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ON THE POLARIZATION OF THE CERENKOV RADIATION FROM A FAST PARTICLE CARRYING A MAGNETIC MOMENT

Iu. M. LOSKUTOV and A. B. KUBANOV

Moscow State University

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The problem of the polarization of the Cerenkov radiation from a magnetic moment moving in a ferrodielectric material is treated by the methods of quantum electrodynamics. Calculations are made of both the part of the radiated intensity accompanied by flip of the spin $(ss' = -1)$ and the part of the intensity emitted without spin-flip $(ss' = +1)$. It is shown that the radiation is composed of a polarized part (which vanishes at the threshold, $\cos\theta = 1$) and an unpolarized part (which does not vanish at the threshold). The unpolarized part of the radiation is accompanied by spin-flip.

The energy losses are treated by classical methods, and the separation of the losses into Cerenkov loss and ionization loss is indicated.

THE problem of the polarization of the Cerenkov radiation of a charge in a dielectric has been dealt with by Sokolov and Loskutov,[1] who have shown that the radiation is partially polarized and does not vanish at the threshold; this last fact is due to the presence of the spin of the electron. It is not hard to show that in the case of the motion of a charge through a ferrodielectric material one gets for the intensity radiated per unit length the formulas

$$W_3 = \frac{e^2}{c^2}\int_0^{\omega_{\max}} \mu(\omega)\,\omega\left\{\frac{n^2\omega^2\hbar^2}{4c^2p^2}(1-n^{-2}) + (1-\cos^2\theta)\right\}d\omega, \quad (1)$$

$$W_2 = \frac{e^2}{c^2}\int_0^{\omega_{\max}} \mu(\omega)\,\omega\,\frac{n^2\omega^2\hbar^2}{4c^2p^2}(1-n^{-2})\,d\omega, \quad (2)$$

where W_3 and W_2 are the intensities respectively in and perpendicular to the plane $\angle\boldsymbol{\varkappa}\mathbf{k})$ and $n = (\epsilon\mu)^{1/2}$.

Comparing Eqs. (10) and (11) of Ref. 1 with Eqs. (1) and (2), we see that inclusion of the effect of the magnetic susceptibility of the medium does not change the nature of the polarization of the radiation in the dielectric.

1. POLARIZATION OF THE CERENKOV RADIATION OF A MAGNETIC MOMENT

(a) As is well known, the operator for the interaction energy of a magnetic moment $\boldsymbol{\mu}_0$ and the electromagnetic field in a medium is given by

$$W = \mu_0\rho_3(\boldsymbol{\sigma}\mathbf{B}) + \mu_0\rho_2(\boldsymbol{\sigma}\mathbf{E}), \quad (3)$$

where ρ_3, ρ_2, and $\boldsymbol{\sigma}$ are Dirac matrices.

The expression for the vector potential $\mathbf{A}$ of the quantized transverse electromagnetic field in a medium characterized by the constants $\epsilon(\omega)$ and $\mu(\omega)$ can be written in the form (cf. Ref. 2)

$$\mathbf{A} = L^{-3/2}\sum_{\varkappa}(2\pi c''\hbar/\varkappa)^{1/2}[\mathbf{a}\exp(-ic'\varkappa t + i\boldsymbol{\varkappa}\mathbf{r}) + \mathbf{a}^+\exp(ic'\varkappa t - i\boldsymbol{\varkappa}\mathbf{r})], \quad (4)$$

where $c' = c/(\epsilon\mu)^{1/2}$, $c'' = c'\mu$; $\hbar\boldsymbol{\kappa}$ is the momentum of a photon; and the amplitudes $\mathbf{a}$ and $\mathbf{a}^+$ obey the commutation relations

$$a_n a'^+_{n'} - a'^+_{n'} a_n = \delta_{\varkappa\varkappa'}(\delta_{nn'} - \varkappa^{-2}\varkappa_n\varkappa_{n'}). \quad (5)$$

In particular, when there are no photons in the initial state (as we shall assume in what follows) we can set

$$a'^+_{n'}a_n = 0, \quad a_n a'^+_{n'} = \delta_{\varkappa\varkappa'}(\delta_{nn'} - \varkappa^{-2}\varkappa_n\varkappa_{n'}). \quad (6)$$

