

This equation agrees with the result that Fröhlich obtained by using perturbation theory.

Equation (34) shows that $v_0 > 0$ for all values of λ , and the rearrangement of the Fermi distribution which Fröhlich predicted does not occur.

It follows from (30) that for $\lambda_0 \sim 1$ the excitation attenuation equals the excitation energy in order of magnitude for $\eta \sim 1$, i.e., for the excitation energy

$$E_{p_1 p_2} \sim \omega_0.$$

With further increase of the excitation energy, the attenuation ceases to increase and becomes smaller than the excitation energy. Thus for $\lambda_0 \sim 1$ electron excitations in the region $E_{p_1 p_2} \sim \omega_0$ cannot be described by means of quasi-particles.

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293

COOLING OF AIR BY RADIATION

II. STRONG COOLING WAVE

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Radiation-cooling wave of air accompanied by a large temperature drop, is considered. It is shown that, the radiation is always from the lower edge of the wave, regardless of the value of the upper temperature, and that the radiation transfer inside a strong wave has the character of radiant heat conduction. The strong-wave mode with adiabatic cooling is considered.

IN the first part of this work (reference 1)* we have described qualitatively the cooling of a large volume of hot air by radiation. We have found in this case that a unique temperature profile is developed in the air in the form of a step or a cooling wave (CW) propagating towards the hotter air. The air in the wave cools down from a high temperature T_1 to a lower temperature T_2 . At the lower temperature T_2 the air becomes transparent, i.e., stops absorbing and emitting radiation.

In reference 1 we have considered the limiting case of a weak CW, in which the upper and lower temperatures T_1 and T_2 are not greatly different, and consequently the flux from the CW is close

enough to either σT_1^4 or to σT_2^4 . In this article we present the theory of a strong CW, in which the upper temperature can be unlimited. The fundamental problem consists obviously of determining the radiation flux from the front of the CW to infinity. Another problem is to find the temperature distribution in the front of the CW.

1. DETERMINATION OF THE RADIATION FLUX FROM THE FRONT OF THE COOLING WAVE

It was indicated in reference 1 that to find the stationary mode of the CW it is necessary to employ one of two procedures. In the first we introduce a constant adiabatic-cooling term into the energy equation. In the second we determine at the very outset the transparency temperature T_2 , using formula (I.4). We then assume that when

*Henceforth, when referring to the formulas of the first part of this article, we shall precede the number of the formula by I [e.g. (I.4), (I.10)].

$T < T_2$ the air is absolutely transparent ($l = \infty$), thereby excluding from consideration the region of air already cooled by the radiation, which absorbs the light rather weakly.

The first procedure gives a more complete picture of the temperature distribution, since it permits an investigation of the course of the temperature in the cooled air and accounts for the absorption of light in this air. It leads, however, to excessive mathematical complications in the analysis of the temperature profile inside the CW (i.e., at temperatures above the transparency temperature) and in the determination of the flux from the front of the CW. Meanwhile, adiabatic cooling plays a very insignificant role inside the CW. It is therefore preferable to consider the internal structure of the CW by using the second procedure. Here the energy equation (I.6) becomes

$$u\rho_1c_p\frac{dT}{dx} + \frac{dS}{dx} = 0 \quad \text{or} \quad u\rho_1c_p\frac{dT}{d\tau} + \frac{dS}{d\tau} = 0, \quad (1)$$

and its integral becomes

$$u\rho_1c_p(T_1 - T) = S. \quad (2)$$

Referring (2) to the lower edge of the CW, we obtain an equation for the energy balance in the CW

$$u\rho_1c_p(T_1 - T_2) = S_2. \quad (3)$$

The most general considerations show that the flux S_2 from the front of the CW can be bounded both from above and from below.

