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Translated by R. T. Beyer
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SOVIET PHYSICS JETP VOLUME 34 (7), NUMBER 3 SEPTEMBER, 1958

# FRACTIONAL PARENTAGE COEFFICIENTS FOR THE WAVE FUNCTION OF FOUR PARTICLES

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Submitted to JETP editor September 26, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 694-699 (March, 1958)

General formulas are obtained for the fractional parentage expansion of type $\langle n | n-2, 2 \rangle$ for the wave function of four nucleons in j-j coupling, with inclusion of effects of isotopic spin. The normalized fractional parentage coefficients (both for nonequivalent and for equivalent particles) are expressed in terms of the Hope $\chi$ functions, i.e., essentially in terms of Wigner 9j symbols. The results can also be applied directly to the case of $\tilde{LS}$ coupling in atoms.

WHEN two-particle interactions are taken into account in the individual-particle nuclear model it turns out to be necessary to calculate the matrix elements of symmetric two-particle operators (of the type $G = \sum_{i<k} g_{ik}$) between antisymmetric states of n particles with prescribed total angular momentum and total isotopic spin. The wave functions of these states are constructed by vector composition from the functions for the individual particles. When the number n of particles is larger than two, the functions obtained by vector composition are not automatically antisymmetric, so that subsequent antisymmetrization is necessary.

In calculating the matrix elements of operators of the type G by the methods of the Racah algebra of tensor operators[1] it is convenient to possess a representation of the antisymmetric wave functions of n particles in the form of an expansion in terms of functions formed by vector composition from antisymmetric functions of the first $n-2$ particles and of the last two particles. The coefficients in this expansion are called fractional parentage coefficients of the type $\langle n | n-2, 2 \rangle$. Together with the analogous coefficients of the type $\langle n | n-1, 1 \rangle$, they were first introduced by Racah[2] for the case of equivalent electrons.

For small values of n general expressions for the fractional parentage coefficients can be obtained in terms of the Racah coefficients (Wigner 6j symbols) and more complicated invariants formed from the Clebsch-Gordan coefficients. The general expression for the coefficients $\langle 3 | 2, 1 \rangle$ for three equivalent or nonequivalent nucleons with inclusion of isotopic spin effects was given by Redlich;[3] Schwartz and de-Shalit[4] gave the formula for the case of four equivalent particles in the j-j coupling scheme. The problem of the fractional parentage coefficients $\langle 4 | 2, 2 \rangle$ is dealt with in a paper by Jahn[5] (see also the related paper of Englefield[6]). In this paper the fractional parentage expansion is indicated for a function of arbitrary symmetry (belonging to an arbitrary representation of the permutation group, but depending on only one type of spin). Jahn's fractional parentage expansion enables one, for example, to obtain directly separate expressions for spatial-spin function in j-j coupling and for isotopic spin functions. By forming an antisymmetric combination of these one can obtain the complete wave function for four nucleons. But such a construction obviously does not lead to a single general formula, since the expressions obtained by Jahn do not show explicitly what the necessary relation is between the symmetry types of the functions from which the complete antisymmetric function is to be constructed.

In the present note we wish to show that the general fractional parentage coefficient $\langle 4 | 2, 2 \rangle$ for the complete wave function of four equivalent or nonequivalent nucleons can be obtained directly, without previous separate construction of the spatial-spin and isotopic spin expansions. For this we shall not require detailed use of the apparatus of the theory of permutation groups, on which Jahn bases his exposition.

To obtain the antisymmetric wave function of four nucleons we choose a function

$$\Psi\{[j_1j_2(J_1T_1)]_a,\ [j_3j_4(J_2T_2)]_a,\ JTMM_T\}, \tag{1}$$

constructed by vector composition:

$$\mathbf{j}_1+\mathbf{j}_2=\mathbf{J}_1,\ \mathbf{t}_1+\mathbf{t}_2=\mathbf{T}_1,\ \mathbf{j}_3+\mathbf{j}_4=\mathbf{J}_2,\ \mathbf{t}_3+\mathbf{t}_4=\mathbf{T}_2,$$
$$\mathbf{J}_1+\mathbf{J}_2=\mathbf{J},\ \mathbf{T}_1+\mathbf{T}_2=\mathbf{T}\ \ (t_1=t_2=t_3=t_4=1/2).$$

The notation $[\ ]_a$ means that the function in the brackets is antisymmetrized. The indices on the j's number the different angular momenta. If the numbering of the particles is not indicated explicitly, then $j_ij_k\ldots j_\ell j_m$ means $j_i(1)\,j_k(2)\ldots j_\ell(3)\,j_m(4)$.