To study the polarization of the radiation we resolve the amplitude $\mathbf{A}$ of the vector potential into components (cf. Ref. 3) which characterize definite states of polarization:

$$\mathbf{a} = \mathbf{a}_2 + \mathbf{a}_3 = \boldsymbol{\beta}_2 q_2 + \boldsymbol{\beta}_3 q_3,$$
$$\boldsymbol{\beta}_2 = [\boldsymbol{\varkappa}^0\mathbf{k}^0]/\sqrt{1-(\boldsymbol{\varkappa}^0\mathbf{k}^0)^2}, \quad \boldsymbol{\beta}_3 = [\boldsymbol{\varkappa}^0\boldsymbol{\beta}_2] \quad (7)$$

in the case of linear polarization, and

$$\mathbf{a} = \mathbf{a}_1 + \mathbf{a}_{-1} = \beta_1 q_1 + \beta_{-1} q_{-1}, \quad \sqrt{2}\,\beta_\lambda = \beta_2 + i\lambda\beta_3, \quad \lambda = 1, -1 \tag{8}$$

in the case of circular polarization. Here $\boldsymbol{\varkappa}^0$ is a unit vector in the direction of $\boldsymbol{\varkappa}$ and $\mathbf{k}^0$ is a unit vector along the direction of motion of the magnetic moment. In Eqs. (7) and (8) the quantum part of the amplitudes satisfies the relations

$$q_{j'}^+ q_j = 0, \quad q_j q_{j'}^+ = \delta_{jj'}; \quad j, j' = 2, 3, 1, -1. \tag{9}$$

Using the fact that

$$\mathbf{B} = \operatorname{curl} \mathbf{A}, \quad \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t},$$

and also recalling the absence of photons in the initial state, we find for the interaction energy operator W^+ (cf. Ref. 3)

$$W_j^+ = -i\mu_0 L^{-3/2} \sum_{\varkappa} (2\pi c''\hbar/\varkappa)^{1/2} \exp\{i(c'\varkappa t - \boldsymbol{\varkappa}\mathbf{r})\} \times \left\{\rho_3(\boldsymbol{\sigma}[\boldsymbol{\varkappa}\mathbf{a}_j^+]) + \frac{c'}{c}\varkappa\rho_2(\boldsymbol{\sigma}\mathbf{a}_j^+)\right\}. \tag{10}$$

By the use of the methods of perturbation theory it is not hard to find (cf. Ref. 4) the probability of emission of radiation by the magnetic moment:

$$W_j = \frac{2\pi}{c\hbar^2} \sum_{\mathbf{k}',\varkappa} R_j^+ R_j \delta_{\mathbf{k},\mathbf{k}'+\boldsymbol{\varkappa}} \delta(K' + \varkappa/n - K), \tag{11}$$

where

$$R_j = -i\mu_0 L^{-3/2}(2\pi c''\hbar/\varkappa)^{1/2} b'^+ \left\{\rho_3(\boldsymbol{\sigma}[\boldsymbol{\varkappa}\mathbf{a}_j^+]) + \frac{c'}{c}\varkappa\rho_2(\boldsymbol{\sigma}\mathbf{a}_j^+)\right\} b,$$

$\mathbf{k}$, $K = (k^2 + k_0^2)^{1/2}$ and $\mathbf{k}'$, $K' = (k'^2 + k_0^2)^{1/2}$ correspond to the magnitudes of the momenta ($\hbar\mathbf{k}$, $\hbar\mathbf{k}'$) and energies ($c\hbar K$, $c\hbar K'$) of the magnetic moment respectively before and after the act of emission. Calculating out the required matrix elements (cf. Ref. 5, Sec. 21) to find W_j by Eq. (11), we get the expression