Let us consider the lower edge of the CW, where the temperatures are close to T_2 . According to the condition assumed by us, the air is not cooled at any point of the wave to a temperature below T_2 , for it stops absorbing and radiating light when $T < T_2$. Consequently, once it reaches a temperature T_2 , the air cannot be cooled further, so that T_2 is the lowest of all possible temperatures in the wave. Therefore, $dT/d \geq 0$ at the edge of the CW, and from (1) we have $dS/d\tau \leq 0$. It follows from (I.11) that the radiation density at the edge of the CW is less than the equilibrium value, $U_{\text{eq}2} = 4\sigma T_2^4/c$. Therefore, in the diffusion approximation, the effective temperature of the radiation going to "infinity" from the boundary of the CW is determined by the formula

$$S_2 = \sigma T_{\text{eff}}^4 \quad (4)$$

and cannot be lower than the lowest temperature T_2 in the CW. Consequently, the flux S_2 and the effective temperature T_{eff} remain within very narrow limits:

$$\sigma T_2^4 < S_2 < 2\sigma T_2^4, \quad (5)$$

$$T_2 < T_{\text{eff}} < \sqrt[4]{2} T_2. \quad (6)$$

Thus, regardless of the amplitude of the CW, which can be characterized by the ratio T_1/T_2 , no matter how high the upper temperature, it is always the lower edge of the CW that radiates. This conclusion follows from the stationary nature of the profile of the CW.

The radiation from the surface of a heated body bordering on a transparent region is generated in a surface layer of optical thickness τ on the order of several units, since the quanta produced in the deeper layers are almost all absorbed in the outer layer. The effective radiation temperature T_{eff} is obviously equal to a certain mean temperature of the radiating layer. It follows from formula (6) that the temperature cannot vary much in a radiating layer with an optical thickness on the order of several units. This is the condition for the existence of a local thermodynamic equilibrium between the radiation and matter, or for the existence of radiant heat conduction. The greater the amplitude of the CW, i.e., the closer U_2 is to $U_{\text{eq}2}$ and the closer S_2 is to $2\sigma T_2^4$, the better is this condition satisfied.

In fact, if the change in temperature in the radiating layer is of the order T_2 , then the change in flux in this layer, according to (2), is $|\Delta| \sim u\rho_1c_p T_2$. However, in a strong wave, according to (3), $S_2 \sim u\rho_1c_p T_1$, since $T_2 \ll T_1$. It is then possible, with the aid of (I.11) and (5), to estimate also the relative deviation of the radiation density U from the equilibrium value U_{eq}

$$\left(\frac{U_{\text{eq}} - U}{U_{\text{eq}}}\right)_2 = -\frac{1}{cU_{\text{eq}2}} \left(\frac{dS}{d\tau}\right)_2.$$

Since $\tau \sim 1$, in the radiating layer, the derivative $(dS/d\tau)_2$ is on the order of $|\Delta S|$ and

$$\left(\frac{U_{\text{eq}} - U}{U_{\text{eq}}}\right)_2 \sim \frac{|\Delta S|}{cU_{\text{eq}2}} \sim \frac{S_2}{cU_{\text{eq}2}T_1} \sim \frac{T_2}{T_1}.$$

Inasmuch as the flux in the radiating layer of a strong CW is almost constant, $|\Delta S|/S_2 \sim T_2/T_1 \ll 1$, the situation on the lower edge of a strong CW is quite analogous to the situation in photospheres of stationary stars. The problem of determining the connection between the flux S_2 with the transparency temperature T_2 in an exact calculation of the angular distribution of the radiation is equivalent, in the limit of the strong CW, to the well-known Milne problem,² the exact solution of which

$$S_2 = \frac{4}{\sqrt{3}} \sigma T_2^4 \quad (7)$$

differs only little from the diffusion solution adopted by us

$$S_2 = 2\sigma T_2^4. \tag{8}$$

2. TEMPERATURE DISTRIBUTION IN THE COOLING WAVE

It was shown above that the radiation density is quite close to equilibrium at the lower edge of a strong CW. It is natural to assume that the local equilibrium extends over the entire wave, and to put $U \approx U_{eq}$ in Eq. (I.12):

$$S = \frac{c}{3} \frac{dU_{eq}}{d\tau} = \frac{16\sigma T^3}{3} \frac{dT}{d\tau}. \tag{9}$$

Inserting the flux (9) into the energy integral (2), we obtain an equation for the temperature

$$dT/d\tau = 3u_0 c_p (T_1 - T) / 16\sigma T^3, \tag{10}$$

from which it is seen that, the derivative $dT/d\tau$ and, consequently, the deviation from the condition of local equilibrium, diminish monotonically with increasing temperature as the distance from the lower edge of the CW increases. Thus, the energy transferred by radiation in a strong CW has the character of radiant heat conduction. This indeed proves the correctness of the method we have used to average the free path over the spectrum² inside the CW (see reference 1).

