spin-nuclear transitions for Cl^{37} are less than the corresponding frequencies for Cl^{35} by approximately a factor of 1.2, which corresponds, within experimental error, to the ratio $\mu_{Cl^{35}}/\mu_{Cl^{37}}$. This should be, according to (5) and (6a).

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POLARIZATION IN HIGH-ENERGY ELASTIC SCATTERING

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A study is made of polarization phenomena in elastic scattering at high energies. It is shown that nuclear beams with considerable polarization can be obtained by small-angle elastic scattering. The applicability of the "black nucleus" and "gray absorbing nucleus" approximations to high-energy p — p scattering is discussed.

1. The present paper contains a discussion of the problem of polarization phenomena at high energies. We determine what peculiarities appear in the polarization phenomena in the approximation in which "diffraction" expressions appear for the scattering cross-sections averaged over the spins, and what sort of information can be obtained from the results of experiments on the polarization at high energies, at which the elastic scattering is to a large extent determined by the presence of inelastic processes.

We consider first the scattering of particles with spin $\frac{1}{2}$ by spinless particles [the case $(0, \frac{1}{2})$].

In most of the published works,* after introducing the effective potential, one makes various assumptions about the radial variation of the potentials, and discusses the results of comparison with the experimental data from the point of view of determining the parameters of the effective potential

*There are many papers in which polarization phenomena in scattering by nuclei are discussed by the use of the concepts of the optical model. We mention the papers of Riesenfeld and Watson[1] and of Brown,[2] which provide references to other papers. The writer takes occasion to thank Dr. Brown for sending him a number of unpublished notes.

(cf., e.g., Ref. 3).

2. We shall proceed differently. Without introducing the interaction potential explicitly, we determine, in analogy with the procedure used in the spinless case (cf., e.g., Refs. 4, 5), the absorption coefficients K_1 and K_2 and the indices of refraction n_1 and n_2. The quantities so introduced are related to the four functions V_{CI}, V_{SI}, V_{CR}, V_{SR} of the paper of Riesenfeld and Watson.[1] For example, the quantity K_1 is proportional to the imaginary part, and $(n-1)$ to the real part, of the average amplitude for nucleon-nucleon scattering at $\theta = 0$. In the notation of Ref. 1, K_1 and $(n_1 - 1)k$ are proportional to the imaginary and real parts of the quantity

$$\overline{M}_0 = {}^1/_8 \{[B+N+G]_{pp} + [B+N+G]_{np}\}_{\theta=0}, \quad (1)$$

and similarly K_2 and $(n_2 - 1)k$ are proportional to the imaginary and real parts of

$$\overline{M}_1 = -\frac{1}{2i}\left[\frac{1}{\sin\theta}(C_{pp}+C_{np})\right]_{\theta=0} = -\frac{1}{2i}\left[\frac{d}{d\cos\theta}(C_{pp}+C_{np})\right]_{\theta=0}. \quad (2)$$

If we use the usual limiting transition from Legendre polynomials $P_l(\cos\theta)$ to Bessel functions $J_0(l\theta)$, then for the coefficients $A(\theta)$ and $B(\theta)$ of the amplitude

$$M = A(\theta) + B(\theta)(\sigma n), \quad \mathbf{n} = \mathbf{k}_0 \times \mathbf{k}/|\mathbf{k}_0 \times \mathbf{k}| \quad (3)$$

we get

$$2A(\theta) = ik\int_0^R bdbJ_0(kb\theta)\{2 - e^{-[K_1 - 2i(n_1-1)k]s} \times [e^{-[K_2-2i(n_2-1)k]s} + e^{[K_2-2i(n_2-1)k]s}]\}$$

$$2B(\theta) = -k\int_0^R bdbJ_1(kb\theta)\,e^{-[K_1-2i(n_1-1)k]s} \times \{e^{-[K_2-2i(n_2-1)k]s} - e^{[K_2-2i(n_2-1)k]s}\}, \quad (4)$$

where $s^2 = R^2 - b^2 = R^2 - l^2\lambda^2$, the remaining notation is obvious, and

$$K_1 > K_2 \geqslant 0. \quad (5)$$

Conditions (5) replace the requirement of definiteness of sign of the imaginary parts of the scattering phase shifts in the spinless case.

