[11] V. L. Ginsburg, J. Exptl. Theoret. Phys. (U.S.S.R.) **29**, 748 (1955); Soviet Phys. JETP **2**, 589 (1956).

[12] M. Laue, Theorie der Supraleitung, Berlin (1949).

[13] D. Schoenberg, Proc. Roy. Soc. **175**, 49 (1940).

[14] N. V. Zavaritskii, Dokl. Akad. Nauk. SSSR **78**, 665 (1951); **82**, 229 (1952).

[15] J. M. Lock, Proc. Roy. Soc. **208**, 391 (1951).

[16] O. S. Lutes, Phys. Rev. **105**, 1451 (1957).

Translated by J. L. Loferski
18

SOVIET PHYSICS JETP VOLUME 34(7), NUMBER 1 JULY, 1958

MOTION OF IONS IN A MIXTURE OF ISOTOPES

Iu. M. KAGAN and V. I. PEREL'

Leningrad Institute of Precision Mechanics and Optics

Submitted to JETP editor July 12, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) 33, 126-128 (January, 1958)

Expressions are derived for the drift velocity of ions of isotopes in a mixture of isotopes. The principal interaction between the ions and atoms is assumed to be pure charge exchange.

In connection with the problem of the distribution of isotopes in a direct current discharge, the question of the mobility of an ion of an isotope in an isotopic mixture becomes of interest. In view of the possibility of charge exchange between an ion of one isotope and an atom of another isotope, Blank's rule for the mobility of ions in a mixture is not applicable in the given case.

Let there be a mixture of two isotopes with concentrations of neutral atoms N_1 and N_2. We shall denote the Maxwellian velocity distributions of the atoms by $n_1(v)$ and $n_2(v)$. Let the concentration of ions be N_1^+ and N_2^+ and their velocity distribution functions be $f_1(\mathbf{v})$ and $f_2(\mathbf{v})$. As is usual in problems on mobility, we shall disregard the effect of the ions on the velocity distribution function of the atoms and the interaction of the ions among themselves. The chief process of interaction between the ions and atoms is, in the given case, the exchange of charge without an exchange of momentum (pure charge exchange model), for which the charge-exchange cross section $q(u)$ can be considered the same in all four processes A^+A, A^+B, B^+B and B^+A. In the presence of a constant homogeneous electric field E, directed along the z axis, the velocity distribution function for the ions is found from a system of two kinetic equations, the first of which has the form

$$\frac{eE}{M_1} N_1^+ \frac{\partial f_1}{\partial v_z} = N_1 N_1^+ \int uq(u)\,[n_1(v) f_1(\mathbf{v}') - f_1(\mathbf{v}) n_1(v')]\,d\mathbf{v}' + N_1 N_2^+ \int uq(u)\, n_1(v) f_2(\mathbf{v}')\,d\mathbf{v}' - N_1^+ N_2 \int uq(u)\, f_1(\mathbf{v}) n_2(v')\,d\mathbf{v}', \tag{1}$$

where M_1 is the mass of an atom of the first isotope and $u = |\mathbf{v} - \mathbf{v}'|$. The second and third terms on the right describe the appearance of A ions as a result of impacts of the type B^+A and their disappearance upon impacts of the type A^+B. The second kinetic equation is obtained from Eq. (1) by an interchange of indices.

Let us solve the system of kinetic equations in the limiting cases of weak and strong fields. For small fields, when the energy acquired by an ion over a mean free path is much less than the thermal energy, we apply the method of Langevin, who assumes the velocity distribution of the ions to be Maxwellian with a small superimposed drift in the direction of the field:

$$f_1(\mathbf{v}) = A_1 \exp\left\{-\frac{M_1}{2kT}\,[v_x^2 + v_y^2 + (v_z - v_1)^2]\right\} \approx n_1(v) + \frac{M_1}{kT} v_1 v_z n_1(v). \tag{2}$$

An analagous expression is obtained for $f_2(\mathbf{v})$.