$$\begin{aligned} W_j = {} & \frac{c\mu_0^2}{16\pi}\int \frac{\varkappa^2}{n^2}\delta(K' + \varkappa/n - K)\,d^3\varkappa \\ & \times \sum_{s,s'}\left\{\left(1 - ss'\frac{kk'}{KK'} + \frac{k_0^2}{KK'}\right)\left[\left(1 - ss'\frac{(\mathbf{kk}')}{kk'}\right) + 2ss'\frac{(\mathbf{k}[\boldsymbol{\varkappa}^0\mathbf{a}_j])(\mathbf{k}'[\boldsymbol{\varkappa}^0\mathbf{a}_j^+])}{kk'}\right]\right. \\ & + \frac{c'^2}{c^2}\left(1 - ss'\frac{kk'}{KK'} - \frac{k_0^2}{KK'}\right)\left[\left(1 - ss'\frac{(\mathbf{kk}')}{kk'}\right) + 2ss'\frac{(\mathbf{ka}_j)(\mathbf{k}'\mathbf{a}_j^+)}{kk'}\right] \\ & \left. + 2\frac{c'}{c}\left(s\frac{k}{K} - s'\frac{k'}{K'}\right)\left(\frac{1}{kk'}(sk'\mathbf{k} - s'k\mathbf{k}')\left[[\boldsymbol{\varkappa}^0\mathbf{a}_j]\,\mathbf{a}_j^+\right]\right)\right\}, \\ & (j = 2, 3), \end{aligned} \tag{12}$$

where s and s' are the spins of the magnetic-moment particle respectively before and after the emission of the photon.

(b) For the intensity of the radiation per unit length we find after averaging over the spin states

$$W_j = \mu_0^2 \int_0^{\omega_{max}} \frac{n^2\omega^3}{\beta^2 c^4}\mu(\omega)\{(j-2)\beta^2(1-\cos^2\theta) + (1-\beta^2)\}(1-n^{-2})\,d\omega, \tag{13}$$

$$(j = 2, 3); \quad W_1 = W_{-1} = {}^1/_2(W_2 + W_3), \tag{14}$$

θ is the angle between the directions of travel of the emitted photon and of the magnetic moment.

(c) For the intensities of the radiation per unit length in the cases of spin-flip ($ss' = -1$, upper sign) and of absence of spin-flip ($ss' = +1$, lower sign) we get

$$\begin{aligned} W_j(\mp) = {} & \frac{1}{4}\mu_0^2 \int_0^{\omega_{max}} \frac{n^2\omega^3}{\beta c^4}\mu(\omega)\left[\frac{2}{\beta}\left\{(j-2)\beta^2(1-\cos^2\theta) + (1-\beta^2)\right\}(1-n^{-2}) \pm \Gamma\left\{\beta(1+n^{-2}) - \frac{2}{n}\cos\theta\right\}\right. \\ & \pm\Gamma^{-1}\left\{\left(1 + (j-3)\,2\sin^2\theta - \frac{n\omega\hbar}{cp}\cos\theta\right)(2/\beta - \beta - \omega\hbar/cp)\right. \\ & + \frac{1}{n^2}\left(1 - (j-2)\,2\sin^2\theta - \frac{n\omega\hbar}{cp}\cos\theta\right)(\beta - \omega\hbar/cp) \\ & \left.\left. - \frac{2}{n}(\cos\theta - n\omega\hbar/cp)\left(1 - \beta\frac{\omega\hbar}{cp}\right)\right\}\right]d\omega, \quad (j = 2, 3), \end{aligned} \tag{15}$$

$$W_1(\mp) = W_{-1}(\mp) = {}^1/_2(W_2(\mp) + W_3(\mp)), \tag{16}$$

where

$$\Gamma = (1 - (2n\omega\hbar/cp)\cos\theta + n^2\omega^2\hbar^2/c^2p^2)^{1/2}.$$

According to Eq. (15) we get for the radiation at the threshold

$$W_j(+) = 0, \tag{17}$$

$$W_j(-) = \mu_0^2 \int_0^{\omega_{max}} \frac{n^2\omega^3}{\beta^2 c^4}\mu(\omega)(1-\beta^2)(1-n^{-2})\,d\omega. \tag{18}$$

It can be seen from Eqs. (13) and (14) that the radiation is partially polarized and is nonvanishing at the threshold; Eqs. (17) and (18) show that this last fact is due to the spin-flip, i.e., is a purely quantum effect. Also not without importance is the fact that the polarized and unpolarized parts of the radiation are of the same order of magnitude, and for $1/n < \beta < (2/3 + 1/n^2)^{1/2}$ the unpolarized part even exceeds the polarized. We note in passing that the threshold radiation vanishes in the ultrarelativistic approximation, just as in the classical approximation. The result of Eq. (18), that the radiation at the threshold is finite, is a consequence of the fact that, as is shown by calculation [by means of Eq. (12)], in the case of absence of spin-flip the probability of emission of radiation at threshold is zero, but with spin-flip it remains finite.

