$\epsilon(\omega)$ and, for small absorption, the corrections connected with the medium, will be anomalously large.

In the experimental study of radiative corrections, it must be kept in mind that the accidental coincidence of $\omega_m$ with a zero of the real part of $\epsilon(\omega)$ can substantially distort the results for small scattering angles.

With diminishing momentum transfer $\xi$, i.e., with decreasing angle of scattering, the ratio $(d\sigma - d\sigma_{vac})/d\sigma_0$ decreases to zero, from which it follows that corrections of large magnitude coming from the medium can be expected only at certain scattering angles.

Thus, in the experimental measurement of the cross section for Compton scattering at small angles, it is necessary to take into account the possibility of a significant change in the differential cross section as a result of the influence of the medium.

In conclusion, I would like to use this opportunity to express deep gratitude to E. L. Feinberg for suggesting this problem and constant interest in this work, and M. L. Ter-Mikaelian for valuable discussion.

[1] L. D. Landau and I. Ia. Pomeranchuk, Dokl. Akad. Nauk SSSR **92**, 735 (1953).

[2] M. L. Ter-Mikaelian, Izv. Akad. Nauk SSSR, Ser. Fiz. **19**, 657 (1955) [Columbia Tech. Transl. p. 595].

[3] A. B. Migdal, Dokl. Akad. Nauk SSSR **96**, 49 (1954); **105**, 77 (1955).

[4] M. I. Riazanov, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 1244 (1957); Soviet Phys. JETP **5**, 1013 (1957).

[5] R. P. Feynman, Phys. Rev. **76**, 749, 769 (1949).

[6] J. Schwinger, Phys. Rev. **74**, 1439 (1948).

[7] N. N. Bogoliubov and D. V. Shirkov, Введение в теорию квантованных полей (Introduction to the Theory of Quantized Fields), M. 1957.

[8] F. J. Dyson, Phys. Rev. **75**, 1736 (1949).

[9] L. M. Brown and R. P. Feynman, Phys. Rev. **85**, 231 (1952).

[10] A. I. Akhiezer and V. B. Berestetskii, Квантовая электродинамика (Quantum Electrodynamics), M. 1953. [Transl., publ. by U.S. Dept. of Commerce]

[11] A. A. Abrikosov, J. Exptl. Theoret. Phys. (U.S.S.R.) **30**, 96 (1956), Soviet Phys. JETP **3**, 71 (1956).

[12] E. L. Feinberg, Usp. Fiz. Nauk **58**, 198 (1956).

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# *ON THE THEORY OF THE THERMAL CONDUCTIVITY AND ABSORPTION OF SOUND IN FERROMAGNETIC DIELECTRICS*

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The temperature dependence of the thermal conductivity and the coefficient of absorption of sound in ferromagnetic dielectrics is determined. It is shown that spin waves play the principal role in these processes at low temperatures.

1. As is well known, the kinetic properties of ordinary dielectrics are determined by the phonon spectrum. In ferromagnetic dielectrics the elementary excitations consist of spin waves in addition to phonons. It is therefore of interest to ascertain the role of the spin waves in thermal conduction and sound absorption in these substances.

We will show that at low temperatures the thermal conductivity in an unbounded ferromagnetic dielectric which contains no impurities is determined principally by the interactions of spin waves with each other and with phonons. If the Curie temperature $\Theta_C$ is lower than the Debye temperature $\Theta$, the temperature dependence of the thermal conductivity is determined not by the exponential factor $\exp(\Theta/2T)$, as in ordinary dielectrics,[1] but by the factor $\exp(\Theta_C/4T)$.

The dissipation function of a ferromagnetic dielectric at low temperatures in the presence of a sound field is also determined by the interaction of spin waves with each other, and turns out not to depend on the temperature, while in ordinary dielectrics it is inversely proportional to the temperature.

**2.** The principal elementary interaction processes in a system of spin waves and phonons, which we shall take into account here, are the conversion of two phonons into one phonon, the conversion of two spin waves into one spin wave, the scattering of a spin wave by a phonon, and the conversion of two spin waves into one phonon. The respective probabilities of these processes are as follows:[2] •

$$W_{f,-f',-f''} = (\Theta a\hbar/\rho)\, ff'f'', \tag{1}$$

$$W_{k,-k',-k''} = \frac{64\pi^3 w^2 a^3}{\hbar}\,|\sin 2\vartheta' e^{i\varphi'} + \sin 2\vartheta'' e^{i\varphi''}|^2, \tag{2}$$

