replacement (14) and (15) will not be mutually exclusive, and the theory is again invariant under the transformation $\Lambda \to -\Lambda$, as in the case of the strong interaction.

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Translated by E. J. Saletan
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SOVIET PHYSICS JETP VOLUME 34 (7), NUMBER 3 SEPTEMBER, 1958

# *DAMPING OF OSCILLATIONS IN A CYCLIC ELECTRON ACCELERATOR*

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Submitted to JETP editor September 6, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **34**, 651-657 (March, 1958)

Damping factors are derived for radial and phase oscillations, taking account of variation of the magnetic field along the orbit. In the case of a strong-focusing accelerator, in contrast to the case of weak focusing, the damping is independent of the variation of the gradient $\partial H_z/\partial r$ along the orbit if the field $H_z$ is the same in all magnet sectors.

## 1. EQUATIONS OF MOTION

To derive the equations of motion of an electron in a cyclic accelerator, we use the well-known relations

$$\Delta L / L_s = \alpha \Delta E / E_s, \qquad \Delta E = E - E_s, \qquad E_s \gg mc^2; \quad (1)$$

$$\dot{\Phi} = -(2\pi q c \alpha / L_s) \Delta E / E_s, \qquad \alpha = d \ln L / d \ln E, \quad (2)$$

where $E_s$ and $L_s$ are the equilibrium values of the electron energy and the orbit length, $q$ is the harmonic number (the ratio of the rf frequency to the frequency of revolution), $\Phi$ is the phase of the accelerating voltage at the moment when the particles pass the middle of the accelerating gap. On the right side of (2), we have dropped some terms which are unimportant for the effects in which we are interested: the perturbation $\Delta\omega_r$ of the frequency of the accelerating field and the transient perturbation $2\pi q c \alpha L_s^{-1} \Delta H(t)/H_s$ of the magnetic field.

Differentiating (2) with respect to the time, we get[1]

$$\ddot{\Phi} + \frac{2\pi q c \alpha}{L_s E_s} \frac{d}{dt}(\Delta E) + \frac{\dot{E}_s}{E_s} \dot{\Phi} = 0, \quad (3)$$

where

$$\frac{d}{dt}(\Delta E) = P_0 - \left(1 - \frac{r}{\rho}\right) P_\gamma - \dot{E}_s, \qquad P_0 = \frac{ceV}{L_s} \sin \Phi, \quad (4)$$

where $\rho$ is the radius of curvature of the orbit and $P_\gamma$ the power in the radiation. Dropping the unimportant term describing the perturbation $\Delta V/V$, we have, in the linear approximation,

$$P_0 \approx P_{0s} [1 + \cot \Phi_s (\Phi - \Phi_s)], \qquad P_{0s} = ceV_0 \sin \Phi_s / L_s; \quad (5)$$

$$P_\gamma = 2e^4 E^2 H^2 / 3m^4 c^7 + p(t) = \bar{P}_\gamma + p(t), \quad (6)$$

where $p(t)$ describes the fluctuations of the radiation, $\bar{P}_\gamma$ is the (frequency) average of the power of the radiation at a given point on the orbit; this last quantity depends on both betatron and phase oscillations. According to (6),

$$\bar{P}_\gamma = \bar{P}_{\gamma s} [1 - 2L_s \dot{\Phi} / 2\pi q c \alpha - 2nr / \rho_s], \quad (7)$$

$$n = -(\rho_s / H_s) \partial H_s / \partial r.$$

Substituting (5) and (7) in (3), we get the following equation for the phase oscillations in linear approximation:[2]

$$\ddot{\phi} + \frac{\dot{E}_s + 2\bar{P}_{\gamma s}}{E_s}\dot{\phi} + \Omega^2\phi + \frac{2\pi q c\alpha}{L_s}\frac{\bar{P}_{\gamma s}}{E_s}(2n-1)\frac{r}{\rho_s} = \frac{2\pi q c\alpha}{L_s E_s}p(t), \tag{8}$$

$$\phi = \Phi - \Phi_s, \quad \Omega^2 = 2\pi q c\alpha P_{0s}\cot\Phi_s / L_s E_s.$$

Here we have made use of the fact that if $\langle P_{\gamma s}\rangle$ is the power of the radiation, averaged over the unperturbed orbit, $P_{0s} = \langle P_{\gamma s}\rangle + \dot{E}_s$.

