STRONG AND WEAK INTERACTIONS INVOLVING HYPERONS

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Gell-Mann's universal strong interaction theory is extended to weak interactions. It is found that the results are verified by experiment.

GELL-MANN has recently suggested[1] that the interactions of π mesons with nucleons and hyperons are universal interactions with a coupling constant about 15 times as great as the coupling constant for the interaction between K mesons and baryons. According to his model, the different baryons can be thought of as forming four degenerate doublets, which could not be distinguished if not for their interaction with K mesons. In the present article we determine the relation of this symmetry property to the charge-independent nature of the pion-baryon interaction. This same symmetry is then extended to weak interactions, and the form of the interaction function is determined. This leads in a natural way to the well known selection rule

$$\Delta T = +{}^1/_2.$$

We have not considered the possibility of parity nonconservation in weak interactions. Its inclusion, however, will not lead to any difficulties.

Let us proceed from the expressions for the meson and baryon wave functions in spinor notation. By using the representation

$$+\tau_1^{\dot{\alpha}\beta} = -\tau_{1\alpha\dot{\beta}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad +\tau_2^{\dot{\alpha}\beta} = -\tau_{2\alpha\dot{\beta}} = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix},$$

$$+\tau_3^{\dot{\alpha}\beta} = -\tau_{3\alpha\dot{\beta}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad (1)$$

for the Pauli matrices, we may write

$$+\pi^{\dot{\alpha}\beta} = -\pi_{\alpha\dot{\beta}} = \boldsymbol{\tau}^{\dot{\alpha}\beta}\boldsymbol{\pi} = \begin{pmatrix} \pi^0 & \sqrt{2}\pi^+ \\ \sqrt{2}\pi^- & -\pi^0 \end{pmatrix},$$

$$N_\alpha = \begin{pmatrix} N^+ \\ N^0 \end{pmatrix}, \quad \Xi^{\dot{\alpha}} = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}, \quad K_\alpha = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix},$$

$$\psi_{\alpha\dot{\beta}} = \Lambda - \boldsymbol{\tau}_{\alpha\dot{\beta}}\Sigma = \begin{pmatrix} Z^0 & \Sigma^+ \\ \Sigma^- & Y^0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} \Lambda + \Sigma^0 & \Sigma_1 + i\Sigma_2 \\ \Sigma_1 - i\Sigma_2 & \Lambda - \Sigma^0 \end{pmatrix},$$

$$\psi^{\dot{\alpha}\beta} = \Lambda - \boldsymbol{\tau}^{\dot{\alpha}\beta}\Sigma = +\begin{pmatrix} Y^0 & -\Sigma^+ \\ -\Sigma^- & Z^0 \end{pmatrix}, \qquad (2)$$

where the dotted indices transform as the complex conjugate of the undotted indices, and π^0, $\pi^\pm$, N^+, N^0, Σ^0, Λ, etc. are the wave functions of the corresponding particles. Since we shall consider only rotations in a three-dimensional isobaric spin space, the dotted indices will transform contravariantly with respect to the undotted indices. The expressions

$$C_{\dot{\alpha}}C_\alpha, \quad C^\alpha C_\alpha, \quad C^{\dot{\alpha}}C^\alpha, \quad C^{\alpha *}C^\alpha,$$

$$C^*_\alpha C_\alpha, \quad C^{\dot{\alpha}*}C_\alpha, \quad C^{\alpha *}C_{\dot{\alpha}} \text{ etc.} \qquad (3)$$

therefore, are invariant. Here the summation convention is used over repeated indices α, which can take on the values 1 and 2.

According to d'Espagnat and Prentki,[2] the strong interactions of π mesons with nucleons and cascade particles are given by the expressions

$$H_1 = g_1\bar{N}_\alpha i\gamma_5\pi^{\alpha\dot{\beta}}N_\beta, \qquad H_2 = g_2\bar{\Xi}^{\dot{\alpha}} i\gamma_5\pi^{\alpha\dot{\beta}}\Xi^{\dot{\beta}}. \qquad (4)$$

Similarly, the interaction of π mesons with Σ and Λ particles is given by

$$H_3' = g_3\bar{\psi}_{\alpha\dot{\beta}} i\gamma_5\pi^{\dot{\alpha}\lambda}\psi_{\lambda\dot{\beta}} + g_4\bar{\psi}^{\dot{\alpha}\beta} i\gamma_5\pi^{\dot{\alpha}\lambda}\psi^{\dot{\lambda}\beta}, \qquad (5)$$