Equation (10) is integrated by quadratures and gives the temperature profile in the CW

$$t = 1 - (1 - t_2) \exp[-\tau/\tau_e + q(t)], \tag{11}$$

where

$$q(t) = (t - t_2) [1 + 1/2(t + t_2) + 1/3(t^2 + tt_2 + t_2^2)]; \tag{11'}$$

$$\tau_e = 8(1 - t_2) / 3t_2^4, \quad t = T/T_1, \quad t_2 = T_2/T_1.$$

Near the lower edge $T \sim T_2$, so that $T \ll T_1$ in a strong CW. The numerator of the right half of (10) can then be replaced with the aid of (3) and (8), yielding the approximate solution at the lower edge of the wave

$$T^4 = T_2^4 \left(1 + \frac{3}{2} \tau\right). \tag{12}$$

This expression, naturally, coincides with the diffusion solution of the Milne problem.

The asymptotic form of the profile for $\tau \gg \tau_e$ can be obtained by putting $q(t) \approx q(1)$ in formula (11). When $t_2 \ll 1$, this quantity is $11/6$. From the formula obtained thereby it follows that τ_e is the effective optical thickness of the CW.

If we extrapolate the approximate formula (12) all the way to the upper temperature T_1 , the op-

tical thickness of the CW becomes approximately four times smaller than τ_e . According to (11') the optical thickness of the CW increases very rapidly with increasing amplitude of the CW [as $(T_1/T_2)^4$]. Figure 1 shows the distribution of $t(\tau)$ for $t_2 = 0.2$.

Let us find now the temperature distribution along the geometric coordinate x . We place the origin of coordinates at the lower edge of the wave, where $T = T_2$ and $\tau = 0$. Rewriting Eq. (I.10) in terms of a new variable temperature instead of the optical thickness τ , and substituting $dT/d\tau$ according to formula (10), we obtain with the aid of expressions (3) and (8)

$$-x = \int_0^\tau l(T) d\tau = \frac{8(T_1 - T_2)}{3T_2^4} \int_{T_2}^{T_1} \frac{l(T) T^3 dT}{T_1 - T}. \tag{13}$$

At temperatures not too close to the upper temperature T_1 , it is possible to put in (13) approximately $T_1 - T \approx T_1 - T_2$. Inserting into (13) the free path obtained from formula (I.3),* we obtain

$$-\frac{x}{l_2} = \frac{8}{3} z_2^7 \int_z^{z_2} e^{z-z_2} \frac{dz}{z^8}, \quad z = \frac{l}{kT}, \quad z_2 = \frac{l}{kT_2}; \quad l_2 = l(T_2). \tag{14}$$

Formula (14) confirms the premise of reference 1, that the temperature in the CW has a sharp step on the high-temperature side. Figure 2 shows the distribution of $T(x)$ on the lower edge, as given by (14).

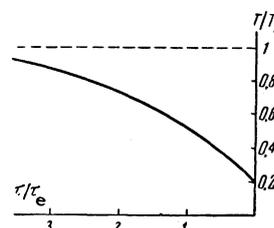


FIG. 1. $T_2/T_1 = 1/5$; $\tau_e = 1670$

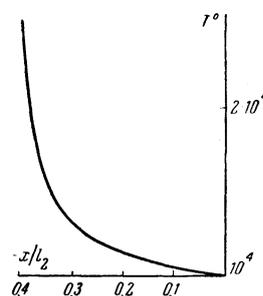


FIG. 2. $T_2 = 10,000^\circ$

*According to (I.3), $l \sim (T^2/\rho) \exp [I/kT]$. As a consequence of the constant pressure in the CW (see reference 1), we have $\rho \sim T^{-1}$ and $l \sim T^3 \exp [I/kT]$.

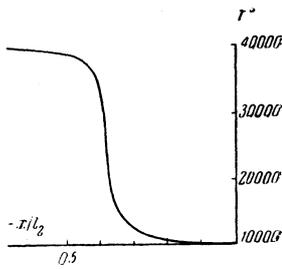


FIG. 3. $T_1 = 40,000^\circ$,
 $T_2 = 10,000^\circ$.