3. For infinite absorption ("black nucleus") $K_1 \to \infty$, and

$$A(\theta) = ik\int_0^R J_0(kb\theta)\,bdb = \frac{iR}{\theta}J_1(kR\theta); \quad B = 0;$$

$$\sigma_t = \frac{4\pi}{k}\operatorname{Im}A(0) = 2\pi R^2; \quad \sigma_s = \pi R^2 \quad (6)$$

and the polarization $\mathbf{P}_{uu}$ from the elastic scattering of an unpolarized beam becomes zero. In virtue of time reversibility the cross-section for elastic scattering of a polarized beam does not contain any azimuthal asymmetry and is the same as the cross-section $I_0(\theta)$ for the scattering of an unpolarized beam. For the polarization $\mathbf{P}_{pu}$ (Ref. 6) after scattering of a polarized (P^{in}) beam we get

$$I_0(\theta)\mathbf{P}_{pu} = P^{in}|A(\theta)|^2\mathbf{n}\times\mathbf{k}_0; \quad \mathbf{P}_{pu} = P^{in}\,\mathbf{n}\times\mathbf{k}_0 = \mathbf{P}^{in}, \quad (7)$$

if we choose $\mathbf{P}^{in}$ perpendicular to $\mathbf{k}_0$ and the normal n. From Eq. (7) it follows that in the approximation of the "black nucleus" there is also no rotation of the polarization. Thus the absence of any polarization effects at all is characteristic of the "black nucleus". Consequently, the observation of a nonvanishing polarization $\mathbf{P}_{uu}$ at high energy can serve as a good "indicator" of the non-applicability of the concept of the "black nucleus." We remark at once that the solution of this problem by the study of unpolarized cross-sections only is a difficult task.

4. In the absence of refraction, i.e., for $n_1 = n_2 = 1$ ("gray absorbing nucleus"),

$$A(\theta) = ik\int_0^R bdbJ_0(kb\theta)(1 - e^{-K_1 s}\cosh K_2 s),$$

$$B(\theta) = k\int_0^R bdbJ_1(kb\theta)\sinh K_2 s; \quad (8)$$

$$\sigma_t = \frac{4\pi}{k}\operatorname{Im}A(0) = 2\pi R^2\{1 - [(K_1-K_2)R]^{-2}[1 - e^{-(K_1-K_2)R} \times (1 + K_1R - K_2R)] - [(K_1+K_2)R]^{-2} \times [1 - e^{-(K_1+K_2)R}(1 + K_1R + K_2R)]\}. \quad (9)$$

The expression for the differential cross-section for elastic scattering

$$I_0(\theta) = |A(\theta)|^2 + |B(\theta)|^2 \quad (10)$$

follows from Eq. (8). Integrating Eq. (10), we get for the total cross-section for elastic scattering

$$\sigma_s = \pi R^2\{1 - {}^1/_2(K_1R)^{-2}[1 - e^{-2K_1R}(1 + 2K_1R)] - 2(K_1-K_2)^{-2}R^{-2}[1 - e^{-(K_1-K_2)R}(1 + K_1R - K_2R)] - 2(K_1+K_2)^{-2}R^{-2}[1 - e^{-(K_1+K_2)R}(1 + K_1R + K_2R)]\}. \quad (11)$$

The expression for the total cross-section σ_c for inelastic processes is obtained by taking the difference of the expressions (9) and (11).

Going on to consider the polarization phenomena, we note that $A(\theta)$ is purely imaginary and $B(\theta)$ is purely real, i.e.,

$$A^+(\theta) = -A(\theta), \quad B^+(\theta) = B(\theta). \quad (12)$$

The expression for the polarization P_{uu}

$$I_0(\theta)P_{uu} = (A^+B + AB^+) \quad (13)$$

vanishes for nonvanishing A and B. Since the azimuthal asymmetry in the cross-section for scattering of a polarized beam is proportional to the right member of Eq. (13), such asymmetry is also absent for the region of energies for which the scattering phases are purely imaginary.

