energy in the undesired direction is covered with an absorbing film we obtain a unidirectional system.

For $\gamma_3 d \leq 1$ we must take into account the interaction of the boundaries and investigate the characteristic equation. In the general case $(\epsilon_0 \neq \tilde{\epsilon},\ \mu_0 \neq \tilde{\mu})$ this equation is

$$h^2\Gamma^2 + h\Gamma(\gamma_1 P_1 - \gamma_2 P_2) - \gamma_1\gamma_2 P_1 P_2 - \gamma_3^2 = \gamma_3(\gamma_1 P_1 + \gamma_2 P_2)\coth\gamma_3 d, \tag{4}$$

$$P_1 = \frac{\mu_\perp}{\mu_0}; \quad P_2 = \frac{\mu_\perp}{\tilde{\mu}}; \quad \gamma_1^2 = h^2 - \frac{\omega^2}{c^2}\varepsilon_0\mu_0;$$

$$\gamma_2^2 = h^2 - \frac{\omega^2}{c^2}\tilde{\varepsilon}\tilde{\mu}; \quad \gamma_3^2 = h^2 - \frac{\omega^2}{c^2}\varepsilon\mu_\perp.$$

This equation contains a term which is linear in h, so that the direct and reverse waves differ not only with respect to the field distribution but also with respect to the phase velocity and critical velocity. For $\epsilon\mu_\perp > \epsilon_0\mu_0 > \tilde{\epsilon}\tilde{\mu}$

$$\omega_{cr} = \frac{c}{d}\,\frac{\alpha P_1 + \beta P_2}{(\alpha P_1 + \Gamma)(\beta P_2 - \Gamma)}, \tag{5}$$

$$\alpha = \left(1 - \frac{\varepsilon_0\mu_0}{\varepsilon\mu_\perp}\right)^{1/2}; \quad \beta = \left(1 - \frac{\tilde{\varepsilon}\tilde{\mu}}{\varepsilon\mu_\perp}\right)^{1/2}.$$

When $\omega > \omega_{cr}$, a sufficient condition for the propagation of s.w. is $M < 0$, where $M = (\Gamma + P_1 + 1)(\Gamma - P_2 - 1)$. When $\omega < \omega_{cr}$, the sufficient condition is $M > 0$. Equation (5) applies to the direct wave; for the reverse wave the sign of Γ must be reversed, giving different conditions for propagation of the two different waves.

4. Analogous conflicting properties are characteristic of a channel 3 ($0 < x < d$; $\epsilon = \epsilon_0$; $\mu = \mu_0$) between two gyrotropic media 1 ($x < 0$; $\mu = \mu_{ik}$) and 2 ($x > d$; $\mu = \tilde{\mu}_{ik}$). With $\mu_\perp < 0$ and $\tilde{\mu}_\perp < 0$, the gyrotropic plates 1 and 2 insulate channel 3 from the surrounding medium like the walls of a metal waveguide. The characteristic equation which corresponds to (4) is

$$h^2\Gamma\tilde{\Gamma} + h(\gamma_1\tilde{\Gamma} - \gamma_2\Gamma) - \gamma_1\gamma_2 - \gamma_3^2 Q\tilde{Q} = \gamma_3\,[h(\Gamma\tilde{Q} - \tilde{\Gamma}Q) + \gamma_2 Q + \gamma_1\tilde{Q}]\coth\gamma_3 d, \qquad (6)$$

where $Q = \mu_\perp/\mu_0$; $\tilde{Q} = \tilde{\mu}_\perp/\mu_0$. The critical frequency is

$$\omega_{cr} = \frac{c}{d}\left(\frac{\tilde{Q}}{\tilde{\Gamma} - \beta} - \frac{Q}{\Gamma + \alpha}\right),$$

$$\alpha = \left(1 - \frac{\varepsilon\mu_\perp}{\varepsilon_0\mu_0}\right)^{1/2}, \quad \beta = \left(1 - \frac{\tilde{\varepsilon}\tilde{\mu}_\perp}{\varepsilon_0\mu_0}\right)^{1/2}.$$

Sufficient conditions for the propagation of the direct s.w. are $N < 0$ for $\omega > \omega_{cr}$ and $N > 0$ for $\omega < \omega_{cr}$, where $N = (\Gamma + Q + 1)(\tilde{\Gamma} - \tilde{Q} - 1)$. By interchanging the signs of Γ and $\tilde{\Gamma}$ we obtain the conditions for the propagation of the reverse wave.

Waves of the waveguide type (by which we mean waves for which $\gamma_3^2 < 0$) can also propagate in this channel. In such waves the energy maximum is not at the walls as in s.w. but in the middle of the channel, so that we can expect smaller loss.

5. The boundary of a gyrotropic medium and the plate and channel discussed in Secs. 3 and 4 are of considerable interest as retarding systems. Their advantages are the possibility of modifying (and specifically, modulating) the retardation coefficient in time (by changing H_0) and in space, and the absence of distortions.

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Translated by I. Emin

ENERGY SPECTRUM OF NUCLEAR-ACTIVE PARTICLES IN EXTENSIVE AIR SHOWERS

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Submitted to JETP editor February 26, 1958

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 1637-1638 (June, 1958)

As a continuation of our earlier work,[1] we studied the energy spectrum of nuclear-active particles in extensive air showers (EAS) of cosmic radiation at 3860 m above sea level. Nuclear-active particles of the shower were identified by production of electron-nuclear showers in lead plates of a large rectangular cloud chamber.* Total thickness of the lead plates amounted to ~ 100 g/cm^2. As a criterion of the nature of secondary showers we used the presence of penetrating or heavily ionizing particles in electron-nuclear showers, which corresponds to that used in reference 3.

The experiment was carried out in two variants, one with no absorber above the chamber, and another with an absorber of ~ 100 g/cm^2 Al. A diagram of the arrangement is shown in Fig. 1.

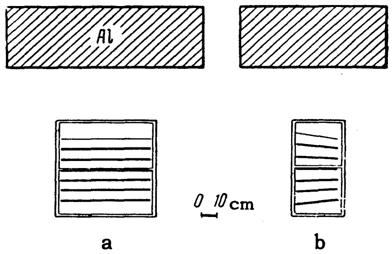

FIG. 1. Diagram of the array, a – front view, b – side view.

EAS with total number of particles N between 10^4 to 10^6 were recorded. The average shower size was $N \sim 10^5$ in the first variant of the experiment, $N \sim 2 \times 10^5$ in the second. A hodoscope consisting of a large number of self-quenching counters made it possible to select EAS, the axes of which fell within 9 m from the cloud chamber, and to determine the total number of particles in a shower.[4] The error in the axis location amounted to ~ 1 m. The energy of nuclear-active particles was determined from the energy of the electron-photon component produced by these particles.[3]

Integral energy spectra of nuclear-active particles for the energy region 2 to 50 Bev at dis-