$$W_{f,k,-k'} = \frac{2\pi\Theta_c^2 a^3}{\Theta\rho}\frac{\hbar}{f}\,|2(\mathbf{kf})(\mathbf{e}_f, \mathbf{k}+\mathbf{f}) + \mathbf{e}_f(\mathbf{k}f^2 + \mathbf{f}k^2)|^2, \tag{3}$$

$$W_{f,-k',-k} = \frac{4\pi^3 w^2}{\Theta a}\frac{\hbar}{\rho}\frac{1}{f}\Big| f^- e_f^- + 2\mathbf{k}\mathbf{e}_f \sin^2\vartheta e^{-2\varphi i} + 2\mathbf{k}'\mathbf{e}_f \sin^2\vartheta' e^{-2i\varphi'}\Big|^2, \tag{4}$$

where **f** and **k** are the wave vectors of phonons and spin waves in different states: a is the lattice constant; $\rho$ is the density of the substance; $w = \beta^2/a^3$ is the magnetic interaction energy of the two spins of neighboring atoms ($\beta$ is the Bohr magneton); $\mathbf{e}_f$ is the polarization vector of phonon **f**; $f^- \equiv f_x - if_y$; $e_f^- = e_{fx} - ie_{fy}$; and $\theta$, $\varphi$, $\theta'$, $\varphi'$, and $\theta''$, $\varphi''$ are the polar angles of the vectors **k**, **k'**, and **k''**.

We now write the kinetic equations for the spin-wave and phonon distribution functions, taking these interaction processes into account. Setting

$$\begin{aligned} n_k &= n_k^0 + (\varphi_k/T)/(e^{\varepsilon_k/T}-1)(e^{-\varepsilon_k/T}-1),\\ N_f &= N_f^0 + (\Phi_f/T)/(e^{E_f/T}-1)(e^{-E_f/T}-1), \end{aligned} \tag{5}$$

where $n_k^0$ and $N_f^0$ are equilibrium Planck functions, it is a simple matter to obtain the following linearized equations for determining the unknown functions $\varphi_k$ and $\Phi_f$:

$$\begin{aligned} &\int W_{f,-f',-f''}\,\delta(E_f - E_{f'} - E_{f''})\frac{\Phi_{f''}+\Phi_{f'}-\Phi_f}{(e^{E_{f''}/T}-1)(e^{E_{f'}/T}-1)(e^{-E_f/T}-1)}\,d\tau_{f'}\\ &+\int W_{f,+k,-k'}\,\delta(E_f + \varepsilon_k - \varepsilon_{k'})\frac{\Phi_f+\varphi_k-\varphi_{k'}}{(e^{\varepsilon_k/T}-1)(e^{-\varepsilon_k/T}-1)(e^{-E_f/T}-1)}\,d\tau_k\\ &+\int W_{f,-k',-k}\,\delta(E_f - \varepsilon_k - \varepsilon_{k'})\frac{\varphi_k+\varphi_{k'}-\Phi_f}{(e^{\varepsilon_k/T}-1)(e^{\varepsilon_{k'}/T}-1)(e^{-E_f/T}-1)}\,d\tau_k\\ &= \frac{(\partial T/\partial z)\, v_z^{(f)} E_f/T}{(e^{E_f/T}-1)(e^{-E_f/T}-1)}. \end{aligned} \tag{6}$$

$$\begin{aligned} &\int W_{k,-k',-k''}\,\delta(\varepsilon_k - \varepsilon_{k'} - \varepsilon_{k''})\frac{\varphi_{k''}+\varphi_{k'}-\varphi_k}{(e^{\varepsilon_{k''}/T}-1)(e^{\varepsilon_{k'}/T}-1)(e^{-\varepsilon_k/T}-1)}\,d\tau_{k'}\\ &+\int W_{f,+k,-k'}\,\delta(E_f + \varepsilon_k - \varepsilon_{k'})\frac{\Phi_f+\varphi_k-\varphi_{k'}}{(e^{\varepsilon_{k'}/T}-1)(e^{-\varepsilon_k/T}-1)(e^{-E_f/T}-1)}\,d\tau_{k'}\\ &+\int W_{f,-k,-k'}\,\delta(E_f - \varepsilon_k - \varepsilon_{k'})\frac{\varphi_k+\varphi_{k'}-\Phi_f}{(e^{\varepsilon_k/T}-1)(e^{\varepsilon_{k'}/T}-1)(e^{-E_f/T}-1)}\,d\tau_{k'}\\ &= \frac{(\partial T/\partial z)\, v_z^{(k)} \varepsilon_k/T}{(e^{\varepsilon_k/T}-1)(e^{-\varepsilon_k/T}-1)}, \end{aligned} \tag{6'}$$

where $v_z^{(f)}$ and $v_z^{(k)}$ are the projections of the group velocities of the phonons and of the spin waves onto the z axis, along which the temperature gradient is directed; E and $\epsilon$ are the energies of the phonons and spin waves in their respective states;

$$d\tau_f = (2\pi)^{-3} f^2 df\, do_f, \quad d\tau_k = (2\pi)^{-3} k^2 dk\, do_k,$$

($do_f$, $do_k$ are elements of solid angle of the vectors **f** and **k**).