For the polarization $\mathbf{P}_{pu}$ after the scattering of a polarized beam we have instead of Eq. (7)

$$I_0(\theta)\mathbf{P}_{pu} = P^{\text{in}}\{(a_0^2 - B^2)\mathbf{n}\times\mathbf{k}_0 - 2a_0 B\mathbf{k}_0\}\ . \quad (14)$$

($A = ia_0$). The quantities A and B are given by the formulas (8).

We note that the vanishing of the polarization $\mathbf{P}_{uu}$ for nonvanishing $A(\theta)$ and $B(\theta)$ also occurs, as is well known, in the Born approximation (for real potentials), since, as can be seen from the expressions for $A(\theta)$ and $B(\theta)$ in terms of the scattering phase shifts, for small real phase shifts

$$A_b^+(\theta) = A_b(\theta),\ B_b^+(\theta) = -B_b(\theta).$$

The polarization (even for real potentials) becomes different from zero if one makes the expressions for A and B obtained in the Born approximation exactly unitary.

5. From consideration of limiting cases it can be seen that the presence of a polarization $\mathbf{P}_{uu}$ is connected with deviations of the indices of refraction n_1 and n_2 from the value 1. The maximum polarization will occur when $n_1 = 1$, $K_2 = 0$. In this case

$$A(\theta) = ik\int_0^R \{1 - e^{-K_1 s}\cos[2(n_2 - 1)ks]\} J_0(kb\theta)\, bdb$$
$$= -A^+ = ia_0,$$
$$B(\theta) = -ik\int_0^R bdbJ_1(kb\theta)\, e^{-K_1 s}\sin[2(n_2 - 1)ks]$$
$$= -B^+ = ib_0, \quad (15)$$

and the polarization reaches 100% at a small angle, where $|a_0| = |b_0|$. The fact that such a point can exist in the case under consideration follows from the fact that in the region of small angles $A(\theta)$ is a decreasing function of the angle and $B(\theta)$ is an increasing function. The intersection $a_0 = b_0$ occurs near the first diffraction minimum. As has been shown by the analysis of Brown,[2] precisely this case is found in the interaction of protons of energy ~ 1 Bev with carbon. To describe the scattering one needs here two parameters K_1 and n_2 (if the radius R is known), and the study of the polarization $\mathbf{P}_{pu}$ can serve as a check on the interpretation adopted. For $\mathbf{P}_{pu}$ we have in this case instead of Eq. (14)

$$I_0(\theta)\mathbf{P}_{pu}(\theta) = P^{\text{in}}\ (a_0^2 - b_0^2)\mathbf{n}\times\mathbf{k}_0 + P_{uu}\mathbf{n}I_0(\theta). \quad (16)$$

The polarization experiments necessary for the determination of the parameters of the effective potential for nonvanishing K_1, K_2, $n_1 - 1$, and $n_2 - 1$ (V_{CI}, V_{SI}, V_{CR}, V_{SR}) have been discussed by Riesenfeld and Watson.[1]

Electromagnetic effects have not been included in the present discussion. As is shown by an examination of the spinless case,[4] it is essential in some cases to take the electromagnetic interaction into account. Of particular importance for the polarization $\mathbf{P}_{uu}$ is the change of phase of the amplitude, which manifests itself at very small angles.

The expression for the amplitude with inclusion of the effect of the magnetic moment, as obtained in the Born approximation,[7,8] has the disadvantage that the expression for $\mathbf{P}_{uu}$ does not go to zero for $\theta \to 0$. The study of the electromagnetic effects on the polarization may be of great interest in getting information about the electromagnetic properties of nucleons (relaxation of the magnetic moments).

6. We have considered above the elastic scattering of particles with spin $\frac{1}{2}$ by a spinless target [the case $(0, \frac{1}{2})$]. The qualitative results relating to $\mathbf{P}_{uu}$ remain valid also for the cases $(\frac{1}{2}, \frac{1}{2})$ and $(1, 0)$ (nucleon-nucleon scattering and scattering of deuterons by spinless nuclei). We represent the amplitude M for the nucleon-nucleon scattering in the form

$$M = BS + C(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2, \mathbf{n}) + \{{}^1/_2 G[(\boldsymbol{\sigma}_1\Delta)(\boldsymbol{\sigma}_2\Delta) + (\boldsymbol{\sigma}_1\boldsymbol{\pi})(\boldsymbol{\sigma}_2\boldsymbol{\pi})] + {}^1/_2 H[(\boldsymbol{\sigma}_1\Delta)(\boldsymbol{\sigma}_2\Delta) - (\boldsymbol{\sigma}_1\boldsymbol{\pi})(\boldsymbol{\sigma}_2\boldsymbol{\pi})] + N(\boldsymbol{\sigma}_1\mathbf{n})(\boldsymbol{\sigma}_2\mathbf{n})\}T. \quad (17)$$

