

to estimate its lifetime. With the help of the rule  $\Delta T = \frac{1}{2}$  we easily obtain

$$\omega_1 = \omega_2 = \omega_7, \quad \omega_3 = \omega_4 = \omega_8; \quad (1)$$

$$\omega_6/\omega_5 = 3/2, \quad \omega_{10}/\omega_9 = 1/4, \quad (2)$$

$$(\omega_5 + \omega_6)/(\omega_9 + \omega_{10}) = 1.$$

The ratios (2), however, do not take into account the mass difference of  $\pi^\pm$  and  $\pi^0$  mesons. The correction due to this mass difference was considered by Dalitz,<sup>4</sup> who allowed for it not only in the phase volumes, but also in the corresponding matrix elements, using essentially perturbation theory. In this work we consider the corrections only in the statistical weights. The statistical weight for the decay of a particle with mass  $M$  into three particles with masses  $m_1, m_2, m_3$  is proportional to

$$\rho \sim m_1 m_2 m_3 (m_1 + m_2 + m_3)^{-1} (M - m_1 - m_2 - m_3)^2.$$

Denoting the statistical weights of the respective decays by  $\rho_n$ , we obtain:

$$\rho_6/\rho_9 = 1.09, \quad \rho_{10}/\rho_9 = 1.20, \quad \rho_8/\rho_9 = 1.31.$$

Using these ratios we obtain, instead of (2),;

$$\omega_6/\omega_5 = 3\rho_6/2\rho_5 \approx 2, \quad \omega_{10}/\omega_9 \approx \rho_{10}/4\rho_9 \approx 0,30, \quad (3)$$

$$(\omega_5 + \omega_6)/(\omega_9 + \omega_{10}) \approx (2/5\rho_5 + 3/5\rho_6)/(4/5\rho_9 + 1/5\rho_{10}) = 1,2.$$

Using the data on the lifetime of the  $K^+$  meson<sup>5</sup> ( $\tau_{K^+} = 1.17 \times 10^{-8}$  sec) and on the abundance of the different types of  $K^+$  decays<sup>6</sup> ( $K_{\mu 3} \sim 5.9\%$ ,  $K_{e 3} \sim 5.1\%$ ,  $K_{\pi 3} \sim 7.9\%$ ), we find that the lifetime of the  $K_2^0$  meson must be equal to

$$\tau_{K_2^0} = \tau_{K^+} \cdot 100 / (2 \cdot 5.9 + 2 \cdot 5.1 + 1.2 \cdot 7.9)$$

$$= 3.8 \cdot 10^{-8} \text{ sec}, \quad (4)$$

and the probabilities of the different decays must add up to the total disintegration probability of the  $K_2^0$  meson in the percentages, respectively,

$$\omega_1 = \omega_2 \sim 16\%; \quad \omega_5 = \omega_4 \sim 19\%;$$

$$\omega_3 \sim 10\%; \quad \omega_6 \sim 20\%. \quad (5)$$

The experimental verification of these results [relations (1), (3), (4), and (5)] could be useful for the clarification of the validity of the rule  $\Delta T = \frac{1}{2}$  for the leptonic and non-leptonic decays of  $K$  mesons. We note that the experimentally established lower limit for the lifetime of the  $K_2^0$  meson is equal to  $3 \times 10^{-8}$  sec, (Ref. 5) which is very close to the value obtained by us, but still below it.

In our investigation we neglected the probability of the decay  $K \rightarrow 2\pi + \gamma$  and of other possible decays of the  $K_2^0$  meson. The inclusion of these

decays lowers, of course, somewhat the value for  $\tau_{K_2^0}$  obtained by us.

<sup>1</sup>M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1957).

<sup>2</sup>Ioffe, Okun', and Rudik, J. Exptl. Theoret. Phys. (U.S.S.R.) **32**, 396 (1957); Soviet Phys. JETP **5**, 328 (1957); Lee, Yang, and Oehme, Phys. Rev. **106**, 340 (1957); R. Gatto, Phys. Rev. **106**, 168 (1957); A. Pais and S. B. Treiman, Phys. Rev. **106**, 1106 (1957).