In the usual notation, this can be written

$$H_3' = (g_3 - g_4)\bar{\Lambda}i\gamma_5(\boldsymbol{\pi}\Sigma)$$

$$+ (g_3 + g_4)[\Sigma i\gamma_5\Sigma]\boldsymbol{\pi} + \text{compl. conj.} \qquad (5a)$$

According to Gell-Mann's model, we need now consider only the two cases, (i) $g_4 = 0$ and $g_3 \neq 0$ and (ii) $g_3 = 0$ and $g_4 \neq 0$. This follows from the fact that for all other cases, as is obvious from (5a), the mass increase due to the interaction with π mesons will not be the same for Λ and for Σ, so that it would make no sense to include both these particles in a single wave function $\psi_{\alpha\dot{\beta}}$. We see that only the two above-mentioned cases lead to different signs in the first term of (5a). If we bear in mind that all wave functions are given only up to an arbitrary factor of modulus 1, it is clear that these two cases transform into each other by the replacement of Λ by $-\Lambda$. They are therefore essentially the same, and in the future we shall consider only case (i). We can then write (5) in the form

$$H_3 = g_3 \bar{\psi}_{\alpha\dot{\beta}}\, i\gamma_5 \pi^{\dot{\alpha}\lambda}\, \psi_{\lambda\dot{\beta}}\,. \tag{6}$$

Using (2), we see that (3) and (6) are invariant under rotations in isobaric spin space. This invariance property, however, is stronger than the requirement that the pion-baryon interaction be independent of the charge of the meson. For this latter requirement it is sufficient that (6) be invariant under transformations with respect to those indices contained in $\pi^{\dot{\alpha}\lambda}$. To illustrate this, let us write (6) as the sum of two terms, namely

$$H_3 = g_3 \bar{\psi}_{\alpha\dot{1}}\, i\gamma_5 \pi^{\dot{\alpha}\lambda} \psi_{\lambda\dot{1}} + g_3 \bar{\psi}_{\alpha\dot{2}}\, i\gamma_5 \pi^{\dot{\alpha}\lambda} \psi_{\lambda\dot{2}} \tag{7}$$

The interaction will be independent of the meson charge if the two terms in (7) are separately invariant under spinor transformations with respect to the indices $\dot{\alpha}$ and λ, i.e., if $\psi_{\alpha\dot{1}}$ and $\psi_{\alpha\dot{2}}$ transform as two first-rank spinors. These ideas can be stated more exactly in the following way. Writing out (6) in detail, we obtain

$$\begin{aligned} H_3 = g_3 \{ (\bar{\Sigma}^+ i\gamma_5 \Sigma^+ - \bar{Y}^0 i\gamma_5 Y^0)\, \pi^0 \\ + \sqrt{2}\,(\bar{\Sigma}^+ i\gamma_5 Y^0 \pi^+ + \bar{Y}^0 i\gamma_5 \Sigma^+ \pi^-) + (\bar{Z}^0 i\gamma_5 Z^0 - \bar{\Sigma}^- i\gamma_5 \Sigma^-)\, \pi^0 \\ + \sqrt{2}\,(\bar{\Sigma}^- i\gamma_5 Z^0 \pi^- + \bar{Z}^0 i\gamma_5 \Sigma^- \pi^+) \}\,. \end{aligned} \tag{8}$$

We see from this that the interaction causes a transition between Σ^+ and Y^0, as well as between Σ^- and Z^0, although it causes no transition from the doublet Σ^+, Y^0 to the doublet Σ^-, Z^0. This is sufficient to prove that (if one is speaking of π-meson interactions) these two doublets differ from each other just as the doublet Ξ^0, Ξ^- differs from N^+, N^0. The hypothesis of charge invariance now means that when a π meson interacts with a hyperon, it "does not recognize" the difference between Σ^+ and Y^0 or between Σ^- and Z^0.

Thus the hyperon may be in a state which is an arbitrary superposition of Σ^+ and Y^0 or Σ^- and Z^0 without changing the interaction. There is no necessity for treating a superposition of Σ^+, Y^0, Σ^-, and Z^0, since the interaction causes no transition from the state Σ^+, Y^0 to Σ^-, Z^0. Mathematically this means that from the point of view of the π-meson interaction, Σ^+, Y^0 and Σ^-, Z^0 should be treated as two spinors, rather than as a single spinor matrix.