The law of conservation of the number of particles is obtained from Eq. (1) by integration over the velocities. As a result, using Eq. (2), we obtain

$$N_1^+/N_1 = N_2^+/N_2. \tag{3}$$

This result is a consequence of the fact that, in our setting up of the problem the formation and disappearance of ions proceeds, only with charge exchange and that the charge-exchange cross sections for impacts of the type A^+B and B^+A are assumed equal. The unknown drift velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ are determined from the momentum balance equations that are obtained if each of the kinetic equations is multiplied by $v_z d\mathbf{v}$ and integrated over the velocities. After simple calculations, using Eqs. (2) and (3), we obtain a system of two equations, the first of which has the form

$$\frac{3\sqrt{\pi}}{16}\frac{eE}{M_1}\sqrt{\frac{M_1+M_2}{2kT}}$$
$$= v_1\left[N_1P_1\sqrt{\frac{1+\gamma_{21}}{2}} + \frac{3}{4}N_2R\sqrt{\gamma_{12}} + \frac{1}{2}N_2P\sqrt{\gamma_{21}}\right]$$
$$+ v_2N_2\sqrt{\gamma_{21}}\left[\frac{1}{2}P - \frac{3}{4}R\right]. \tag{4}$$

The second equation is obtained from the first by an interchange of indices. Here the following notation has been introduced:

$$\gamma_{12} = \frac{1}{\gamma_{21}} = \frac{M_1}{M_2}; \quad P = \frac{1}{2}\int_0^\infty e^{-x}x^2q(u)\,dx;$$
$$R = \int_0^\infty e^{-x}\,xq(u)\,dx; \qquad x = \frac{M_1M_2}{M_1+M_2}\frac{u^2}{2kT}.$$

P_1 and P_2 are obtained from P by replacing the reduced mass by $M_1/2$ and $M_2/2$ respectively. The expressions for v_1 and v_2 are easy to obtain from the system of Eqs. (4), but we shall not write them out because of their unwieldiness. For the case where the charge exchange cross section can be regarded as independent of velocity, $P_1 = P_2 = P = R = q$, and we obtain

$$v_1 = \frac{3\sqrt{\pi}}{16}\frac{eE}{M_1q(N_1+N_2)\sqrt{kT/M_1}}\,\frac{(4\sqrt{(1+\gamma_{21})/2}+1)c_2 + (2+3\gamma_{21})c_1}{c_1^2(2+3\gamma_{21}) + c_2^2(3\sqrt{\gamma_{12}}+2\sqrt{\gamma_{21}}) + 2^{-1/2}c_1c_2\; 3\sqrt{\gamma_{12}}(1+\gamma_{21})+4](1+\gamma_{21})^{1/2}}, \tag{5}$$

where $c_1 = N_1/(N_1 + N_2)$, $c_2 = N_2/(N_1 + N_2)$.

We obtain the expression for v_2 from Eq. (5) by an interchange of indices. In the special cases where $c_2 = 0$ or $M_1 = M_2$, Eq. (5) goes over to the ordinary formula for the drift velocity of ions in their respective gas.[1] For the case where $c_1 \to 0$,

$$v_1 = \frac{3\sqrt{\pi}}{16}\frac{eE}{M_1qN_2\sqrt{kT/M_2}}\,\frac{4\sqrt{(1+\gamma_{21})/2}+1}{3+2\gamma_{21}}. \tag{6}$$

In the general case for the ratio of the drift velocities of ions of two isotopes, we obtain

$$\frac{v_1}{v_2} = \frac{(4\sqrt{(1+\gamma_{21})/2}+1)c_2 + (2+3\gamma_{21})c_1}{(4\sqrt{(1+\gamma_{12})/2}+1)c_1 + (2+3\gamma_{12})c_2}. \tag{7}$$

From Eq. (7) it is possible to find the limits within which the ratio of the drift velocities of ions in an isotopic mixture varies with a change in concentration from $c_2 = 0$ to $c_1 = 0$. For example, for a mixture of hydrogen with deuterium we find that v_1/v_2 lies between the limits 1.8 and 1.7; for a mixture of He^3 with He^4, it is within the limits 1.27 and 1.25. The results obtained refute the assertion to the effect that as a result of frequent charge exchanges a homogeneous drift velocity is set up for ions of both isotopes.[2] The drift velocity of an ion of the lighter isotope noticeably exceeds the drift velocity of an ion of the heavier isotope.