We write the equations for betatron oscillations in the form

$$\ddot{r} + \frac{\dot{E}_s + \bar{P}_{\gamma s}}{E_s}\dot{r} + \frac{c^2}{\rho_s^2}(1-n)r + \frac{L_s}{2\pi q c\alpha}\frac{c^2}{\rho_s^2}\dot{\phi} = \frac{c^2}{H\rho}\frac{\partial H_z}{\partial z}z - \frac{c}{H\rho}H_x\dot{z}; \tag{9}$$

$$\ddot{z} + \frac{\dot{E}_s + \bar{P}_{\gamma s}}{E_s}\dot{z} + \frac{c^2}{\rho_s^2}nz = \frac{c^2}{H\rho}\frac{\partial H_z}{\partial z}r + \frac{c}{H\rho}H_x\dot{r}. \tag{10}$$

$H_x$ is the longitudinal component of the magnetic field.

## 2. DAMPING OF FREE RADIAL OSCILLATIONS*

The radiation fluctuations $p(t)$ cause phase oscillations to be set up and, because of the coupling of radial and phase oscillations [the term with $\dot{\phi}$ in (9)], also excite radial oscillations.[3] The vertical oscillations are not directly coupled to the phase oscillations. If $\partial H_z/\partial z = H_x = 0$, they are damped according to the formula

$$\exp\left(-\frac{1}{2}\int_0^t dt' \frac{\dot{E} + P_\gamma}{E}\right)$$

(cf. Ref. 2). The damping associated with the radiation is classical in character, and is caused by the fact that in the accelerating intervals the particles pass through an electric field which is intended to compensate for the radiation loss. For this reason the damping contains the term $P_{0s}/E$ in place of the usual $\dot{E}/E$. If there were no coupling of radial and phase oscillations, the free radial oscillations would, for this same reason, have an additional damping (cf. Ref. 2) with decrement $-\langle P_{\gamma s}\rangle/2E$ and the phase oscillations a damping with decrement $-\langle P_{\gamma s}\rangle/E$, as is evident from Eqs. (8) and (9).*

The coupling of the oscillations leads to a redistribution of the damping strength. If no special measures are taken, the radial oscillations in a strong focusing accelerator are anti-damped with decrement $\frac{1}{2}\langle P_{\gamma s}\rangle/E$, while the phase oscillations are, correspondingly, more strongly damped with decrement $-2\langle P_{\gamma s}\rangle/E$. To get rid of the build-up of the radial oscillations, we can make use of their coupling to the vertical oscillations (cf. Ref. 4), for example, by introducing magnets in which there is a component $H_x$. However, this method is not the best, since it obviously does not enable us to obtain the oscillation damping which is needed for marked reduction of particle losses (since this method leaves the sum of the damping factors for vertical and radial oscillations equal to zero).

Later we shall show that in a strong-focusing accelerator (more precisely, when $\alpha = d\ln L/d\ln E \ll 1$) the damping factors do not depend on the variation of $n$ along the orbit (if $H_z$ is the same in all the magnets), and we shall obtain a general formula for the damping of radial and phase oscillations. As we shall see, the damping depends on the variation of $H_z$ along the orbit, and that for any values of $H_z$ those magnets which have low $n$ have no influence on the result (contrary to the erroneous statements in Refs. 2 and 4).

Since the coupling of $r$ and $z$ oscillations is unimportant for this problem, we shall assume that $\partial H_z/\partial z = H_x = 0$.