We must completely antisymmetrize the function (1) and then expand it in terms of functions analogous to (1), but in general, with different angular momenta.

It is not hard to show that the operator of complete antisymmetrization

$$A(1234)=\frac{1}{\sqrt{4!}}\sum_P(-1)^P P \tag{2}$$

can be written in the form

$$A(1234)=BA(12)\,A(34), \tag{3}$$

where

$$A(12)=(1-P_{12})/\sqrt{2},\ A(34)=(1-P_{34})/\sqrt{2}; \tag{4}$$

$$B=(1+P_{24}P_{13}-P_{13}-P_{14}-P_{23}-P_{24})/\sqrt{6} \tag{5}$$

($P_{ik}$ means transposition of particles i and k).

For what follows it is essential that

$$A(12)\,A(34)\,B=BA(12)\,A(34). \tag{6}$$

The function (1) can be written in the form

$$\Psi\{[j_1j_2(J_1T_1)]_a,\ [j_3j_4(J_2T_2)]_a,\ JTMM_T\}$$
$$=A(12)\,A(34)\,\Psi\{j_1j_2(J_1T_1),\ j_3j_4(J_2T_2),\ JTMM_T\}. \tag{7}$$

The function on the right is simply the product of a spatial-spin function by a function of the isotopic spins

$$\Psi\{j_1j_2(J_1T_1),\ j_3j_4(J_2T_2),\ JTMM_T\}$$
$$=\varphi(j_1j_2(J_1),\ j_3j_4(J_2),\ JM)\,\varphi_T(1/2\,1/2(T_1),\ 1/2\,1/2(T_2),\ TM_T). \tag{8}$$

We can now define a completely antisymmetric function of the four particles by the expression

$$\Psi\{j_1j_2(J_1T_1),\ j_3j_4(J_2T_2),\ JTMM_T\}_a$$
$$=B\Psi\{[j_1j_2(J_1T_1)]_a,\ [j_3j_4(J_2T_2)]_a,\ JTMM_T\}$$
$$=A(12)\,A(34)\,B\Psi\{j_1j_2(J_1T_1),\ j_3j_4(J_2T_2),\ JTMM_T\}. \tag{9}$$

The action of the operator B on a function of the type (8) is easily expressed by means of the Hope $\chi$ function,[7-9] which can be defined by the relation

$$P_{23}\varphi(j_1j_2(J_1),\ j_3j_4(J_2),\ JM)\equiv\varphi(j_1(1)\,j_2(3)(J_1),\ j_3(2)\,j_4(4)\ (J_2),\ JM)=\sum_{J_3J_4}\varphi(j_1j_3(J_3),\ j_2j_4(J_4),\ JM)\,\chi\begin{pmatrix} j_1j_2J_1\\ j_3j_4J_2\\ J_3J_4J\end{pmatrix}. \tag{10a}$$

The function $\chi$ is a normalized Wigner 9j symbol and can be expressed in terms of 6j symbols (or Racah W coefficients):

$$[(2J_1+1)(2J_2+1)(2J_3+1)(2J_4+1)]^{-1/2}\chi\begin{pmatrix} j_1j_2J_1\\ j_3j_4J_2\\ J_3J_4J\end{pmatrix}=\begin{Bmatrix} j_1j_2J_1\\ j_3j_4J_2\\ J_3J_4J\end{Bmatrix}=\sum_\lambda(-1)^{2\lambda}(2\lambda+1)\begin{Bmatrix} j_1j_3J_3\\ J_4J\lambda\end{Bmatrix}\begin{Bmatrix} j_2j_4J_4\\ j_3\lambda J_2\end{Bmatrix}\begin{Bmatrix} J_1J_2J\\ \lambda j_1j_2\end{Bmatrix};$$

$$\begin{Bmatrix} j_1j_2j_3\\ l_1l_2l_3\end{Bmatrix}=(-1)^{j_1+j_2+l_1+l_2}W(j_1j_2l_2l_1;\ j_3l_3). \tag{11}$$