Here

$$S = (1 - \boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)/4, \quad T = (3 + \boldsymbol{\sigma}_1\boldsymbol{\sigma}_2)/4$$

are the singlet and triplet projection operators, and

$$\boldsymbol{\pi} = (\mathbf{k}_0 + \mathbf{k})/|\mathbf{k}_0 + \mathbf{k}|,\ \Delta = (\mathbf{k}_0 - \mathbf{k})/|\mathbf{k}_0 - \mathbf{k}|.$$

From the expressions for the coefficients B, C, . . . obtained by Wright[9] it can be seen that for imaginary phases B, G, H, and N are imaginary quantities and C is real. We note particularly that just this case for $\overline{M}_0$ and $\overline{M}_1$ in Eqs. (1) and (2) leads to the maximum polarization in the scattering of nucleons by nuclei.

For imaginary phase shifts in N-N scattering the polarization $\mathbf{P}_{uu}$ goes to zero, and the cross-section for scattering of a polarized beam by an unpolarized target has no azimuthal symmetry. Thus here too the study of the polarization $\mathbf{P}_{uu}$ can be a good way of checking the correctness of the diffraction approach, with imaginary phases, to the analysis of the experimental data. A number of other polarization effects (including the correlation of polarizations) are nonvanishing, and the correlation of the polarizations, when the beam is polarized, does not differ from the case of an unpolarized beam with an unpolarized target. The addition to the polarization

$$I_0(\theta)\, P_{ipq} = \frac{1}{4}\, \mathrm{Sp}\, M\sigma_{1i}M^{+}\sigma_{1p}\sigma_{2q}$$

goes to zero.

For imaginary scattering phase shifts the number of independent experiments is the same as the number of components in the amplitude, since the phases of the coefficients become equal to 0 and $\pi/2$.

To get an idea of what happens in the elastic scattering of particles with higher spins, let us consider briefly the case (1, 0), again confining ourselves to imaginary phase shifts. For this case the amplitude M is essentially identical with the M for the triplet scattering of neutrons by protons, i.e., we can get it by setting $B = 0$, $T = 1$ in Eq. (17) and replacing $(\sigma_1\mathbf{a})(\sigma_2\mathbf{a})$ by $2(\hat{\mathbf{S}}\mathbf{a})^2 - 1$ in the remaining expressions, where $\hat{\mathbf{S}}$ is the operator for spin 1. Then, as follows from the work of Wright[9] and Cheishvili,[10] the polarization $\mathbf{P}_{uu}$ also goes to zero, while the average values of the tensors

$$\hat{D}_{ik} = {}^1/_2\,(\hat{S}_i\hat{S}_k + \hat{S}_k\hat{S}_i) - {}^2/_3\delta_{ik},$$

which, together with $\hat{\mathbf{S}}_i$, characterize the state of polarization, are not equal to zero. In general the cross-section for scattering of a polarized beam will contain terms proportional to $\cos\varphi$ and $\cos 2\varphi$, but the term containing $\cos\varphi$ is only proportional to the average value of

$$T_{2.1} = -\frac{\sqrt{3}}{2}\,\{(\hat{S}_x\hat{S}_z + \hat{S}_z\hat{S}_x) + i\,(\hat{S}_z\hat{S}_y + \hat{S}_y\hat{S}_z)\}$$
$$= -\sqrt{3}\,\{\hat{D}_{xy} + i\hat{D}_{yz}\}.$$

7. In the discussion of proposed experiments with nucleons of energies amounting to several Bev it is sometimes assumed that the elastic scattering of nucleons by nucleons and by nuclei will correspond to the simple picture of diffraction by a "black nucleus" or by a "gray nucleus", and that polarization phenomena will be absent. Confirmation for this is found by its proponents in the good agreement of the available experimental data on p-p scattering and the scattering of nucleons by nuclei with the simple formulas for the cross-sections in the approximation of the "black nucleus" or of the "gray absorbing nucleus" with $n = 1$, although the values of the parameters obtained (with the use of "spinless amplitudes") in the energy region around 1 Bev make these writers express amazement at the good agreement.