Suppose that radial oscillations are excited at $t = 0$. Because of the dependence of the radiation in an inhomogeneous field on $r$ [the term $2nr$ in (8)], the excitation will be transferred to the phase oscillations. Usually the strong inequality $\omega \gg \Omega \gg P_\gamma/E$ is satisfied, where $\omega$ and $\Omega$ are the frequencies of betatron and phase oscillations, respectively. The forced oscillation of the phase is therefore of the form

$$\ddot{\phi} \approx -\frac{2\pi q c\alpha}{L_s}\frac{\bar{P}_{\gamma s}}{E_s}(2n-1)\frac{r}{\rho_s};$$

$$\dot{\phi} \approx -\frac{2\pi q c\alpha}{L_s}\int_0^t \frac{2n-1}{\rho_s^2} r \frac{\bar{P}_{\gamma s}\rho_s}{E_s} dt'. \tag{11}$$

We shall use Eq. (9) for $r$, and shall neglect in (11), as we do throughout, the terms containing $P_\gamma^2/E^2$, $\dot{E}P_\gamma/E^2$, $\dot{E}^2/E^2$, $\dot{P}_\gamma/E = 4P_\gamma\dot{E}/E^2$, $\ddot{E}/E^2$.

*The possibility of the existence of a damping mechanism due to radiation was first pointed out by A. M. Budker in 1954 (private communication).

*In addition, $\Omega^2$ is not proportional to $\dot{E}/E$, but rather to $P_{0s}/E$. This also gives an additional damping of the phase oscillations.

In other words, we shall use as our basic equation

$$\ddot{r} + (c/\rho)^2 (1-n) r = 0. \qquad (12)$$

We first write $\overline{P}_{\gamma s}\rho_s$ as

$$\overline{P}_{\gamma s}\rho_s = P_{\gamma 0}\rho_0 + (\overline{P}_{\gamma s}\rho_s - P_{\gamma 0}\rho_0), \qquad (13)$$

where $P_{\gamma 0}\rho_0$ is some constant. If the magnetic field is the same in all magnets with the exception of certain sections, then $P_{\gamma 0}$ and $\rho_0$ are the power of the radiation and the radius of curvature in the magnets of standard type, while $\overline{P}_{\gamma s}\rho_s - P_{\gamma 0}\rho_0$ is the deviation in the non-standard sections. From (12) and (13) we get

$$\dot{\phi} = -\frac{2\pi q c \alpha}{L_s}\left\{2\frac{P_{\gamma 0}\rho_0}{c^2 E}\dot{r} + \int_0^t \frac{\overline{P}_{\gamma s}}{E_s}\frac{r}{\rho_s}dt' + 2\int_0^t \frac{n-1}{\rho_s^2}\frac{\overline{P}_{\gamma s}\rho_s - P_{\gamma 0}\rho_0}{E_s} r\,dt'\right\}.$$

After substituting this expression for $\dot{\phi}$ in Eq. (9) for the radial oscillations, we make the substitution

$$r = a\varphi + a^*\varphi^*, \quad \dot{r} = a\dot{\varphi} + a^*\dot{\varphi}^*, \qquad (14)$$

where $\varphi$ and $\varphi^*$ are two linearly independent solutions of (12), normalized by the condition

$$\varphi^*\dot{\varphi} - \varphi\dot{\varphi}^* = 2iW. \qquad (15)$$

We then find the following equation for $a$:

$$\dot{a} = \frac{\varphi^*}{2iW}\left\{-\frac{\dot{E}_s + \overline{P}_{\gamma s} - 2P_{\gamma 0}\rho_0/\rho_s}{E_s}(a\dot{\varphi} + a^*\dot{\varphi}^*) + \frac{c^2}{\rho_s}\int_0^t \frac{\overline{P}_{\gamma s}}{E_s\rho_s}(a\varphi + a^*\varphi^*)\,dt' + 2\frac{c^2}{\rho_s^2}\int_0^t \frac{\overline{P}_{\gamma s}\rho_s - P_{\gamma 0}\rho_0}{E_s}\frac{n-1}{\rho_s^2}(a\varphi + a^*\varphi^*)\,dt'\right\}. \qquad (16)$$

$\varphi$ and $\varphi^*$ are oscillating functions, which average to zero.