<sup>3</sup>L. B. Okun', J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 469 (1958), Soviet Phys. JETP **7**, 322 (1958). Report at the Venice Conference, 1957.

<sup>4</sup>R. H. Dalitz, Proc. Phys. Soc. A**69**, 527 (1956).

<sup>5</sup>Proc. Seventh Rochester Conference, 1957.

<sup>6</sup>Alexander, Johnston, and O'Ceallaigh, Nuovo cimento **6**, 478 (1957).

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### POLARIZATION EFFECTS IN SCATTERING OF ELECTRONS BY PROTONS

G. V. FROLOV

Radium Institute, Academy of Sciences, U.S.S.R.

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AKHIEZER, Rozentsveig, and Shmushkevich<sup>1</sup> have shown that if the scattering of electrons by protons is considered in the first approximation with respect to  $e$ , but with account of all the meson-radiation corrections, the structure of the proton reduces to two real form factors  $a(q^2)$  and  $b(q^2)$ . Here  $q^2 = (p_1 - p_2)^2$ , where  $p_1$  and  $p_2$  are the four-dimensional momenta of the electrons before and after the collision. In the same article, the authors calculated the cross section for the scattering of polarized electrons by polarized protons and the recoil-proton polarization which occurs when polarized electrons are scattered by unpolarized protons.

In this note we calculate the polarization of electrons,  $\xi_2^0$ , and recoil protons,  $Z_2^0$ , resulting from the scattering of a beam of electrons with polarization  $\xi_1^0$  by protons with polarization  $Z_1^0$ , which makes it possible to determine  $a$  and  $b$  by means of suitable experiments. (The polari-

zation vectors  $\zeta_1^0$ ,  $Z_1^0$ ,  $\zeta_2^0$ , and  $Z_2^0$  are the average values of the particle spin operators in a reference system in which the corresponding particle is at rest.) This makes it possible to determine  $a$  and  $b$  from suitable experiments. The calculations were made by the usual methods. We neglected everywhere the electron mass  $m$

compared with the proton mass  $M$ , and used an initial electron energy  $\epsilon_1$  in the laboratory system. It was also assumed that  $\vartheta \gg m/\epsilon_1$ , where  $\vartheta$  is the electron scattering angle in the laboratory system.

We obtained the following expression for polarization of the recoil protons:

$$\begin{aligned} Z_2^0 &= k [\alpha_{11}(k\zeta_1^0) + A_{11}(kZ_1^0) + A_{13}(IZ_1^0)] + nA_{22}(nZ_1^0) + l[\alpha_{31}(k\zeta_1^0) + A_{31}(kZ_1^0) + A_{33}(IZ_1^0)]; \\ \alpha_{11} &= \frac{\gamma\eta\rho \tan(\vartheta/2)}{\eta+1} \left\{ 1 - \eta + \mu - 3\mu\eta + \xi(1+2\mu) + \frac{1}{\xi} \left( -\eta - 2\mu\eta - \cot^2 \frac{\vartheta}{2} \right) \right\}, \\ A_{11} &= \gamma \left\{ 1 - \frac{\eta\rho \tan(\vartheta/2)(1+\mu)}{\eta+1} \left[ 1 + 2\eta + \cot^2 \frac{\vartheta}{2} + \sin^2 \frac{\vartheta}{2} + (2\eta/\xi) \cos^2 \frac{\vartheta}{2} \right] \right\}, \\ A_{13} &= \frac{\gamma\eta\rho}{\eta+1} \left\{ 1 + \mu\eta - \mu\xi - (1+\mu) \sin^2 \frac{\vartheta}{2} + \frac{1}{\xi} \left[ 1 - \eta(1+\mu) + 2\eta(1+\mu) \sin^2 \frac{\vartheta}{2} + \cot^2 \frac{\vartheta}{2} \right] \right\}, \\ A_{22} &= \gamma \{ 1 - \eta\rho(1+\mu)\tan(\vartheta/2) \}, \quad \alpha_{31} = \frac{\gamma\eta\rho}{\eta+1} \left\{ 2 + \mu - \mu\eta + \frac{1}{\xi} (1 - \eta - 2\mu\eta) \right\}, \\ A_{31} &= \frac{\gamma\eta\rho}{\eta+1} \left\{ -1 - \mu\eta + \mu\xi - (1+\mu) \sin^2 \frac{\vartheta}{2} - \frac{1}{\xi} \left[ 1 + \eta(1+\mu) - 2\eta(1+\mu) \sin^2 \frac{\vartheta}{2} + \cot^2 \frac{\vartheta}{2} \right] \right\}, \\ A_{33} &= \gamma \left\{ 1 - \frac{\eta\rho \tan(\vartheta/2)(1+\mu)}{\eta+1} \left[ 1 + \eta + \cot^2 \frac{\vartheta}{2} + \cos^2 \frac{\vartheta}{2} - (2\eta/\xi) \cos^2 \frac{\vartheta}{2} \right] \right\}. \end{aligned}$$