Using Eq. (10a) and the relation

$$\varphi(j_1(1)\,j_2(2)\,JM) = (-1)^{j_1+j_2-J}\,\varphi(j_2(2)\,j_1(1)\,JM), \tag{12}$$

one easily obtains the relations

$$P_{14}\varphi(j_1j_2(J_1),\ j_3j_4(J_2),\ JM) = \sum_{J_3J_4}\varphi(j_4j_2(J_4),\ j_3j_1(J_3),\ JM)\,\chi\begin{pmatrix} j_1j_2J_1 \\ j_3j_4J_2 \\ J_3J_4J \end{pmatrix}(-1)^{j_1+j_2+j_3+j_4-J}; \tag{10b}$$

$$P_{13}\varphi(j_1j_2\,(J_1),\ j_3j_4(J_2),\ JM) = \sum_{J_3J_4}\varphi(j_3j_2(J_4),\ j_1j_4(J_3),\ JM)\,\chi\begin{pmatrix} j_1j_2J_1 \\ j_4j_3J_2 \\ J_3J_4J \end{pmatrix}(-1)^{j_2+2j_3+j_4-J_2+J_3-J}; \tag{10c}$$

$$P_{24}\varphi(j_1j_2(J_1),\ j_3j_4(J_2),\ JM) = \sum_{J_3J_4}\varphi(j_1j_4(J_3),\ j_3j_2(J_4)\,JM)\,\chi\begin{pmatrix} j_1j_2J_1 \\ j_4j_3J_2 \\ J_3J_4J \end{pmatrix}(-1)^{j_2+2j_3+j_4-J_2-J_4} \tag{10d}$$

$$P_{24}P_{13}\varphi(j_1j_2(J_1),\ j_3j_4(J_2)\,JM) = (-1)^{J_1+J_2-J}\,\varphi(j_3j_4(J_2),\,j_1j_2(J_1),\,JM). \tag{10e}$$

Similar relations are, of course, valid for the functions $\varphi_T$ in Eq. (8).

Substituting the expressions (10) into Eq. (9) we find [after a slight transformation of some of the terms by means of Eq. (12)]

$$\Psi\{j_1j_2(J_1T_1),\ j_3j_4(J_2T_2),\ JTMM_T\}_a = A(12)\,A(34)\frac{1}{\sqrt{6}}\Big\{\Psi\{j_1j_2(J_1T_1),\ j_3j_4(J_2T_2),\ JTMM_T\}$$
$$+(-1)^{J_1+J_2-J+T_1+T_2-T}\Psi\{j_3j_4(J_2T_2),\ j_1j_2(J_1T_1),JTMM_T\} - \sum_{J_3J_4T_3T_4}\chi\begin{pmatrix} j_1j_2J_1 \\ j_3j_4J_2 \\ J_3J_4J \end{pmatrix}\chi\begin{pmatrix} 1/2\,1/2\,T_1 \\ 1/2\,1/2\,T_2 \\ T_3T_4T \end{pmatrix}[\Psi\{j_1j_3(J_3T_3),\ j_2j_4(J_4T_4),\ JTMM_T\}$$
$$+(-1)^{J_3+J_4-J+T_3-T+T_4}\Psi\{j_2j_4(J_4T_4),\,j_1j_3(J_3T_3),\ JTMM_T\}] - (-1)^{j_3+j_4-J_2-T_2}\sum_{J_5J_6T_5T_6}\chi\begin{pmatrix} j_1j_2J_1 \\ j_4j_3J_2 \\ J_5J_6J \end{pmatrix}\chi\begin{pmatrix} 1/2\,1/2\,T_1 \\ 1/2\,1/2\,T_2 \\ T_5T_6T \end{pmatrix}$$
$$\times[\Psi\{j_1j_4(J_5T_5),\,j_2j_3(J_6T_6),\ JTMM_T\} + (-1)^{J_5+J_6-J+T_5+T_6-T}\Psi\{j_2j_3(J_6T_6),\,j_1j_4(J_5T_5),JTMM_T\}]\Big\} \tag{13}$$

If all the j's are different and the original functions are normalized, then the function we have obtained will also be normalized in virtue of the relation

$$\sum_{J_3J_4}\chi\begin{pmatrix} j_1j_2J_1 \\ j_3j_4J_2 \\ J_3J_4J \end{pmatrix}\chi\begin{pmatrix} j_1j_2J_1' \\ j_3j_4J_2' \\ J_3J_4J \end{pmatrix} = \delta(J_1J_1')\,\delta(J_2J_2'), \tag{14}$$

which expresses the unitary property of the (real) transformation (10a).