The frequencies of the betatron oscillations are always chosen so that there is a non-integral number of oscillations around the orbit. Because of this we see that if we are interested in only those terms of (16) which give an increase (or decrease) of $a$ with time, we can drop oscillating functions and so throw out all the terms containing $a^*$. In the remaining integrals, we can take $a$ out from under the integral sign, neglecting terms which are quadratic in the perturbation. After this, making use of the fact that only the real part of (16) gives an increase of $a$ with time, we find by using (15),

$$a = a_0 \exp\left\{-\frac{1}{2}\int_0^t E_s^{-1}dt'\left(\dot{E}_s + \overline{P}_{\gamma s} - \frac{2P_{\gamma 0}\rho_0}{\rho_s}\right) + \frac{c^2}{2W}\,\mathrm{Im}\int_0^t dt'\,\frac{\varphi^*}{\rho_s}\int_0^{t'} dt''\,\frac{\overline{P}_{\gamma s}\varphi}{E_s\rho_s} + \frac{c^2}{W}\,\mathrm{Im}\left(\int_0^t dt'\,\frac{\varphi^*}{\rho_s}\int_0^{t'} dt''\,\frac{\overline{P}_{\gamma s}\rho_s - P_{\gamma 0}\rho_0}{E_s}\frac{n-1}{\rho_s^2}\varphi\right)\right\}. \qquad (17)$$

Now we transform the double integrals:

$$\int_0^t \frac{\varphi^*}{\rho_s}dt'\int_0^{t'} F(t'')\varphi\,dt'' = \int_0^t F(t'')\varphi\,dt''\int_{t''}^t \frac{\varphi^*}{\rho_s}dt' = -\int_0^t F(t'')\varphi\,dt''\int_0^{t''}\frac{\varphi^*}{\rho_s}dt' + K(t). \qquad (18)$$

Concerning the function $K(t)$, which is the product of two integrals (one of which contains $\varphi$ and the other $\varphi^*$), we can say that in any case this function unlike the double integral does not increase with time,* so that it can be dropped as unimportant in the exponential (17). On the other hand, we shall add an unimportant oscillating function to the integrand of (18), replacing the function

$$\frac{c^2}{W}\,\mathrm{Im}\,\varphi\int_0^t dt'\,\varphi^*/\rho_s$$

by the function

$$\psi = \frac{c^2}{W}\,\mathrm{Im}\,\varphi\left[\int_0^t dt'\,\varphi^*/\rho_s + \mathrm{const}\right]. \qquad (19)$$

The constant in the brackets can be chosen so that $\psi$ is a periodic solution (with a period equal to one revolution) of the equation

$$\ddot{\psi} + (c/\rho_s)^2(1-n)\psi = c^2/\rho_s. \qquad (20)$$

The function $\psi$, which describes the forced radial oscillations that result from energy fluctuations, is useful since it occurs frequently in accelerator computations.

The difference $\overline{P}_{\gamma s}\rho_s - P_{\gamma 0}\rho_0$ occurring in (17) is conveniently represented as

$$\overline{P}_{\gamma s}\rho_s - P_{\gamma 0}\rho_0 = P_{\gamma 0}\rho_0 (H_s - H_0)/H_0. \qquad (21)$$

Using (18), (19), and (21), we get the following final expression for the exponential damping of the free radial oscillations:

$$r_f \sim \exp\left\{-\frac{1}{2}\int_0^t \frac{dt'}{E_s}\left[\dot{E}_s + \overline{P}_{\gamma s}\left(1 + \frac{\psi}{\rho_s}\right) - 2\frac{P_{\gamma 0}\rho_0}{\rho_s} + 2P_{\gamma 0}\rho_0\frac{n-1}{\rho_s^2}\frac{H_s - H_0}{H_0}\psi\right]\right\}. \qquad (22)$$

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*More precisely, it is proportional to $P_\gamma/E$, whereas the other terms are proportional to $\int_0^t (P_\gamma/E)\,dt'$.