One frequently encounters the case where the mixture contains a third component besides atoms of isotopes (for example, molecules of isotopes). Here, the interaction of ions with particles of the third component does not have charge-exchange character (the interaction depends essentially upon polarization forces). In this case the kinetic equation (1) contains additional terms on the right side, which take into account collisions of ions of isotopes with particles of the third component. Using Langevin's method, we obtain a system of two linear equations in v_1 and v_2, the first of which differs from Eq. (4) by the additional term

$$\frac{3\sqrt{\pi}}{16}\sqrt{\frac{M_1+M_2}{2kT}}\,\frac{e}{M_1}\,\frac{v_1}{b_{13}}. \tag{8}$$

Here b_{13} is the mobility of an ion of the first isotope in the third component (in the Langevin approximation):

$$b_{13} = \frac{e}{kT}\,\frac{3\sqrt{\pi}}{16N_3}\sqrt{\frac{2kT}{M_1M_3}}\,(M_1+M_3)\,\frac{1}{L};$$
$$L = \frac{1}{2}\int_0^\infty Q_{13}(u)\,x^2e^{-x}dx; \qquad x = \frac{M_1M_3}{M_1+M_3}\frac{u^2}{2kT};$$

N_3 and M_3 are the concentration and mass of particles of the third component and Q_{13} is the diffusion interaction cross section for ions of the first isotope with particles of the third component. The second equation for v_1 and v_2 is obtained from the first, as usual, by the interchange of the indices 1 and 2. In the presence of several additional components, each of them gives an additional term of the type of Eq. (8) in the right side of Eq. (4).

For the case of strong fields where it is possible to disregard the thermal motion of the atoms we obtain, by a method completely analagous to that applied in Ref. 3, an exact result for the drift velocity of an ion of an isotope in an isotopic mixture

$$v_1 = [2eE / \pi M_1 (N_1 + N_2) q]^{1/2}. \tag{9}$$

This result is obtained under the assumption that the charge exchange cross section is independent of velocity (in the method applied this need not necessarily be so, and is assumed for simplicity). The drift velocities in this case are inversely proportional to the square roots of the masses of the ions. This conclusion is physically obvious inasmuch as in the assumption of quiescent atoms the difference in their masses plays no role.

[1] Iu. M. Kagan and V. I. Perel', J. Exptl. Theoret. Phys. (U.S.S.R.) **29**, 884 (1955), Soviet Phys. JETP **2**, (1956).

[2] Beckey, Groth and Welge, Z. Naturforsch., **8a**, 556 (1953).

[3] Iu. M. Kagan and V. I. Perel', Dokl. Akad. Nauk SSSR, **98**, 575 (1954).

Translated by R. Eisner
19

SOVIET PHYSICS JETP VOLUME 34(7), NUMBER 1 JULY, 1958

POLARIZATION OF SLOW NEUTRONS SCATTERED IN CRYSTALS

S. V. MALEEV

Leningrad Physico-Technical Institute

Submitted to JETP editor July 11, 1957

J. Exptl. Theoret. Phys. (U.S.S.R.) **33**, 129-130 (January, 1958)

Formulas are derived for the polarization and cross section of slow neutrons scattered in crystals with polarized nuclei.

ROSE[1] has examined the polarization effects arising in the scattering of slow neutrons in crystals, and has obtained formulas for the scattering cross section of polarized neutrons in crystals with polarized nuclei. He considered only crystals of atoms with a single isotope. In the present note, his results are generalized to include crystals of several isotopes, and expressions obtained for the change in neutron polarization due to scattering.

It is well known[2] that the scattering amplitude for slow neutrons in a crystal is proportional to the quantity

$$\left(f\mu' \left| \sum_j e^{i\mathbf{qR}_j} \left(A_j + \frac{1}{2} B_j \sigma \mathbf{I}_j\right) \right| i\mu\right) = F, \tag{1}$$

where i and f are the initial and final states of the scatterer; μ and μ' are the initial and final neutron spin projections; $\sigma/2$ is the neutron spin; $\mathbf{q}$ is the momentum transferred to the scatterer; $\mathbf{R}_j$ and $\mathbf{I}_j$ are the coordinates and spins of the j-th nucleus; and A_j, B_j are complex constants. The summation is over all the nuclei in the crystal.

From (1) it is easy to obtain the following expression for the coherent and incoherent parts of the scattering cross section, averaged over all orientations of the neutron and nuclear spins, and over all possible distributions of isotopes in the lattice:

$$\sigma_{\mathrm{coh}} = \sigma^{(0)}_{\mathrm{coh}} \{1 + |\langle A\rangle|^{-2} [(\mathbf{pN}) \operatorname{Re} \langle A^*\rangle \langle BIP\rangle + {}^1/_4 |\langle BIP\rangle|^2]\}; \tag{2}$$