We may, with Gell-Mann, assume that the coupling constants g_1, g_2, and g_3 in Eqs. (3) and (7) are equal. Then the strong interactions of π mesons with different hyperons can be written in the universal form

$$H_i = g \bar{\psi}_\alpha^{(i)} i\gamma_5 \pi^{\dot{\alpha}\beta} \psi_\beta^{(i)} \qquad (i = 1, 2, 3, 4); \tag{9}$$

$$\psi^{(1)} = \begin{pmatrix} N^+ \\ N^0 \end{pmatrix}, \quad \psi^{(2)} = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix}, \quad \psi^{(3)} = \begin{pmatrix} Z^0 \\ \Sigma^- \end{pmatrix}, \quad \psi^{(4)} = \begin{pmatrix} \Sigma^+ \\ Y^0 \end{pmatrix}. \tag{9a}$$

If we assume further that the non-renormalized masses of the N^+, N^0, Ξ, and Σ particles are initially equal (it has already been assumed that the Λ and Σ masses are equal), we obtain Gell-Mann's "global symmetry." The baryons form four pairs of degenerate doublets which cannot be differentiated except by the K-meson interaction. We see from the above that the equality of all the baryon masses is not a necessary condition for the universality of the strong interaction with π mesons. In fact, the latter requires only the equality of the non-renormalized Λ and Σ masses.

In the same way we may assume the universality of the weak pion-baryon interactions. As opposed to the strong interactions, in which the π mesons interact with only a single baryon field $\psi^{(i)}$ in an elementary act, in the weak interaction the π mesons must interact simultaneously with two different baryon fields. We assume that the universal weak interaction is given by

$$H_{Wij} = G \psi_\alpha^{(i)} i\gamma_5 \pi^{\dot{\alpha}\beta} \psi_\beta^{(j)} + \text{compl. conj.} \qquad (i \neq j), \tag{10}$$

where G is the universal weak coupling constant. We note that expression (10) transforms in the same way as the strong interaction (9) under rotations in isobaric spin space.

Not all interaction functions (10) with different values of i and j will conserve the electric charge in the system. It is easily seen that the charge is conserved only for the two interactions

$$\begin{aligned} H_{W1} &= -G \bar{N}_\alpha \pi^{\dot{\alpha}\beta} \psi_{\beta\dot{2}} + \text{compl. conj.} \\ H_{W2} &= -G \bar{\Xi}^{\dot{\alpha}} \pi^{\dot{\alpha}\beta} \psi_{\beta\dot{1}} + \text{compl. conj.} \end{aligned} \tag{11}$$

Before moving on, it must be emphasized that there exists also an alternate form for the weak interaction functions, satisfying all the requirements satisfied by (11). This form is

$$\begin{aligned} H_{W1} &= -G \bar{N}_\alpha \pi^{\dot{\alpha}\beta} \psi^{\dot{\beta}2} + \text{compl. conj.}, \\ H_{W2} &= -G \bar{\Xi}^{\alpha} \pi^{\dot{\alpha}\beta} \psi^{\dot{\beta}1} + \text{compl. conj.} \end{aligned} \tag{12}$$

As in the previously treated case, (12) reduces to (11) when Λ is replaced by $-\Lambda$. In view of the fact that the choice of Eq. (6) has eliminated the ambiguity in the sign of the Λ function, Eqs. (11) and (12) should now be different from each other. Writing (11) and (12) in the usual notation, we obtain

$$H_{W1} = G\left\{\left(\bar{N}^{+}i\gamma_5\Sigma^{+} \mp \frac{1}{\sqrt{2}}\bar{N}^{0}i\gamma_5\Lambda + \frac{1}{\sqrt{2}}\bar{N}^{0}i\gamma_5\Sigma^{0}\right)\pi^{0} + (\pm\bar{N}^{+}i\gamma_5\Lambda - \bar{N}^{+}i\gamma_5\Sigma^{0})\pi^{+} + \sqrt{2}\,\bar{N}^{0}i\gamma_5\Sigma^{+}\pi^{-} + \text{compl. conj.}\right\},$$