Let us consider the case where the field in all magnets is the same (in particular, the case of a weak-focusing accelerator). In this case the ratio $\psi/\rho_s$ in the integrand can be replaced by its average value along the orbit. As is well known,

$$\langle\psi/\rho_s\rangle = \lim_{t\to\infty}\frac{1}{t}\int_0^t \frac{\psi}{\rho_s}\,dt' = \alpha = d\ln L/d\ln E \qquad (23)$$

(where we have replaced the momentum by the energy, assuming that $E \gg mc^2$). Thus, for $H_s = H_0$,

$$r \sim \exp\left\{-\frac{1}{2}\int_0^t \frac{dt'}{E_s}\left[\dot{E}_s + \overline{P}_{\gamma s}(\alpha - 1)\right]\right\}. \qquad (24)$$

This special case is the result already given by Kolomenskii and Lebedev.[2] In a strong-focusing accelerator ($\alpha \ll 1$) there is antidamping of the oscillations, while in a weak-focusing accelerator [$\alpha = 1/(1-n)$, $0 < n < 1$] there is damping with the decrement

$$-\frac{1}{2}\frac{n}{1-n}\frac{\langle P_\gamma\rangle}{E_s}.$$

## 3. DAMPING OF FREE PHASE OSCILLATIONS. DISCUSSION OF RESULTS

The result which we have found is confirmed by a simpler computation of the damping of the phase oscillations.

Suppose that phase oscillations are excited at $t = 0$. This leads to the development of radial oscillations. We may choose a periodic solution for $r$, since it differs from any other by an oscillating term which is irrelevant for the damping. Thus

$$r = \psi\Delta E/E_s = -(L_s\psi/2\pi q c\alpha)\,\dot{\phi}. \qquad (25)$$

Substituting (25) in (8) and taking account of the variation in the frequency of the phase oscillations, we get

$$\phi \sim \exp\left\{-\frac{1}{2}\int_0^t\left[\frac{\dot{\Omega}}{\Omega} + \frac{\dot{E}_s + 2\overline{P}_{\gamma s} - \overline{P}_{\gamma s}(2n-1)\psi/\rho_s}{E_s}\right]dt'\right\}. \qquad (26)$$

According to Eq. (20), which $\psi$ satisfies,

$$\int_0^t \frac{\overline{P}_{\gamma s}\rho_s}{E_s}\frac{2n-1}{\rho_s^2}\psi\,dt' = \int_0^t \frac{dt'}{E_s}\left[\overline{P}_{\gamma s}\frac{\psi}{\rho_s} - 2P_{\gamma 0}\frac{\rho_0}{\rho_s} + 2P_{\gamma 0}\rho_0\frac{H_s - H_0}{H_0}\frac{n-1}{\rho_s^2}\psi\right] + \int_0^t d't\cdot\ddot{\psi}P_{\gamma 0}\rho_0/E_s c^2. \qquad (27)$$

The last integral can be dropped since $\ddot{\psi}$ (unlike $\psi$) averages to zero. Finally,

$$\phi_f \sim \exp\left\{-\frac{1}{2}\int_0^t\left[\frac{\dot{\Omega}}{\Omega} + \frac{\dot{E}_s + \overline{P}_{\gamma s}(2-\psi/\rho_s)}{E_s} + 2P_{\gamma 0}\frac{\rho_0}{\rho_s} - 2P_{\gamma 0}\rho_0\frac{H_s - H_0}{H_0}\frac{n-1}{\rho_s^2}\psi\right]dt'\right\}. \qquad (28)$$

As was to be expected, the sum of the damping coefficients for radial and phase oscillations does not depend on the form of $\psi$ and $(H_s - H_0)/H_0$, i.e., it does not depend on the specific form of the radial-phase coupling. The part of the sum which depends on the radiation is always

$$-\frac{3}{2}\int_0^t dt'\,\overline{P}_{\gamma s}/E_s.$$

For the case of weak focusing, where $H_s = H_0$, we get the familiar result of Sands:[1]

$$\phi \sim \exp\left\{-\frac{1}{2}\int_0^t\left[\frac{\dot{\Omega}}{\Omega} + \frac{\overline{P}_{\gamma s}}{E_s}\frac{3-4n}{1-n} + \frac{\dot{E}_s}{E_s}\right]dt',\right. \qquad \left\langle\frac{\psi}{\rho_s}\right\rangle = \alpha = \frac{1}{1-n}. \qquad (29)$$