The fact that the threshold radiation of a magnetic electron described by the Pauli equation is nonvanishing has been pointed by Ginzburg.[6]

2. ENERGY LOSS OF A MAGNETIC MOMENT MOVING THROUGH A FERRODIELECTRIC MEDIUM

In dealing with the energy loss in a ferrodielectric medium we shall assume that the direction of the magnetic moment coincides with the direction of motion. In this case the electric moment **p** appearing because of the motion of the magnetic moment is zero. The equations for the potentials must here be written in the form

$$\Delta \mathbf{A} - (\varepsilon\mu / c^2)\, \partial^2 \mathbf{A} / \partial t^2 = -4\pi\mu \operatorname{curl} \mathbf{M}, \qquad \Delta\varphi - (\varepsilon\mu / c^2)\, \partial^2 \varphi / \partial t^2 = 0, \tag{19}$$

where $\mathbf{M} = \boldsymbol{\mu}_0 \delta(\mathbf{r} - \mathbf{r}_\xi)$ and $\mathbf{r}_\xi(t)$ specifies the position of the magnetic moment.

Setting $\varphi \equiv 0$, we have for the intensities of the electric and magnetic fields

$$\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}, \quad \mathbf{H} = \frac{1}{\mu} \operatorname{curl} \mathbf{A}, \tag{20}$$

and setting $\mathbf{A} = \operatorname{curl} \mathbf{\Pi}$, where $\mathbf{\Pi}$ is the magnetic polarization potential, we get the equation

$$\Delta \mathbf{\Pi} - \frac{\varepsilon\mu}{c^2}\frac{\partial^2 \mathbf{\Pi}}{\partial t^2} = -4\pi\mu\boldsymbol{\mu}_0\, \delta(\mathbf{r} - \mathbf{r}_\xi). \tag{21}$$

In the case of uniform motion of the magnetic moment along the z axis (for method of solution of such equations see, for example, Ref. 7 or Ref. 8) we find

$$\mathbf{\Pi} = \frac{\mu_0}{\pi} \int \mu K_0(\zeta r) \exp\{i\varkappa(z - vt)\}\, d\varkappa, \tag{22}$$

where $K_0 = (\pi i/2) H_0^{(1)}(i\zeta r)$ and $\kappa = \omega/c$. Here

$$\zeta = \varkappa(1 - \varepsilon\mu\beta^2)^{1/2} \operatorname{sign} \operatorname{Re} \varkappa (1 - \varepsilon\mu\beta^2)^{1/2},$$

and $H_0^{(1)}$ is the Hankel function.

For the energy loss W_b in collisions with impact parameter larger than b we compute the flux of the Umov-Poynting vector through the lateral surface of a cylinder of radius b surrounding the z axis:

$$W_b = \frac{c}{4\pi v} \int_{\Sigma} \mathbf{E} \times \mathbf{H}\, dS$$

$$= \frac{b\mu_0^2}{\pi v^2} \operatorname{Re} \int_{-\infty}^{+\infty} i\omega\mu^*(\omega)\, \zeta^* \zeta^2 K_0(\zeta b) K_1(\zeta^* b)\, d\omega, \tag{23}$$

where **dS** is a surface element of the cylinder.

In the derivation of Eq. (23) we have used the relations (20) and (22), and also the formulas

$$K_{n+1}(\xi) - K_{n-1}(\xi) = 2nK_n(\xi)/\xi,$$

$$dK_n(\xi)/d\xi = -K_{n-1}(\xi) - \frac{n}{\xi} K_n(\xi).$$

In proceeding to the consideration of small values of the parameter b (of the order of interatomic distances), regarding the extinction coefficients in the actual expressions for ϵ and μ as finite, we can confine ourselves to the first terms of the expansions of the Bessel functions $K_0(\xi)$ and $K_1(\xi)$ (since $|\zeta b| \ll 1$):

$$K_1 = 1/\zeta^* b, \quad K_0(\zeta b) = \frac{1}{2} \ln(4/3.17\,\zeta^2 b^2).$$

Consequently, at small impact parameters we have for W_b

$$W_b = \frac{\mu_0^2}{2\pi v^2} \operatorname{Re} \int_{-\infty}^{+\infty} i\omega\mu^*(\omega)\, \zeta^2 \ln \frac{4}{3.17\zeta^2 b^2}\, d\omega. \tag{23a}$$