The exponential dependence of $l(T)$, which results in a sharp step in CW, is valid actually only in the temperature range where the first ionization alone is significant, i.e., up to $\sim 30,000$ to $40,000^\circ$. At higher temperatures, l goes through a minimum and then increases (relatively slowly). Therefore, the upper edge of a sufficiently strong wave with $T_1 > 50,000$ to $100,000^\circ$ is quite spread out, more so than $T(\tau)$ of formula (11). [Were l constant in the high-temperature region, the profile of $T(x)$ would coincide exactly in this region with the profile of $T(\tau)$]. An approximate profile of the temperature over the entire wave is shown in Fig. 3.

To estimate the accuracy of the radiant heat-conduction approximation in which the temperature profile has been determined, and to define thereby the concept of a "strong" wave, we can determine the correction to the value of the flux $S_2 = 2\sigma T_2^4$, necessitated by the deviation from local equilibrium. Obviously, this correction gives the accuracy of the approximate solution of the equation for the CW, since the greatest deviation from local equilibrium occurs precisely on the lower boundary of the wave. Calculation of the above correction by the method of successive approximation yields: $1 - S_2/2\sigma T_2^4 = 0.18$ for $T_1/T_2 = 1.5$. Its value becomes 0.1 for $T_1/T_2 = 3$. Thus, the accuracy of the radiant heat-conduction approximation increases rapidly with increasing amplitude of the CW, and a wave with $T_1/T_2 = 3$ can be considered strong to within 5%.

3. LOWEST EDGE OF COOLING WAVE AND TRANSITION TO THE TRANSPARENT ZONE OF THE COOLED AIR

We have considered above the structure of the CW front, i.e., of that layer in which the air cools by radiation from an initial temperature T_1 to the transparency temperature T_2 . We have used from the very outset the general condition (1.2) to determine the transparency temperature, and have assumed that when $T_1 < T_2$ the air is absolutely

transparent. Actually, the absorption of light by air cooled below temperature T_2 , although small, is nevertheless finite.

What happens to the radiation from the front of the CW, and how does the temperature behave in the zone of the cooled air?

The process in this region is essentially non-stationary and depends on the actual conditions, such as dimensions, hydrodynamic motion, or additional mechanisms of light absorption which take place at low temperatures (see reference 1). We shall consider here the important case when the air pressure has not yet dropped to atmospheric and the radiation-cooled air continues to cool adiabatically. Owing to the exceedingly strong dependence of l on T , the air is adiabatically cooled quite rapidly to a temperature, at which the absorption becomes so small that this region of air no longer exerts any influence whatever on the mode of the CW.

Little is changed by the adiabatic cooling in that layer of air, which still can influence the general distribution of the temperature, and in which the temperature drops to $1,000$ or $2,000^\circ$ below the transparency temperature. A process with adiabatic cooling is therefore quasi-stationary over the entire region of interest.

Let us trace the successive changes in the state of an air particle that enters a strong CW or, what is the same, let us move in the positive direction of the x axis at a constant velocity u . Let the particle enter the CW with a high initial temperature T_1 . It will start to cool rapidly by radiation. The radiation density in the particle remains in this case at all times below equilibrium, since the energy absorbed per unit time is less than the radiated energy; the radiation flux increases in the particle. The speed of adiabatic cooling is first considerably below the speed of radiant cooling. This continues until the particle cools down to such a low temperature, that the speed of adiabatic cooling exceeds the speed of radiant heat exchange. As a consequence of the exceedingly sharp drop in absorption (and radiation) with diminishing temperature, even slight adiabatic cooling makes the particle almost transparent after that instant, and the radiant heat exchange soon ceases.

Now the radiation density, which is determined by the flux generated in the hotter layers and passing through the particle, remains almost constant. On the other hand, the equilibrium radiation density, proportional to T^4 , diminishes rapidly. The radiation density in the "transparent" region becomes greater than equilibrium, unlike the "non-transparent" one (the energy absorbed per unit

time becomes greater than the radiated energy). The air then becomes somewhat heated by the radiation, and the flux diminishes. Consequently, there exists on the x axis such a point $x = x_2$ (with corresponding optical thickness and temperature τ_2 and T_2 respectively), which separates the regions of the "nontransparent" air, which is intensely cooled by radiation, from the almost-transparent air, which is slightly heated by the radiation. The radiation density at this point is exactly equal to the corresponding equilibrium value $U_2 = U_{eq2}$, and the flux S_2 in it is a maximum.