The damping factors for $r$ and $\phi$ can also be obtained by finding the characteristic roots of the system of equations (8) and (9). For this purpose, we make the substitution

$$r = x_1\varphi e^{-i\nu t} + x_2\varphi^* e^{i\nu t},\quad x_2 = x_1^*,\quad \dot{r} = x_1\dot{\varphi}e^{-i\nu t} + x_2\dot{\varphi}^* e^{i\nu t},$$

$$\phi = x_3 + x_4,\quad \dot{\phi} = i\Omega(x_3 - x_4),\quad x_4 = x_3^*,$$

where $\nu$ is the betatron oscillation frequency, so that $\varphi e^{-i\nu t}$ and $\dot{\varphi}e^{-i\nu t}$ are periodic functions. We then get in place of (8) and (9) a set of four first order equations with periodic coefficients (which are constants for the case of weak focusing), and using first order perturbation theory we get four fundamental solutions $x_{ik}(t)$ (one index labels the solution, the other the function in the solution). If the $x_{ik}(t)$ are defined by the initial conditions

$$x_{ik}(0) = \delta_{ik},$$

the characteristic equation has the form[5]

$$|x_{ik}(T) - \lambda\delta_{ik}| = 0,$$

where $T$ is the period of the coefficients in the equations for the $x_{ik}(t)$. The damping factors are identical with those obtained above.

Formula (22) shows that in order to get damping of radial oscillations in a strong-focusing accelerator with a decrement equal to, say, $-\langle P_\gamma\rangle/2E_s$, which is sufficient for a marked reduction of particle losses, the approximate equality

$$\left\langle \frac{n-1}{\rho_s^2} \frac{H_s - H_0}{H_0} \psi \right\rangle \approx \left\langle \frac{1}{\rho_s} \right\rangle. \tag{30}$$

must be satisfied. At the same time the damping factor for the phase oscillations is still sufficiently large and is equal to $-\langle P_\gamma \rangle / E_s$.

The results we have found can be visualized as follows. The additional damping of the radial oscillations occurs together with an additional antidamping of the phase oscillations, i.e., oscillations of the energy are built up. For buildup of energy oscillations it is obviously necessary that an increase in the energy of the particle be accompanied by a decrease in the radiation. This will be the case if, when the energy is increased, the trajectory of the particle changes so that the quantity $\langle H^2 \rangle \sim \langle \rho^{-2} \rangle$, averaged along the new trajectory, decreases. For example, in a weak-focusing accelerator $\langle \rho^{-2} \rangle$ always decreases with increasing energy, and the more sharply the closer $n$ is to unity, since for $n \geq 1$ the motion becomes unstable. On the other hand, in a strong-focusing accelerator $\langle \rho^{-2} \rangle$ increases with increasing energy, even though $\langle 1/\rho \rangle$ decreases. This result is explained by the strong bending of the perturbed trajectory when $\Delta E/E > 0$ (inside the radially focusing magnets) compared with the unperturbed orbit, because of the large value of $n$. As a result the radial oscillations are built up in the strong-focusing accelerator (instead of being damped as they are in the weak-focusing case).

Obviously magnets with low $n$, introduced into a strong-focusing system, cannot change this picture, since they have practically no effect on the trajectory and do not change $\rho^{-2}$. As was proven above, another less obvious statement is also valid, namely that the introduction of additional magnets with arbitrarily large $n$ but with the same field as in the other magnets does not change the dependence of $\langle \rho^{-2} \rangle$ on the energy fluctuations. When $n$ is varied along the orbit without varying the field, the equilibrium trajectory is distorted so that

$$\langle \rho^{-2} \rangle \sim \left\langle \frac{1 - 2n\psi / \rho_s}{\rho_s^2} \right\rangle$$

is not changed for a given $\Delta E/E$.

To change the damping, it is necessary to vary the field. According to the qualitative arguments given above, to increase the damping for radial motion the field must be larger in the radially defocusing magnets ($n > 0$), since it is precisely in this case that the equilibrium trajectory straightens with increasing energy (if $\psi > 0$, which is usually the case). Formula (30) corresponds to precisely this result.

In practice, to satisfy condition (30) it is apparently more convenient to have a small number of radially focusing magnets with zero or negative field and high $n < 0$, which are designed so that they perturb the conditions of oscillation as little as possible. These requirements are satisfied, for example, by some of the proposals of Livingston and Robinson.[6]


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