In the first terms of (6) and (6') the following conservation laws of the wave vectors are satisfied:

$$\mathbf{f} - \mathbf{f}' - \mathbf{f}'' = 0,\ \pm 2\pi\mathbf{b},$$
$$\mathbf{k} - \mathbf{k}' - \mathbf{k}'' = 0,\ \pm 2\pi\mathbf{b},$$

in the second terms,

$$\mathbf{f} + \mathbf{k} - \mathbf{k}' = 0,\ \pm 2\pi\mathbf{b},$$

and in the third terms

$$\mathbf{f} - \mathbf{k} - \mathbf{k}' = 0, \ \pm 2\pi \mathbf{b},$$

where $\mathbf{b}$ is a vector of the reciprocal lattice.

We will consider further the case of low temperatures, when $\alpha = \Theta^2/\Theta_c T \gg 1$. Retaining the most essential terms in the equations, we can write (6) and (6′) symbolically in the form

$$L_2\{\Phi, \varphi\} + e^{-\Theta/2T} L_1^u\{\Phi\} + e^{-\Theta_c/4T}\alpha^{1/2} L_2^u\{\Phi, \varphi\} = \frac{\frac{\partial T}{\partial z}\frac{a}{\hbar}\frac{\Theta_c}{\Theta}\alpha\cos\vartheta_f\eta}{(e^{\eta}-1)(e^{-\eta}-1)},$$

$$\alpha^{1/2} M_1\{\varphi\} + e^{-\Theta_c/4T} M_1^u\{\varphi\} + e^{-\Theta/2T}\alpha^{1/2} M_2^u\{\Phi, \varphi\} = \frac{\frac{\partial T}{\partial z}\frac{a}{\hbar}\frac{\Theta_c}{\Theta}x^{3/2}\cos\vartheta_k}{(e^{x}-1)(e^{-x}-1)}, \quad (7)$$

where $\eta = E_f/T$, $\kappa = \epsilon_k/T$; $L_2\{\Phi, \varphi\}$ is an operator, corresponding to the second term of (6) when the conservation law of the wave vector is strictly satisfied, describing the scattering of spin waves by phonons; $M_1\{\varphi\}$ is an operator, corresponding to the first term of (6′), describing the interaction of spin waves with each other also when the wave-vector conservation law is strictly fulfilled; and the operators with the index u correspond to terms taking account of transfer processes. These operators do not contain the temperature explicitly.

Multiplying (6) by $f_z$ and (6′) by $k_z$ and adding both equations we obtain, after integrating over the phonon states f and the spin-wave states k:

$$\frac{2\pi}{aT}\int W_{f,-f',-f''}\,\delta(E_f - E_{f'} - E_{f''})\frac{\Phi_{f''}+\Phi_{f'}-\Phi_f}{(e^{E_{f''}/T}-1)(e^{E_{f'}/T}-1)(e^{-E_f/T}-1)}d\tau_{f'}d\tau_f$$
$$+\frac{2\pi}{aT}\int W_{k,-k',-k''}\,\delta(\varepsilon_k-\varepsilon_{k'}-\varepsilon_{k''})\frac{\varphi_{k''}+\varphi_{k'}-\varphi_k}{(e^{\varepsilon_{k''}/T}-1)(e^{\varepsilon_{k'}/T}-1)(e^{-\varepsilon_k/T}-1)}d\tau_{k'}d\tau_k$$
$$+\frac{2\pi}{aT}\int W_{f,+k,-k'}\,\delta(E_f+\varepsilon_k-\varepsilon_{k'})\frac{\Phi_f+\varphi_k-\varphi_{k'}}{(e^{\varepsilon_k/T}-1)(e^{-\varepsilon_{k'}/T}-1)(e^{-E_f/T}-1)}d\tau_f d\tau_k \quad (8)$$
$$+\frac{2\pi}{aT}\int W_{f,-k,-k'}\,\delta(E_f-\varepsilon_k-\varepsilon_{k'})\times\frac{\varphi_k+\varphi_{k'}-\Phi_f}{(e^{\varepsilon_k/T}-1)(e^{\varepsilon_{k'}/T}-1)(e^{-E_f/T}-1)}d\tau_f d\tau_k = R_f + R_k,$$