Obviously, the point at which the cooling of the air by radiation ceases should indeed be considered the lower boundary of the CW, and the temperature in this point should be considered the transparency temperature for a given value of adiabatic cooling A . The flux S_0 that goes to infinity is somewhat less than the flux S_2 from the surface of the CW front, owing to absorption in the "transparent" zone. This absorption turns out to be small: the optical thickness of the "transparent" zone, as estimated in the Appendix, is approximately $\tau_2 \approx 0.16$, so that S_0 is only little less than S_2 .

The temperature and flux profiles $T(x)$ and $S(x)$, corresponding to the above qualitative description of the process, are shown schematically in Figs. 4 and 5. At low temperatures the curve

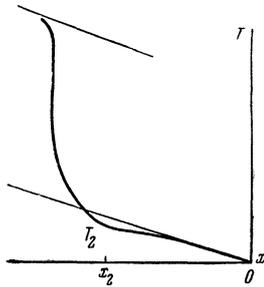


FIG. 4.

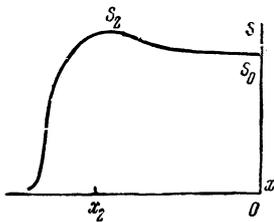


FIG. 5.

$T(x)$ follows very closely the lower straight line corresponding to the constant adiabatic cooling A and to the flux S_0 going to infinity. The curve approaches this line from below, since the air in this region is heated by radiation. On the high tempera-

ture side, the $T(x)$ curve deviates greatly from the upper straight line corresponding to adiabatic cooling A and zero flux. On the lower edge of the CW, where the flux is a maximum, the downward deviation of the temperature from the lower straight line is also a maximum, as can be seen from the energy equation (I.6).

As shown in the Appendix, the transparency temperature depends only logarithmically on the value of the adiabatic cooling and on the amplitude of the CW.

We are grateful to Academician N. N. Semenov for stimulating discussions.

APPENDIX

We consider the stationary mode of a strong CW with adiabatic cooling A . The integration constant of the energy integral (I.17), like that in the case of a weak CW, equals the flux that goes to infinity.

$$u\rho_1c_pT + S = -Ax + S_0. \tag{15}$$

On the low-temperature side, when the flux S tends to S_0 , the temperature curve approximates the lower straight line

$$u\rho_1c_pT = -Ax, \tag{16}$$

and on the high-temperature side, as $S \rightarrow 0$, the temperature is asymptotic to the upper line*

$$u\rho_1c_pT = -Ax + S_0. \tag{17}$$

In the "transparent" region $|x| > |x_2|$, where the adiabatic cooling can be neglected, the solution of the equations coincides with the solution obtained in Sec. 2. One need merely write, in lieu of the particular integral (11) passing through the point $\tau = 0, T = T_2$, the general integral passing through the still arbitrary point (τ_2, T_2) . By extrapolating this solution to the transparency temperature T_2 , we obtain the previous connection between the flux and the temperature T_2 , namely $S_2 = 2\sigma T_2^4$.

In the "transparent" region $|x| < |x_2|$, the radiating ability is very small at low temperatures, the flux becomes unilateral: all the quanta move only "forward" and leave the region of sufficiently high

*In a mode without adiabatic cooling, the condition $S \rightarrow 0$ on the high-temperature side is equivalent to the condition $T \rightarrow T_1 = \text{const}$. On the other hand, there the temperature gradient is not only different from zero, but tends to a constant value at high temperatures. To satisfy the condition $S \rightarrow 0$ at $T \rightarrow \infty$, and for the mode to exist, it is essential that $l \rightarrow 0$ sufficiently rapidly at $T \rightarrow \infty$. In the problem of the weak CW, this condition was automatically satisfied through the use of the approximation formula (I.23).

temperature. The integral expressions (I.13) and (I.14) now become

$$S = cU/2 = S_0 e^{2\tau}. \quad (18)$$

Extrapolation of this solution, which is valid for $U_{\text{eq}} \ll U$, to the point x_2 where $U_2 = U_{\text{eq}2}$ also yields a flux $S_2 = 2\sigma T_2^4$.