$$H_{W2} = G\left\{\left(-\bar{\Xi}^{-}i\gamma_5\Sigma^{-} \mp \frac{1}{\sqrt{2}}\bar{\Xi}^{0}i\gamma_5\Lambda + \frac{1}{\sqrt{2}}\bar{\Xi}^{0}i\gamma_5\Sigma^{0}\right)\pi^{0} + (\mp\bar{\Xi}^{-}i\gamma_5\Lambda + \bar{\Xi}^{-}i\gamma_5\Sigma^{0})\pi^{-} + \sqrt{2}\,\bar{\Xi}^{0}i\gamma_5\Sigma^{-}\pi^{+} + \text{compl. conj.},\right. \quad (13)$$

where the upper and lower signs on Λ correspond to interactions (11) and (12). It should be noted that (11) and (12) are spinor components in isobaric spin space. Several authors[3] have already suggested and investigated the possibility that the weak interaction Hamiltonians may be spinors in isobaric spin space. This type of Hamiltonian will change the isotopic spin T by $\frac{1}{2}$. We note that no term of (13) will cause the transition $\Sigma^{-} \to N^{0} + \pi^{-}$. From the form of (13), we see also that Σ particles can undergo transitions only to states with $T = +\frac{1}{2}$ (which means that $\Delta T = -\frac{1}{2}$). At first sight this would mean that (13) is not in agreement with experiment, since we know that the lifetime for the $\Sigma^{-} \to N^{0} + \pi^{-}$ decay mode is comparable to that for the decay of Σ^{+}. We shall show that the strong interaction makes it possible for the weak interaction (13) to lead to decay of Σ^{-}.*

The decay process may take place through the strong interaction in the following way:

$$\Sigma^{-} \to \pi^{-} + \Lambda \to \pi^{-} + \pi^{-} + N^{+} \to \pi^{-} + N^{0}; \quad (14)$$

$$\Sigma^{-} \to \pi^{-} + \Sigma^{0} \to \pi^{-} + \pi^{-} + N^{+} \to \pi^{-} + N^{0}. \quad (15)$$

If (11) is taken as the weak-interaction Hamiltonian and (3) and (6) are taken as the strong-interaction Hamiltonians, direct calculation shows that the transition amplitude for (14) is equal and opposite to the transition amplitude for (15). It is easily seen that for more complicated decay schemes the contribution from virtual Λ particles always annuls a similar contribution from virtual Σ^{0} particles, so that the resulting transition amplitude for Σ^{-} decay is always zero. If, however, we take (12) as the interaction Hamiltonian, we find that the two amplitudes are always in the same direction. We therefore obtain a nonvanishing transition amplitude for Σ^{-} decay, with a lifetime comparable to the lifetime for Σ^{+} decay.

From the above result we conclude that (12) [and not (11)] should be chosen as the universal weak interaction Hamiltonian.

We shall show below that (12) also leads to the decay of the K^{0} meson. The most general form of the K-meson-baryon interaction Hamiltonian is

$$H' = f_1\bar{\Xi}^{\dot{\alpha}}\psi_{\alpha\dot{\beta}}K^{\dot{\beta}} + f_2\bar{\Xi}^{\dot{\alpha}}\psi^{\dot{\alpha}\beta}K^{\dot{\beta}} + f_3\bar{N}_{\alpha}\psi_{\alpha\dot{\beta}}K_{\beta} + f_4\bar{N}_{\alpha}\psi^{\dot{\alpha}\beta}K_{\beta} + \text{compl. conj.} \quad (16)$$

or, in the usual notation

$$H' = (f_1 + f_2)\bar{\Xi}\tau_2K^{*}\Lambda + (f_1 - f_2)\bar{\Xi}(\boldsymbol{\tau}\Sigma)K^{*} + (f_3 + f_4)\bar{N}K\Lambda + (f_3 - f_4)\bar{N}(\tau\Sigma)K + \text{compl. conj.} \quad (17)$$

According to Gell-Mann, the coupling constants f_i are one order of magnitude less than g in Eq. (9), and the mass difference between the nucleon, Σ, Λ, and Ξ is due to the interaction (16). The decay of K^{0} into $\pi^{+} + \pi^{-}$ and $\pi^{0} + \pi^{0}$ can go by way of the strong interactions (3) and (6), the moderately strong interaction (16), and the weak interaction (12). The simplest diagrams for such processes contain three vertices, one for each form of the interaction. The fact that the f_i are small compared to g may increase the decay time of the K^{0} meson compared to the decay time of Λ and Σ, but this is compensated by the fact that the number of virtual paths is relatively large (there are about ten). Another compensating factor is the large volume in phase space of the final state. One may therefore expect that the lifetime of K^{0} decay is of the same order as the Λ and Σ lifetimes, which is in agreement with experiment.