Thus we immediately obtain the fractional parentage expansion for the case of different j's:

$$\Psi\{j_1j_2(J_1T_1),\ j_3j_4(J_2T_2),\ JTMM_T\}_a = \frac{1}{\sqrt{6}}\Big\{\Psi\{[j_1j_2(J_1T_1)]_a,\ [j_3j_4(J_2T_2)]_a,\ JTMM_T\}$$
$$+(-1)^{J_1+J_2-J+T_1+T_2-T}\Psi\{[j_3j_4(J_2T_2)]_a,\ [j_1j_2(J_1T_1)]_a,\ JTMM_T\}$$
$$-\sum_{J_3J_4T_3T_4}\chi\begin{pmatrix} j_1j_2J_1 \\ j_3j_4J_2 \\ J_3J_4J \end{pmatrix}\chi\begin{pmatrix} 1/2\,1/2\,T_1 \\ 1/2\,1/2\,T_2 \\ T_3T_4T \end{pmatrix}\Big[\Psi\{[j_1j_3(J_3T_3)]_a,\,[j_2j_4(J_4T_4)]_a,\ JTMM_T\}$$
$$+(-1)^{J_3+J_4-J+T_3+T_4-T}\Psi\{[j_2j_4(J_4T_4)]_a,\ [j_1j_3(J_3T_3)]_a,JTMM_T\}\Big] - (-1)^{j_3+j_4-J_2-T_2}\sum_{J_5J_6T_5T_6}\chi\begin{pmatrix} j_1j_2J_1 \\ j_4j_3J_2 \\ J_5J_6J \end{pmatrix}\chi\begin{pmatrix} 1/2\,1/2\,T_1 \\ 1/2\,1/2\,T_2 \\ T_5T_6T \end{pmatrix} \tag{15a}$$
$$\times\Big[\Psi\{[j_1j_4(J_5T_5)]_a,\ [j_2j_3(J_6T_6)]_a\ JTMM_T\} + (-1)^{J_5+J_6-J+T_5+T_6-T}\Psi\{[j_2j_3(J_6T_6)]_a,[j_1j_4(J_5T_5)]_a,JTMM_T\}\Big]\Big\}.$$

If there are equal values among the angular momenta j, then in carrying out the application of the antisymmetrizing operators in Eq. (13) we must note that, for example, for equivalent particles 1 and 2

$$A(12)\psi(j^2J_1T_1) = \delta_p(T_1,\ 2j - J_1)\sqrt{2}\,\psi(j^2J_1T_1), \tag{16}$$

where the function $\delta_p(x, y)$ is defined for integer values of its arguments by the relation[5]

$$\delta_p(x, y) = {}^1/_2(1 + (-1)^{x+y}). \tag{17}$$

When equal j's occur the functions obtained from Eq. (13) will not be automatically normalized, so that the normalizing factors have to be determined by further calculations. We give the final normalized expressions

$$\Psi\{j_1j_2(J_1T_1), j_3^2(J_2T_2), JTMM_T\}_a = \frac{1}{\sqrt{6}}\delta_p(T_2, 2j_3 - J_2)\Big\{\Psi\{[j_1j_2(J_1T_1)]_a, j_3^2(J_2T_2), JTMM_T\}$$
$$+ (-1)^{J_1+J_2-J+T_1+T_2-T}\Psi\{j_3^2(J_2T_2), [j_1j_2(J_1T_1)]_a, JTMM_T\} - \sqrt{2}\sum_{J_3J_4T_3T_4}\chi\begin{pmatrix} j_1j_2J_1 \\ j_3j_3J_2 \\ J_3J_4J \end{pmatrix}\chi\begin{pmatrix} {}^1/_2\,{}^1/_2\,T_1 \\ {}^1/_2\,{}^1/_2\,T_2 \\ T_3T_4T \end{pmatrix}$$
$$\times\Big[\Psi\{[j_1j_3(J_3T_3)]_a, [j_2j_3(J_4T_4)]_a, JTMM_T\} + (-1)^{J_3+J_4-J+T_3+T_4-T}\Psi\{[j_2j_3(J_4T_4)]_a, [j_1j_3(J_3T_3)]_a, JTMM_T\}\Big]\Big\}; \tag{15b}$$