The axes were chosen in the following manner:  $\mathbf{k}$  is the unit vector in the direction of  $\mathbf{p}_1$  of the incident electron beam,  $\mathbf{n} = (\mathbf{p}_1 \times \mathbf{p}_2) / (\mathbf{p}_1 \times \mathbf{p}_2)$  is a unit vector normal to the scattering plane,  $\mathbf{l} = \mathbf{k} \times \mathbf{n}$ . The same notation is used as in Ref. 1:

$$\begin{aligned} \eta &= q^2/4M^2, \quad \xi = \epsilon_1/M, \quad \mu = b/a, \\ \gamma &= \{ 1 + (\zeta_1^0 \mathbf{k}) [M_{11}(Z_1^0 \mathbf{k}) + M_{13}(Z_1^0 \mathbf{l})] \}^{-1}, \\ \rho &= 2(1+\mu)\tan(\vartheta/2) / [1 + \mu^2\eta + 2\eta(1+\mu)^2 \tan^2(\vartheta/2)], \\ M_{11} &= \rho \tan(\vartheta/2) [\gamma(1-\mu+1/\xi) - \xi], \quad M_{13} = \rho\eta(1/\xi - \mu). \end{aligned}$$

For the polarization of the electrons, we obtained the following expression

$$\zeta_2^0 = k [\beta_{11}(k\zeta_1^0) + \beta_{13}(l\zeta_1^0) + B_{11}(kZ_1^0) + B_{13}(IZ_1^0)] + n\beta_{22}(n\zeta_1^0) + l[\beta_{31}(k\zeta_1^0) + \beta_{33}(l\zeta_1^0) + B_{31}(kZ_1^0) + B_{33}(IZ_1^0)],$$

where

$$\begin{aligned} \beta_{11} &= \gamma \cos \vartheta, \quad \beta_{31} = -\gamma \sin \vartheta, \\ \beta_{22} &= \gamma [1 - \eta\rho(1+\mu)\tan(\vartheta/2)]; \\ \beta_{13} &= \beta_{22} \sin \vartheta, \quad \beta_{33} = \beta_{22} \cos \vartheta, \\ B_{11} &= \gamma \cos \vartheta M_{11}, \quad B_{31} = -\gamma \sin \vartheta M_{11}, \\ B_{13} &= \gamma \cos \vartheta M_{13}, \quad B_{33} = -\gamma \sin \vartheta M_{13}. \end{aligned}$$

An examination of the resultant expressions leads to certain qualitative conclusions. It is easy to show that when polarized electrons are scattered by unpolarized protons, the longitudinal component of the electron polarization remains unchanged. On the other hand, the polarization of the recoil protons is in this case entirely due to the longitudinal component of the electron polarization. In scattering of unpolarized electrons by polarized protons, the electrons are longitudinally-polarized.

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<sup>1</sup>Akhiezer, Rozentsveig, and Shmushkevich, J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 765 (1957), Soviet Phys. JETP **6**, 588 (1958).

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