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PROTON WAVE EQUATIONS

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THE existing techniques of treatment of the electromagnetic field do not allow to handle the interaction of photons with other fields in terms of quantum field theory in a number of cases. These problems include the whole complex of gravitation-electromagnetic interactions: graviton-photon scattering, graviton bremsstrahlung by photons, etc. In order to treat such problems one has to formulate the wave equation of light quanta in matrix form.

The fundamental difficulty in formulating a matrix theory of the photon field lies in the fact that the rest mass of the photon vanishes and further that the wave function contains both electromagnetic potentials and fields. This makes the application of the Kemmer formalism exceedingly difficult.[1,2] However, by applying the Dirac algebra[3] one can remove this difficulty and it is possible to formulate a photon theory analogously to the Lee-Yang theory[4] of rest-mass-zero fermions.

In Ref. 5 it was shown explicitly how to achieve a representation of the 16-row Kemmer algebra by 8- or 4-row representations of the Dirac algebra. These same representations will have to be applied to the photon theory. (A detailed investigation of these algebrae will be published in Nuovo cimento.)

For the photon wave function we shall take the half-undor ψ which includes, besides the fields E and H, two new quantities, a scalar, ψ, and pseudoscalar, $\tilde{\psi}$. Using an 8-row representation of the Dirac algebra one can write the free field wave equation in the form

$$(\boldsymbol{\alpha},\ \nabla + \partial / c\,\partial t)\,\psi(\mathrm{x},\ t) = 0 \qquad (1)$$

or

$$(\boldsymbol{\alpha}^*,\ \nabla + \partial / c\,\partial t)\,\psi(\mathrm{x},\ t) = 0, \qquad (2)$$

where

$$\tfrac{1}{2}\{\alpha_i\alpha_k\} - \delta_{ik}I = \tfrac{1}{2}\{\alpha_i^*\alpha_k^*\} - \delta_{ik}I = [\alpha_i\alpha_k^*] = 0. \qquad (3)$$

We define a matrix $\alpha_L \neq I$ with the properties

$$[\alpha_L\alpha_i] = [\alpha_L\alpha_i^*] = [r_i\alpha_L] = 0, \quad \alpha_L^2 = I \qquad (4)$$

where $r_i = \alpha_i\alpha_i^*$ are reflection matrices (here one does not sum over the indices i). It leads to the Larmor transformation for ψ: $\psi' = \alpha_L\psi$. The corresponding transformation in the neutrino theory is the Salam transformation[6] $\varphi' = \gamma_5\varphi$.

An explicit expression for α_L is

$$\alpha_L = i\alpha_1\alpha_2\alpha_3 = i\alpha_1^*\alpha_2^*\alpha_3^*, \qquad (5)$$

Besides α_L there exists another pseudoscalar operator, $i\alpha_0 = r\alpha_L$, where r has the properties

$$\{r\alpha_i\} = \{r\alpha_i^*\} = [r_i r] = 0. \qquad (6)$$

Equations (1) and (2) are invariant under Larmor transformations. In order to go over to a 4-row representation one introduces the Larmor-invariant wave function[7] $(I + \alpha_L)\psi$. Then both anticommutative groups G_α and G_{α^*} go over into the group G_γ of the Dirac matrix theory of the electron in the representation where charge conjugation is represented by complex conjugation.[5,8]

It is interesting to note that these matrices are identical with the matrices describing the two internal degrees of freedom of Fock's electron.[9,10] However, they enter linearly the operator of van Wyk's generalized gauge transformation.[11]

The Larmor photons can have different parity and can have a spin of ħ even in the case of longitudinal polarization (longitudinal-magnetic photons). In order to describe Larmor-nonsymmetrical, Maxwell photons one has to go over to a wave function which is a simultaneous solution of (1) and (2), or, of the following system of equations which is equivalent to (1), (2) in this particular case:

$$(\beta^{(+)},\ \nabla + \partial / c\,\partial t)\,\psi(\mathrm{x},\ t) = 0, \quad \beta^{(-)},\ \nabla\,\psi(\mathrm{x},\ t) = 0,$$
$$\beta^{(\pm)} = (\boldsymbol{\alpha} \pm \boldsymbol{\alpha}^*)/2. \qquad (7)$$

The wave equations (1) and (2) are derived from the Lagrangian

$$L \sim \bar{\psi}(\boldsymbol{\alpha},\ \nabla + \partial / c\,\partial t)\,\psi \qquad (8)$$

(for ordinary photons $\boldsymbol{\alpha}$ here has to be replaced by $\beta^{(+)}$).