$$\Psi\{j_1^2(J_1T_1), j_2^2(J_2T_2), JTMM_T\}_a = \frac{1}{\sqrt{6}}\delta_p(T_1, 2j_1 - J_1)\,\delta_p(T_2, 2j_2 - J_2)\Big\{\Psi\{j_1^2(J_1T_1), j_2^2(J_2T_2), JTMM_T\} + (-1)^{2j_1+2j_2+J+T}$$
$$\times\{j_2^2(J_2T_2), j_1^2(J_1T_1), JTMM_T\} - 2\sum_{J_3J_4T_3T_4}\chi\begin{pmatrix} j_1j_1J_1 \\ j_2j_2J_2 \\ J_3J_4J \end{pmatrix}\chi\begin{pmatrix} {}^1/_2\,{}^1/_2\,T_1 \\ {}^1/_2\,{}^1/_2\,T_2 \\ T_3T_4T \end{pmatrix}\times\Psi\{[j_1j_2(J_3T_3)]_a, [j_1j_2(J_4T_4)]_a, JTMM_T\}\Big\}; \tag{15c}$$

$$\Psi\{j_1^2(J_1T_1), j_1j_2(J_2T_2), JTMM_T\}_a = N_1\frac{1}{\sqrt{6}}\delta_p(T_1, 2j_1 - J_1)\Big\{\Psi\{j_1^2(J_1T_1), [j_1j_2(J_2T_2)]_a, JTMM_T\}$$
$$+ (-1)^{J_1+J_2-J+T_1+T_2-T}\Psi\{[j_1j_2(J_2T_2)]_a, j_1^2(J_1T_1), JTMM_T\} - 2\sum_{J_3J_4T_3T_4}\chi\begin{pmatrix} j_1j_1J_1 \\ j_1j_2J_2 \\ J_3J_4J \end{pmatrix}\chi\begin{pmatrix} {}^1/_2\,{}^1/_2\,T_1 \\ {}^1/_2\,{}^1/_2\,T_2 \\ T_3T_4T \end{pmatrix}\delta_p(T_3, 2j_1 - J_3)$$
$$\times\Big[\Psi\{j_1^2(J_3T_3), [j_1j_2(J_4T_4)]_a, JTMM_T\} + (-1)^{J_3+J_4-J+T_3+T_4-T}\Psi\{[j_1j_2(J_4T_4)]_a, j_1^2(J_3T_3), JTMM_T\}\Big]\Big\}; \tag{15d}$$

$$\Psi\{j^2(J_1T_1), j^2(J_2T_2), JTMM_T\}_a = N_2\frac{1}{\sqrt{6}}\delta_p(T_1, 2j - J_1)\,\delta_p(T_2, 2j - J_2)$$
$$\times\Big\{\Psi\{j^2(J_1I_1), j^2(J_2T_2), JTMM_T\} + (-1)^{J+T}\Psi\{j^2(J_2T_2), j^2(J_1T_1), JTMM_T\}$$
$$-4\sum_{J_3J_4T_3T_4}\chi\begin{pmatrix} j\,j\,J_1 \\ j\,j\,J_2 \\ J_3J_4J \end{pmatrix}\chi\begin{pmatrix} {}^1/_2\,{}^1/_2\,T_1 \\ {}^1/_2\,{}^1/_2\,T_2 \\ T_3T_4T \end{pmatrix}\delta_p(T_3, 2j - J_3)\,\delta_p(T_4, 2j - J_4)\times\Psi\{j^2(J_3T_3), j^2(J_4T_4), JTMM_T\}\Big\}; \tag{15e}$$

$$N_1 = \left[1 - 2\chi\begin{pmatrix} j_1j_1J_1 \\ j_1j_2J_2 \\ J_1J_2J \end{pmatrix}\chi\begin{pmatrix} {}^1/_2\,{}^1/_2\,T_1 \\ {}^1/_2\,{}^1/_2\,T_2 \\ T_1T_2T \end{pmatrix}\right]^{-1/2}; \tag{18a}$$

$$N_2 = \left[1 + (-1)^{J+T}\delta(J_1, J_2)\,\delta(T_1, T_2) - 4\chi\begin{pmatrix} j\,j\,J_1 \\ j\,j\,J_2 \\ J_1J_2J \end{pmatrix}\chi\begin{pmatrix} {}^1/_2\,{}^1/_2\,T_1 \\ {}^1/_2\,{}^1/_2\,T_2 \\ T_1T_2T \end{pmatrix}\right]^{-1/2}. \tag{18b}$$