where

$$R_f = -\sum_f \frac{\partial T}{\partial z}\frac{v_z^{(f)}f_z}{T}\frac{E_f/T}{(e^{E_f/T}-1)(e^{-E_f/T}-1)} \approx \frac{1}{(2\pi)^3}\frac{1}{\hbar}\frac{\partial T}{\partial z}\frac{1}{a^3}\left(\frac{T}{\Theta}\right)^3,$$

$$R_k = -\sum_k \frac{\partial T}{\partial z}\frac{v_z^{(k)}k_z}{T}\frac{\varepsilon_k/T}{(e^{\varepsilon_k/T}-1)(e^{-\varepsilon_k/T}-1)} \approx \frac{1}{(2\pi)^3}\frac{1}{\hbar}\frac{\partial T}{\partial z}\frac{1}{a^3}\left(\frac{T}{\Theta_c}\right)^{3/2}.$$

Equation (8) is satisfied if we set

$$\varphi_k = \Psi k_z, \quad \Phi_f = \Psi f_z, \quad (9)$$

where the quantity $\Psi$ does not depend on f and k and has the following form:

$$\Psi \approx \frac{a^2\Theta^2}{w^2}\frac{1}{\alpha}\frac{\partial T}{\partial z}e^{\Theta_c/4T} \quad \text{for} \quad \Theta \gg \Theta_c, \ \Theta^2/\Theta_c \gg T,$$

$$\Psi \approx \frac{\rho}{\hbar^2}\Theta a^7\frac{\partial T}{\partial z}\alpha^{1/2}e^{\Theta/2T} \quad \text{for} \quad \Theta \ll \Theta_c, \ T \ll \Theta^2/\Theta_c.$$

The functions (9) transform $L_2$ and $M_1$ to zero. Hence, following Ref. 3, we readily conclude that these functions are the principal parts of the solutions of the kinetic equations.

Using these functions we calculate the heat currents $\Pi_f$ and $\Pi_k$ carried by the phonons and by the spin waves:

$$\Pi_f = \int v_z^{(f)}\Phi_f\frac{(E_f/T)d\tau_f}{(e^{E_f/T}-1)(e^{-E_f/T}-1)},$$

$$\Pi_k = \int v_z^{(k)}\varphi_k\frac{(\varepsilon_k/T)d\tau_k}{(e^{\varepsilon_k/T}-1)(e^{-\varepsilon_k/T}-1)}.$$

Corresponding to these equations, the coefficients of thermal conductivity of the phonon and spin-wave gases are, for $\Theta \gg \Theta_c$, $\Theta^2/\Theta_c \gg T$:

$$\varkappa_f \sim \frac{\Theta_c\Theta^2}{\hbar a w^2}\left(\frac{T}{\Theta}\right)^5 e^{\Theta_c/4T}, \quad \varkappa_k \sim \frac{\Theta_c^3}{\hbar a w^2}\left(\frac{T}{\Theta_c}\right)^{7/2} e^{\Theta_c/4T}.$$

In this case $\kappa_k \gg \kappa_f$.

If $\Theta_c \gg \Theta$, $\Theta^2/\Theta_c \gg T$, then

$$\varkappa_f \sim \frac{\rho a^4}{\hbar^3}\frac{\Theta_c^3}{\Theta}\left(\frac{T}{\Theta_c}\right)^{7/2}e^{\Theta/2T}, \quad \varkappa_k \sim \frac{\rho a^4}{\hbar^3}\Theta^{1/2}\Theta_c^{3/2}\left(\frac{T}{\Theta}\right)e^{\Theta/2T}.$$

In this case also $\kappa_k \gg \kappa_f$, that is, the thermal conductivity, as in the preceding case, is determined principally by the spin waves. In contrast to the previous case, however, it is the Debye temperature and not the Curie temperature which enters into the exponent.

**3.** We now study sound absorption in a ferromagnetic dielectric. To this end it is necessary, as in the case of ordinary dielectrics,[4] to find the deviations of the phonon and spin-wave distribution functions from their equilibrium values and to determine the entropy increase of the crystal due to these deviations.