Some objections against the applicability of such an argument for the p-p scattering in the region of energies ~ 1 Bev have been presented by Rarita.[11]

The results of the discussion in the present paper show that also at high energies, when the elastic scattering is to a large extent determined by the presence of inelastic processes, it may turn out to be possible to obtain beams of nucleons with considerable polarization. The existence of such beams makes possible the performance of additional experiments. Polarization experiments give a sensitive method for studying spin effects in elastic scattering, the existence of which might not be revealed by the study of the differential cross-sections.

The relevance of the predictions of the "black nucleus" or "gray absorbing nucleus" approximations to polarization phenomena appears doubtful, since the quantities $(n_{1,2} - 1)k$ are proportional to the real part of the forward scattering amplitude and to its derivative with respect to the angle at $\theta \to 0$, divided by the momentum k. But even in the high-energy limit these quantities are not zero, as is shown by the dispersion relations, but approach constant nonvanishing values. The decisive quantity is the relation between the limiting values of the real and imaginary parts of the amplitude. In addition to this, the presence of a magnetic moment of the nucleon leads to the presence in the amplitude (3) of a coefficient $B(\theta)$ with a nonvanishing imaginary part.

As for the agreement of the diffraction formulas for the cross-sections with the experimental data in the region ~ 1 Bev, this is evidently due to the fact that the main features of the elastic scattering at high energies (the scattering is strongly directed forward and is concentrated in the region of small angles) are expressed by the simple inequality[11-13]

$$I_0(0) \geqslant (k\sigma_t/4\pi)^2, \tag{18}$$

from which one has the result that[12]

$$\pi\theta^2 \leqslant (4\pi/k\sigma_t)^2\,\sigma_s. \tag{19}$$

For the n-p scattering one has added to this the quantity

$$I_0(\pi) \geqslant (k/4\pi)^2\,(\sigma_{pp}^t - \sigma_{np}^t)^2. \tag{20}$$

The inequalities (18) to (20) are based on the optical theorem, i.e., they follow from the general unitary property of the S matrix, which also is preserved in the optical model. In the framework of the optical model (for $n = 1$) Eq. (18) becomes an equality, and from (19) it follows that the main range of scattering angles is given by

$$\theta^2 \leqslant (2/kR)^2.$$

These last facts also evidently explain the good agreement of the simple "diffraction" formulas with the experimental data on the elastic scattering at high energies.

Arguments analogous to those given by Okun' and Pomeranchuk[14] lead to the conclusion that in the amplitude (3) the quantity $A(\theta)$ is in general predominant in magnitude as compared with $B(\theta)$, so that the latter can be neglected in the discussion of the unpolarized cross-sections. As is shown by the arguments that have been given, in the range of angles where $A(\theta)$ and $B(\theta)$ turn out to be comparable, a considerable degree of polarization is nevertheless possible. This will also occur for the inelastic processes.

Although the discussion was primarily in terms of the interaction of nucleons with nucleons and nuclei, it of course applies to beams of other nuclei, and also to antinucleons, hyperons, etc.

We remark that beams of antinucleons, hyperons, and antihyperons are in general partially polarized by the processes that produce them. This provides the possibility of using the azimuthal dependence of the cross-sections for interaction of such beams with nucleons or nuclei to obtain information about the spin values for hyperons and antihyperons.[15]

An interesting point for study is the polarization P_{pu}. As can be seen from Eq. (16), $P_{pu}(\theta)$ is practically identical with P^{in} in the range of angles in which $a^2 \gg b^2$. The rotation of the polarization is greatest in the region of angles where $a_0 = b_0$. There P_{uu} is at its maximum, and P_{pu}, given by the second term in Eq. (16), is turned through an angle $\pi/2$ relative to P^{in}.

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