All the normalizing factors are easily obtained by means of Eq. (14) and the relations

$$\sum_{J_3J_4}\chi^2\begin{pmatrix} j_1j_1J_1 \\ j_1j_2J_2 \\ J_3J_4J \end{pmatrix}(-1)^{J_3} = (-1)^{2j_1}\chi\begin{pmatrix} j_1j_1J_1 \\ j_1j_2J_2 \\ J_1J_2J \end{pmatrix}; \tag{19}$$

$$\sum_{J_3J_4}\chi^2\begin{pmatrix} j\,j\,J_1 \\ j\,j\,J_2 \\ J_3J_4J \end{pmatrix}(-1)^{J_3+J_4} = (-1)^J\delta(J_1, J_2). \tag{20}$$

The relation (19) can be verified by using the expansion (11). It is easy to prove Eq. (20), starting with Eq. (14) and the symmetry properties of $\chi$ functions.

The fractional parentage expansions given here have been obtained for the purpose of using them for calculations of two-particle interactions in nuclei. They can also be applied in atomic spectroscopy (after the replacements $j_i \to \ell_i$, $J \to L$, $T \to S$).

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Translated by W. H. Furry
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SOVIET PHYSICS JETP VOLUME 34 (7), NUMBER 3 SEPTEMBER, 1958

# *ON THE THEORY OF PHOTONUCLEAR REACTIONS*

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Submitted to JETP editor September 30, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 700-706 (March, 1958)

The cross section is computed for the capture of gamma rays by nuclear matter at giant resonance energies.

IN theoretical investigations of giant resonance in photonuclear reactions, extensive use has been made of two models for interactions between gamma rays and nuclei. Migdal',[1] Goldhaber and Teller[2] and others regarded giant resonance as the result of an interaction between gamma rays and the collective dipole vibrations of nuclei. In contrast with this collective aspect, Wilkinson[3] and Burkhardt[4] have used the shell model in a detailed study of the mechanism of gamma-ray capture as the result of the excitation of single nucleons. However, neither model provides an explanation of all of the experimental data. For example, recent calculations based on the collective model[5] give a width of giant resonance in ($\gamma$, n) reactions which is much smaller than the observed width. On the other hand, the shell model gives incorrect frequencies for giant resonance.[4*] The principal defect of the calculations that have been mentioned is apparently the use of incorrect wave functions to describe highly excited nuclear states.

It is shown in several papers[6-9] that experiments on slow neutron scattering by medium and heavy nuclei can be interpreted satisfactorily if in constructing wave functions for highly excited states account is taken of the possibility that the excitation energy of a single particle is redistributed among other degrees of freedom. In our calculation of the photonuclear absorption cross section we shall use herein the method of Lane, Thomas, and Wigner[7] for constructing the wave functions of excited states, taking the above mentioned possibility into account.

## 1. CALCULATION OF THE PHOTONUCLEAR ABSORPTION CROSS SECTION

The nonrelativistic operator for the interaction between an electromagnetic field and a system of nucleons is

$$H' = -\sum_n \Big[\Big(\frac{e}{Mc}\Big)\mathbf{pA}\Big(\frac{1}{2} - t_{z_n}\Big) + \Big\{\mu_p\Big(\frac{1}{2} - t_{z_n}\Big) + \mu_n\Big(\frac{1}{2} + t_{z_n}\Big)\Big\}\sigma\nabla \times \underline{A}\Big]. \qquad (1)$$

If $\Psi_0$ and $\Psi_{E_\gamma}$ are the wave functions of the nu-

*Note added in proof. The computed giant-resonance frequencies are close to the observed frequencies when the effective nucleon mass is set equal to half of the true mass for all excitation energies,[19] but there is no other basis for this assumption.