$$\lambda = -2a_1^2\varepsilon^{-2}(k_3 - k_b) R_{31}\sqrt{R_{12}/\varkappa_{1b}},$$
$$\varepsilon = \sqrt{a_1 a_2/\alpha_0} \quad (\varepsilon \ll 1), \qquad (3)$$
$$R_{31}^{-1} = -\partial^2 k_b/\partial\varkappa_{1b}^2, \quad R_{21}^{-1} = -\partial^2\varkappa_{1b}/\partial k_2^2.$$

Here κ_{1b} is the value of κ_1 on the boundary of cells in k-space (for the boundary section κ_{1b} and R_{12} vanish but their ratio remains constant). Substituting (2) in (1), integrating over k_1 and dropping small quantities, we obtain for the integral in (1)

$$\frac{a_2}{2\alpha_0^2}\int_{-k_{3\max}}^{k_{3\max}} e^{-ip\alpha_0^2}\,[e^{(k_b-k_3)/k_0}+1]^{-1}\,dk_3, \qquad (4)$$

where

$$k_0 = (\varepsilon^2/2\pi a_1^2 R_{31})\sqrt{\varkappa_{1b}/R_{12}}. \qquad (5)$$

For the calculation of (4) it is necessary to know the dependence of the area of a section S on $x = (k_b - k_3)/k_0$. Assuming that in the vicinity of the "isthmus" shown in the figure the surface (a corrugated cylinder) can be replaced by a one-sheet hyperboloid, we obtain this dependence in the form

$$S = S_b - (1/\pi\alpha_0^2)\,x\ln|ax|;$$
$$a = (\varepsilon^2/4\pi^3)(a_2/a_1)^2\sqrt{R_{12}/\varkappa_{1b}}. \qquad (6)$$

The integral (4) with the single parameter $|\ln a|$ was calculated numerically, assuming $|\ln a| \sim 1 - 10$.

Passing from the state sum to the thermodynamic potential Ω, we obtain

$$\Omega_{\rm osc} = \frac{a_2 kT}{\pi a_1\alpha_0^4 R_{31}}\sqrt{\frac{\varkappa_{1b}}{R_{12}}}\frac{1}{(\ln a)^2}\left(1+\frac{3\pi}{|\ln a|}\right)$$
$$\times\sum_{p=1}^{\infty}\frac{\cos(p\alpha_0^2 S_b(\zeta)-\gamma)\cos\left(\frac{1}{2}\mu_0 H\frac{\partial S_b}{\partial\zeta}p\alpha_0^2\right)}{p^3\sinh\left(\pi p\alpha_0^2\frac{\partial S_b}{\partial\zeta}kT\right)}, \qquad (7)$$
$$\gamma = \arctan\frac{3p|\ln a|+9\pi}{p|\ln a|+6\pi}-\frac{\pi}{2},$$

which differs from the usual potential (for closed surfaces) as follows: (1) The period of the oscillations is determined not by S_m but by S_b and the phase γ is weakly dependent on the magnetic field; (2) the factor $|\partial^2 S_m/\partial k_3^2|^{-1/2}$ of the amplitude is replaced by

$$\frac{1}{\alpha_0 R_{31}}\frac{1}{(\ln a)^2}\left(1+\frac{3\pi}{|\ln a|}\right)\sqrt{\frac{\varkappa_{1b}}{R_{12}}}.$$

This denotes multiplication of the amplitude by $H^{1/2}$, so that the oscillating terms in χ will contain H^{-1} instead of $H^{-3/2}$. In absolute magnitude the amplitude is generally smaller by the factor ϵ^{-1} than for electrons in closed trajectories. But when, for example, we have a field $H \sim 10^4$ oersted and thus $\epsilon \sim 10^{-2}$, the reduction by the factor ϵ^{-1} can be concealed by other factors.

Other oscillatory effects such as oscillations of the resistance (the Shubnikov—de Haas effect), oscillations of the thermoelectric power or of the Hall field, etc. will also be determined in the present case by boundary and not by extremal sections.

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CONCERNING THE THEORY OF RAYLEIGH SCATTERING OF LIGHT IN LIQUIDS

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In a recently published article, S. M. Rytov[1] develops a theory of scattering of light in liquids under the assumption that this scattering is due entirely to fluctuation deformations $u_{\alpha\beta}$ and to fluctuations of the temperature Θ. Actually, only the fluctuations of $u_{\alpha\beta}$ and Θ are considered in Ref. 1, when it is indicated at the same time that the theory includes all internal processes in the medium and describes the entire spectrum of the scattered light. Next, it is emphasized, in the remarks on p. 518 of Ref. 1 [p. 404 in the translation] that the spectral amplitudes of the fluctuations of any internal parameter can be expressed linearly in terms of $u_{\alpha\beta}$ and Θ, and there is no scattering by isotropic fluctuations which cannot be reduced to density and temperature fluctuations (the possibility of the occurrence of such a scattering was noted by the author[2]).