From this we get

$$W_b = \frac{\mu_0^2}{\pi v^4} \int_0^\infty \omega^3 \Big\{ [\operatorname{Im}\mu + \beta^2 |\mu|^2 \operatorname{Im}\varepsilon] \ln \frac{4v^2}{3.17 b^2 \omega^2 |1 - \varepsilon\mu\beta^2|} + \varphi [\operatorname{Re}\mu - \beta^2 |\mu|^2 \operatorname{Re}\varepsilon] \Big\}\, d\omega, \tag{24}$$

where

$$\varphi = \tan^{-1} \frac{-\beta^2 \operatorname{Im} \varepsilon\mu}{1 - \beta^2 \operatorname{Re} \varepsilon\mu}$$

for the Bohr frequencies $(1 - \beta^2 \operatorname{Re} \epsilon\mu > 0)$ and

$$\varphi = -\pi + \tan^{-1} \frac{\beta^2 \operatorname{Im} \varepsilon\mu}{-1 + \beta^2 \operatorname{Re} \varepsilon\mu}$$

for the Cerenkov frequencies $(1 - \beta^2 \operatorname{Re} \epsilon\mu < 0)$.

In making the separation of the losses into Cerenkov and ionization losses we use the fact that the frequency at which the Cerenkov radiation breaks off $(\omega = \omega_{max})$ is not a characteristic frequency for the ionization losses. Consequently, the mathematical formula for the latter losses must not change on passage through this frequency. Thus we can write

$$W_b^{cer} = -\frac{\mu_0^2}{v^4} \int_{\operatorname{Re}\varepsilon\mu\beta^2 > 1} \omega^3 (\operatorname{Re}\mu - \beta^2 |\mu|^2 \operatorname{Re}\varepsilon)\, d\omega \tag{25}$$

for the Cerenkov loss and

$$W_b^{ion} = \frac{\mu_0^2}{\pi v^4} \int_0^\infty \omega^3 \Big\{ (\operatorname{Im}\mu + \beta^2 |\mu|^2 \operatorname{Im}\varepsilon) \ln \frac{4v^2}{3.17 b^2 \omega^2 |1 - \varepsilon\mu\beta^2|} - (\operatorname{Re}\mu - \beta^2 |\mu|^2 \operatorname{Re}\varepsilon) \tan^{-1} \frac{\beta^2 \operatorname{Im}\varepsilon\mu}{1 - \beta^2 \operatorname{Re}\varepsilon\mu} \tag{26}$$

for the ionization loss. For $\mu = 1$, Eq. (25) gives the result of Eq. (16) in Ref. 6.

In conclusion we thank Professor A. A. Sokolov for suggesting this topic and for a discussion of our results.

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GLOW OF AIR DURING A STRONG EXPLOSION, AND THE MINIMUM BRIGHTNESS OF A FIREBALL *

Iu. P. RAIZER

Academy of Sciences, U.S.S.R.

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The optical properties of air at temperatures below 6,000° are considered. It is shown that the radiation and absorption of visible light at temperatures between 6,000° and 2,000° is due to the nitrogen dioxide that is formed in the air at these temperatures. This affords an explanation for several optical phenomena observed in strong explosions: the glow of air in a shock wave at low temperatures (down to 2,000°), the separation of the shock-wave front from the boundary of the fireball when the temperature of the front is close to 2,000°, and the peculiar effect of minimum brightness of the fireball.

A general description of the optical effects observed during strong (atomic) explosions in air is given in an American survey.[2]

A shock wave propagates from the center of the explosion along a trajectory which was shown by Sedov[3] to satisfy, with good approximation, the self-similar law $R \sim t^{2/5}$.

So long as the amplitude of the wave is sufficiently large, the surface of the front of the shock wave (SWF) glows brightly, forming the so-called fireball (FB). The brightness or the effective temperature of the FB, taken to mean the absolute temperature of a black body producing an identical radiation flux as the FB, diminishes with time as the true temperature behind the SWF decreases. At a certain instant of time, t_{min}, which is on the order of 10^{-2} sec for an explosion with energy $E \sim 10^{21}$ ergs, the SWF stops glowing and the boundary of the glowing body separates from the wave front. The brightness of the FB now goes through a minimum, after which it increases again — the FB, so to speak, flares up again. Now the dimensions of the FB increase much slower, while

*The work was performed in 1954. For a brief communication of the results see the review, Ref. 1.