Assuming that the sound wavelength is sufficiently large, we can consider that in a system of phonons and spin waves it is possible for an equilibrium to be established which corresponds to the instantaneous value of the sound field.[4] The influence of the sound field on the phonons and spin waves amounts to a change in their energies. These quantities can be represented in the presence of sound in the form

$$E_f = E_f^0(1 + \Lambda_{ij}u_{ij}), \quad \varepsilon_k = \varepsilon_k^0 + \lambda_{ij}u_{ij},$$

where $E_f^0 = \Theta af$, $\epsilon_k^0 = \Theta_c a^2 k^2$ are the equilibrium values of the phonon and spin-wave energies in the absence of sound, and $u_{ij}$ is the deformation tensor due to the sound.

In contrast to the phonon energies, which become zero for $f = 0$ both in deformed and non-deformed crystals, the spin-wave energies acquire an addition which, generally speaking, does not reduce to zero for $k = 0$.

The changes of the phonon and spin-wave distribution functions due to the sound field are

$$\dot{N}_f^s = \frac{1}{T}\frac{E_f - E_f\dot{T}/T}{(e^{E_f/T}-1)(e^{-E_f/T}-1)},$$

$$\dot{n}_k^s = \frac{1}{T}\frac{\dot{\varepsilon}_k - \varepsilon_k\dot{T}/T}{(e^{\varepsilon_k/T}-1)(e^{-\varepsilon_k/T}-1)}.$$

In contrast to the problem of thermal conductivity, it is possible in this case to ignore transfer processes.[4]

The linearized kinetic equations for the phonon and spin-wave distribution functions in an external sound field for $\alpha \gg 1$ have the form

$$L_2\{\Phi, \varphi\} \sim \eta\Lambda_{ij}\dot{u}_{ij}N_f^0(N_f^0+1),$$
$$\alpha^{1/2}M_1\{\varphi\} \sim \Theta_c\Theta^{-2}\alpha\lambda_{ij}\dot{u}_{ij}n_k^0(n_k^0+1), \tag{10}$$

where $L_2$ and $M_1$ are the operators appearing in Eq. (7). The largest terms have been retained in the left side of (10).

The solutions of these equations have the form

$$\varphi_k = \alpha^{1/2}\lambda_{ij}\dot{u}_{ij}F_1, \quad \Phi_f = \Lambda_{ij}\dot{u}_{ij}F_2, \tag{11}$$

where $F_1$ and $F_2$ are certain functions of **x**, $\eta$, and of the angles determining the directions of **k** and **f**.

The dissipation function of the system of phonons and spin waves is

$$T\dot{S} = T\dot{S}_f + T\dot{S}_k,$$

where $S_f$ and $S_k$ are the phonon and spin-wave entropies:

$$S_f = \sum_f\{(N_f+1)\ln(N_f+1) - N_f\ln N_f\},$$

$$S_k = \sum_k\{(n_k+1)\ln(n_k+1) - n_k\ln n_k\}.$$

Substituting $n_k$ and $N_f$ in the form of (5) here, we obtain

$$T\dot{S} = \frac{1}{(2\pi a)^3}\left\{\left(\frac{T}{\Theta}\right)^3\int\Phi_f\Lambda_{ij}\dot{u}_{ij}N_f^0(N_f^0+1)\eta^3\,d\eta\,do_f + \left(\frac{T}{\Theta_c}\right)^{3/2}\int\varphi_k\frac{\Theta_c}{\Theta^2}\alpha\lambda_{ij}\dot{u}_{ij}n_k^0(n_k^0+1)x^{1/2}dx\,do_k\right\}.$$

Making further use of (11), we readily conclude that when $T \ll \Theta^2/\Theta_c$ the dissipation function does not depend on the temperature and has the form

$$T\dot{S} = C(\bar{\lambda}_{ji}\dot{u}_{ji})^2, \tag{12}$$

where $C$ is a certain constant and $\bar{\lambda}_{ij}$ is the value of $\lambda_{ij}$ averaged with respect to the angles.

It is thus clear that for $T \ll \Theta^2/\Theta_c$ the absorption of sound is determined principally by the spin waves and is independent of the temperature.

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[1] R. Peierls, Ann. Physik (Fifth Series) **3**, 1055 (1929).

[2] A. I. Akhiezer, Journ. of Phys. **10**, 217 (1946).

[3] A. I. Akhiezer, J. Exptl. Theoret. Phys. (U.S.S.R.) **10**, 1934 (1940).

[4] A. I. Akhiezer, J. Exptl. Theoret. Phys. (U.S.S.R.) **8**, 1330 (1938).

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