By definition, the transparency temperature corresponds to the place in the wave, where the speed of radiant heat exchange dS/dx changes sign, i.e., vanishes. It is clear, however, that near this temperature the rate of radiant cooling of the particle drops to a value on the order of the rate of adiabatic cooling. In fact, as already mentioned above, owing to the sharp temperature dependence of the coefficient of absorption, to which the rate of radiant cooling is proportional, even a small adiabatic temperature drop reduces sharply the rate of radiant heat exchange. The transparency temperature T_2 can therefore be determined from the condition that the rate of radiant cooling, obtained from the extrapolated solution in the "nontransparent" region, must be equal to the rate of adiabatic cooling A .

We calculate the rate of radiant cooling at a point with temperature T_2 with the aid of formulas (1), (3), (8), (9) and (I.10):

$$\begin{aligned} \left(\frac{dS}{dx}\right)_2 &= -u\rho_1 c_p \left(\frac{dT}{dx}\right)_2 = u\rho_1 c_p \frac{3S_2}{16\sigma T_2^3 l(T_2)} \\ &= \frac{3}{8} \frac{u\rho_1 c_p T_2}{l(T_2)} = \frac{3}{4} \frac{\sigma T_2^4}{l(T_2)(T_1/T_2 - 1)}. \end{aligned} \quad (19)$$

We thus arrive at a transcendental equation for the transparency temperature in terms of the velocity of the CW or in terms of the upper temperature T_1 of the CW:

$$\frac{8}{3} \frac{Al(T_2)}{u\rho_1 c_p T_2} = \frac{4}{3} \frac{Al(T_2)(T_1/T_2 - 1)}{\sigma T_2^4} = 1. \quad (20)$$

Thanks to the exponential dependence of l on T , the transparency temperature depends only logarithmically on the amplitude of the CW, an amplitude characterized by the velocity or upper temperature of the wave, and on the adiabatic cooling.

It is clear that the temperature defined by Eq. (20) is equal, within logarithmic accuracy, to the "true" transparency temperature, which is defined by the condition that the radiant heat exchange vanish. This has made the above approximation possible. Geometrically, condition (20) signifies that we extrapolate the solution from the "nontransparent" side until the slope of the temperature curve dT/dx coincides with the slope of the line (16), which the temperature curve approaches from below in the "transparent" region (see Fig. 4).

We must still determine the position of the lower edge of the CW, i.e., the coordinates x_2 and τ_2 . For this purpose we determine approximately the optical thickness τ corresponding to some point x in the "transparent" region. Noting that in the low-temperature limit the absorption of the flux is negligible ($S \approx S_0$) and the temperature curve $T(x)$ almost coincides with the lower straight line (17), we obtain

$$\tau = -\int_0^x \frac{dx}{l(T)} = -\int_0^T \frac{dx}{dT} \frac{dT}{l(T)} \approx \frac{u\rho_1 c_p}{A} \int_0^T \frac{dT}{l(T)}. \quad (21)$$

Here we bear in mind the "exact" Kramers formula for the free path instead of approximation (I.23), by which $l = \infty$ when $T = 0$. Inserting the free path given by (I.3) into (21) and recalling that at low temperatures an exponential law is much stronger than a power law, we obtain by approximate integration

$$\tau \approx \frac{u\rho_1 c_p T}{Al(T)} \frac{kT}{T} = \frac{|x| kT}{l(T) T}. \quad (22)$$

By the very nature of its derivation, this formula is valid when $\tau \ll 1$. If it is referred to the lower edge of the CW, i.e., to the point where $T = T_2$, we obtain with the aid of (20)

$$\tau_2 \approx 8kT_2/3l. \quad (23)$$

Since $I = 14$ eV and $T_2 \approx 10,000^\circ$ in air, $\tau_2 \approx 0.164$ turns out to be rather small, and (23) can be considered as the optical thickness of the lower edge of the CW.

The geometric coordinate of the lower edge of the CW, which equals, according to (22) and (23),

$$|x_2| = u\rho_1 c_p T_2/A = {}^8/3 l(T_2), \quad (24)$$

represents in this case the distance, at which the temperature is reduced by adiabatic cooling from T_2 to 0.

As expected, the free path corresponding to the transparency temperature is exactly of the same order as determined from the value of the adiabatic cooling and its time of action.

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