However, we do not believe this point of view to be correct: scattering of light in the isotropic bodies discussed here does not reduce to fluctuations of $u_{\alpha\beta}$ and Θ. In fact, in the absence of temperature fluctuations and for a fixed position of the centers of gravity of the molecules (i.e., going to the macroscopic treatment as the fluctuations $u_{\alpha\beta}$ go to zero), scattering of light will take place in an ideal gas, consisting of anisotropic molecules, as the result of fluctuations in the orientations of the axes of the molecules. As we go over to dense gases or to liquids, the situation does not change qualitatively, and the scattering will thus take place also when $u_{\alpha\beta} = 0$ and $\Theta = 0$.

In the absence of fluctuations of $u_{\alpha\beta}$ and Θ, it is possible also to have scattering due to the formation of temporary complexes in the liquid (this effect is particularly clearly pronounced in scattering by concentration fluctuations in the case of solutions). This can be readily illustrated quite clearly with models pertaining to the radio-frequency band. Let us consider, for example, small solid hollow non-metallic spheres, with scattering dipoles placed in the centers of these spheres. It is clear that a set of such spheres, no matter what their density, will scatter radial waves as a result of the fluctuations in the orientations of the dipoles, even in the total absence of scattering due to the inhomogeneous spatial distribution of the spheres, or due to other factors. Let us note, furthermore, that the general theory[3] leads to the possible existence of scattering which cannot be described merely by introducing the symmetrical tensor $\epsilon_{\alpha\beta}$; yet, the tensor $u_{\alpha\beta}$, and therefore also the tensor $\epsilon_{\alpha\beta}$ considered in Ref. 1, is symmetrical.*

Under real conditions, in the case of weak absorption, the anti-symmetrical portion of the scattering is apparently quite small.[2] It is possible that this holds in most cases also for scattering by isotropic fluctuations that do not reduce to density and temperature fluctuations. However, we see no grounds for assuming that in low-viscosity liquids the fluctuation shear deformations, $u'_{\alpha\beta} = u_{\alpha\beta} - \frac{1}{3}u_{\gamma\gamma}\delta_{\alpha\beta}$, play a major role in the corresponding portion of the scattering [we have in mind scattering for which $\epsilon_{\gamma\gamma} = 0$; see Ref. 2, where this portion of the tensor $\epsilon_{\alpha\beta}$ is denoted by $\Delta\epsilon_{ik}'^{(s)}$].

It is clear from the above that in Ref. 1, and therefore in Ref. 4, only part of the scattering is actually considered. This circumstance limits even more the possibility of a real utilization of the formulas obtained in Refs. 1 and 4. The point is that experimentally, at a fixed frequency ω_0 of the incident light and at a fixed scattering angle φ, one determines either the two frequency functions $j_x(\Omega)$ and $j_z(\Omega)$, or else their combinations — the total intensity $j(\Omega) = j_x + j_z$, and the degree of depolarization $\Delta(\Omega) = j_x/j_z$. At the same time, Refs. 1 and 4 involve sixteen functions of Ω, namely the complex functions $\overline{K}$, $\overline{\mu}$, κ, C, D, X, Y, and Z. There is no doubt that certain of these functions can be assumed to be practically constant, and others can be approximated in a judicious manner. However, under conditions when all these coefficients $\overline{K}$, $\overline{\mu}$, etc. determine only the previously-unknown portion of the scattering, no way is seen for a reliable separation of this portion and for a comparison of the theory of Refs. 1 and 4 with experiment over the entire region of the Rayleigh line. If we bear in mind, however, the most interesting case, that of the Mandel'shtam-Brillouin doublet in low-viscosity liquids, it is difficult, in all probability, to go here beyond accounting for the dispersion of the velocity of sound, as was done successfully by Fabelinskii[5] (see also Ref. 2).

*The tensor $\varepsilon_{\alpha\beta}(\omega)$, which serves to describe completely the Rayleigh scattering, can be called the dielectric-constant tensor at $\omega \neq \omega_0$ only by convention (ω is the frequency of the scattered light, $\omega_0 = \omega - \Omega$ is the frequency of the incident radiation). There is therefore no wonder that the tenso $\varepsilon_{\alpha\beta}$ can be non-Hermitian,[3] in spite of the fact that the dielectric-constant tensor is Hermitian (when $\omega = \omega_0$ the tensor $\varepsilon_{\alpha\beta}$ is Hermitian, but even in this case it need not necessarily be symmetrical, as is already clear from the fact that optical activity exists).

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