Note added in proof (February 20, 1958). A recent experiment has verified the fact that parity is not conserved in strange-particle decay. This means that the weak interaction Hamiltonian will contain terms which are pseudoscalars, rather than scalars. If Λ in (13) is replaced by $i\gamma_5\Lambda$, then all the terms containing Λ will become pseudoscalars. It is easily seen that after such a

*It should be emphasized, that due to the strong interaction, the selection rule for ΔT which follows from the weak interaction Hamiltonian is not generally the observed selection rule. For instance, the unstable strange particle under consideration may be transformed into another unstable strange particle by the strong interaction. For this new particle, the selection rule given by the interaction Hamiltonian will be different. If the initial particle decays through this second one, the selection rule for the process may be modified. As an example, the strange particle may emit virtual mesons so that the isotopic spin T' (thought of as a space-quantized vector) of the particle in this virtual state is antiparallel to the total T. We thus obtain $\Delta T = -\Delta T'$. If the interaction Hamiltonian allows, for instance, only $\Delta T = +\frac{1}{2}$, we see immediately that a decay which takes place through this virtual state may have a change in T equal to $\Delta T = -\frac{1}{2}$.

replacement (14) and (15) will not be mutually exclusive, and the theory is again invariant under the transformation $\Lambda \rightarrow -\Lambda$, as in the case of the strong interaction.

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DAMPING OF OSCILLATIONS IN A CYCLIC ELECTRON ACCELERATOR

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Damping factors are derived for radial and phase oscillations, taking account of variation of the magnetic field along the orbit. In the case of a strong-focusing accelerator, in contrast to the case of weak focusing, the damping is independent of the variation of the gradient $\partial H_z/\partial r$ along the orbit if the field H_z is the same in all magnet sectors.

1. EQUATIONS OF MOTION

To derive the equations of motion of an electron in a cyclic accelerator, we use the well-known relations

$$\Delta L / L_s = \alpha \Delta E / E_s, \qquad \Delta E = E - E_s, \qquad E_s \gg mc^2; \quad (1)$$

$$\dot{\Phi} = -(2\pi q c\alpha / L_s)\,\Delta E / E_s, \qquad \alpha = d \ln L / d \ln E, \quad (2)$$

where E_s and L_s are the equilibrium values of the electron energy and the orbit length, q is the harmonic number (the ratio of the rf frequency to the frequency of revolution), Φ is the phase of the accelerating voltage at the moment when the particles pass the middle of the accelerating gap. On the right side of (2), we have dropped some terms which are unimportant for the effects in which we are interested: the perturbation $\Delta\omega_r$ of the frequency of the accelerating field and the transient perturbation $2\pi q c \alpha L_s^{-1} \Delta H(t)/H_s$ of the magnetic field.

Differentiating (2) with respect to the time, we get[1]

$$\ddot{\Phi} + \frac{2\pi q c\alpha}{L_s E_s}\frac{d}{dt}(\Delta E) + \frac{\dot{E}_s}{E_s}\dot{\Phi} = 0, \quad (3)$$

where

$$\frac{d}{dt}(\Delta E) = P_0 - \left(1 - \frac{r}{\rho}\right)P_\gamma - \dot{E}_s, \qquad P_0 = \frac{ceV}{L_s}\sin\Phi, \quad (4)$$

where ρ is the radius of curvature of the orbit and P_γ the power in the radiation. Dropping the unimportant term describing the perturbation $\Delta V/V$, we have, in the linear approximation,

$$P_0 \approx P_{0s}[1 + \cot\Phi_s(\Phi - \Phi_s)], \qquad P_{0s} = ceV_0 \sin\Phi_s / L_s; \quad (5)$$

$$P_\gamma = 2e^4 E^2 H^2 / 3m^4 c^7 + p(t) = \bar{P}_\gamma + p(t), \quad (6)$$

where $p(t)$ describes the fluctuations of the radiation, $\bar{P}_\gamma$ is the (frequency) average of the power of the radiation at a given point on the orbit; this last quantity depends on both betatron and phase oscillations. According to (6),

$$\bar{P}_\gamma = \bar{P}_{\gamma s}[1 - 2L_s\dot{\Phi} / 2\pi q c\alpha - 2nr / \rho_s], \quad (7)$$

$$n = -(\rho_s / H_s)\,\partial H_s / \partial r.$$