The commutation relations are, as usual,

$$[\psi(\mathrm{x},\ t),\ \bar{\psi}(\mathrm{x}',\ t')] = iS(\mathrm{x} - \mathrm{x}',\ t - t'), \qquad (9)$$

where the commuting function S is different for the Larmor and Maxwell photons:

$$= (\boldsymbol{\alpha},\ \nabla - \partial / c\,\partial t)\, D(\mathbf{x},\ t), \tag{10}$$

$$S_M(\mathbf{x},\ t) = (\boldsymbol{\alpha},\ \nabla - \partial / c\,\partial t)\,(\boldsymbol{\alpha}^*,\ \nabla - \partial / c\,\partial t)\, D(\mathbf{x},\ t). \tag{11}$$

In momentum space S_M reduces to the form of the Cayley-Klein transformation of the unit wave vector $\mathbf{k}^0 = \mathbf{k}/k$

$$S_M = (I + \boldsymbol{\alpha}^*,\ \mathbf{k}^0) / (I - \boldsymbol{\alpha},\ \mathbf{k}^0). \tag{12}$$

For the Maxwell field it is possible to utilize Eqs. (1) and (2) with the reduced wave function $\psi_M = T\psi$, where

$$T = (3 + M)/4, \quad M = r_1 + r_2 + r_3. \tag{13}$$

The photon theory, like the new neutrino theory, is intrinsically three-dimensional, since both groups G_α and G_{α^*} have a diagonal matrix in common: $\alpha_4 = \alpha_4^* = r$. Taking this into account, one can write the wave equations in a symmetric four dimensional form $(-ir\alpha_k \rightarrow \gamma_k,\ r \rightarrow \gamma_4)$.

The interaction of photons with the gravitational field is described by the equations

$$\gamma_\lambda \partial\varphi / \partial x_\lambda = 0, \quad \text{or} \quad \beta_\lambda \partial\varphi / \partial x_\lambda = 0,$$

where

$$\varphi(x) = (I + g\gamma_\mu \gamma_\nu^* h_{\mu\nu}(x))\, \psi(x) \tag{14}$$

($h_{\mu\nu}(x)$ is the gravitational potential). The application of perturbation theory to this equation is facilitated by the smallness of the coupling constant g. The usual formulae apply for the traces of products of α_i and α_i^*. The traces of products $\alpha_i \alpha_k^*$ vanish identically.

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CERTAIN NEW MAGIC NUCLEON NUMBERS

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NUCLEI containing 30 neutrons have, in most cases, some rare distinguishing properties. The nuclide $^{26}Fe_{30}$ is the most abundant of all nuclides having $Z > 10$. The relative abundance of this iron isotope is 91.7%, while, for example, the isotope $^{26}Fe_{28}$ (in spite of its containing 28 neutrons) has an abundance of only 5.8%. The relative abundance of the lightest nickel isotope $^{28}Ni_{30}$ is 67.8%, although in the case of analogous isotopes of other elements it usually does not exceed several percent or fractions of a percent. The nuclide $^{24}Cr_{30}$ is different in having a low effective capture cross-section for thermal neutrons. (0.36 barns), like nuclides containing the usual magic numbers of neutrons. Along with this, the iron and nickel isotopes having $N = 30$ have very high effective cross-sections for coherent scattering (without change in spin) of thermal neutrons compared with other isotopes of the same elements, and very high total scattering cross-section (σ_{free}), multiplied by $[(A + 1)/A]^2$ to reduce it to the case of the nucleus at rest. The corresponding data (in barns) are given in Table I.

TABLE I

Element	N	A	$\sigma_{free}[(A+1)/A]^2$	σ_{coh}
Fe	28	54	2.5	2.20
	30	56	12.8	12.8
	31	57	2.0	0.64
Ni	30	58	24.4	25.9
	32	60	1.0	1.1
	34	62	9.0	9.5

The effective scattering cross-section of $^{28}Ni_{30}$ is particularly high. It is possible that the properties of a 30-neutron configuration manifest themselves differently in different nuclides, depending