$$\frac{1 - M^2 - v_0 M^2/v}{1 - M^2 + v_0 M^2/v} < \left(\frac{\partial V}{\partial p}\right)_H j^2 < \frac{1 + 2M^2 - v_0 M/v}{1 + v_0 M/v}. \tag{6}$$

The whole region of emission is completely determined by the inequalities

$$\frac{1 - M^2 - v_0 M^2/v}{1 - M^2 + v_0 M^2/v} < \left(\frac{\partial V}{\partial p}\right)_H j^2 < 1 + 2M. \tag{7}$$

It is of interest that the inequality (5) extends the region of emission in that direction which is most convenient for experiments, because it corresponds to the least exotic form of the Hugoniot adiabatic.

The condition that the left-hand side of the inequality (7) be negative and, consequently, that the region sought can, in principle, exist on ordinary adiabatics (where $(\partial V/\partial p)_H < 0$), is

$$1 < M^2 (1 + v_0/v). \tag{8}$$

This inequality is realized even for the case of an ideal gas. The region of sound emission is not however, present because the requirement that $(\partial V/\partial p)_H j^2 \equiv -M_0^{-2}$ (for an ideal gas) satisfy (7) leads to $M_0 < 1$, i.e., (7) coincides with the region of instability.

*In the notation of Ref. 1 the components of the wave vector are $q(k, \ell_2, 0)$. The remaining notation is unchanged here.

[1] S. P. D'iakov, J. Exptl. Theoret. Phys. (U.S.S.R.) 27, 288 (1954).

[2] L. D. Landau and E. M. Lifshitz, Механика сплошных сред (Mechanics of Continous Media), GITTL, M. 1953.

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312

REFLECTION AND REFRACTION OF SOUND BY A SHOCK WAVE

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FOR an ideal gas the normal incidence of sound on a shock wave has been treated by Blokhintsev[1] and by Burgers,[2] while Brillouin[3] has considered oblique incidence. However, in the last reference the author obtains erroneous results for the reflection and transmission coefficients as he does not take into account the perturbation of the surface of the shock wave.

The reflection and refraction of sound by a shock wave in an arbitrary medium is considered in this note.

As is well known, the shock wave moves with a supersonic speed with respect to the rarefied medium and with a subsonic speed with respect to the compressed medium. Therefore if a sound wave is incident on the surface of discontinuity from the side of the compressed medium, no transmitted wave is formed, and a reflected wave and an entropy-rotational perturbation carried by the liquid flux appear. Similarly, when sound is incident on the shock wave from the side of the rarefied medium no reflected wave is formed while an entropy-rotational wave appears in addition to the transmitted wave. A perturbation wave of amplitude η moves along the surface of discontinuity with a propagation vector k equal to the projection of the propagation vector of the incident wave $k_i(k, \ell, 0)$ on the surface of discontinuity and with a frequency ω equal to the frequency of the incident sound wave in the system of coordinates in which the surface of discontinuity is at rest.

The equations for the entropy-rotational (1) and the sound (2) disturbances have the form:*

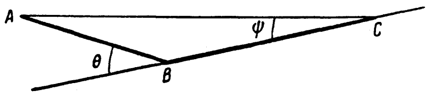
$$k\delta v_x^{(1)} + l^{(1)}\delta v_y^{(1)} = 0;\ \delta p^{(1)} = 0;\ l^{(1)}v = \omega; \tag{1}$$

$$\delta v_i^{(2)} = k_i/\rho\,(\omega - vl^{(2)});\ \delta s^{(2)} = 0;\ (\omega - vl^{(2)})^2 = c^2(k^2 + l^{(2)2}). \tag{2}$$

We must write down the boundary conditions in the system of coordinates in which the normal velocity of the perturbed discontinuity is zero. Linearized with respect to the amplitude of the sound wave they give:

$$[v\delta\rho + \rho\delta v_y + i\omega\rho\eta] = 0;\ [\delta v_x + ik\eta v] = 0;\ [\delta p + v^2\delta\rho + 2\rho v\delta v_y] = 0;\ [\delta w + v\delta v_y + i\omega v\eta] = 0. \tag{3}$$

Equations (1) — (3) form a complete system. In the problem of reflection we must distinguish between the incident $\delta A^{(i)}$ and the reflected $\delta A^{(r)}$ waves. At the same time from the side of the rarefied medium $\overline{\delta A} = 0$ which corresponds to unperturbed flow in front of the surface of discontinuity. In place of one of equations (3) it is more convenient to use a consequence of Hugoniot's equation[4]



$AB/BC = M$

$$\delta\rho = Q\delta p,\ Q = (\partial\rho/\partial p)_H. \tag{4}$$

It is convenient to express the result in terms of the angle $\theta = \tan^{-1}(k/l)$ and the auxiliary angle ψ whose definition is evident from the diagram. The law of refraction can be formulated as the equality $\psi^{(i)} = \psi^{(r)}$:

$$\delta p^{(r)}\{\cos(\theta^{(r)} - \psi) + h\} = -\delta p^{(i)}\{\cos(\theta^{(i)} - \psi) + h\},\quad h = \frac{\cos\psi}{2M}\left\{1 + Qv^2 + \frac{\bar{v}}{v}(Qv^2 - 1)\tan^2\psi\right\}. \tag{5}$$

In the refraction problem we are interested in the ratio of a quantity in the transmitted wave δA to the corresponding quantity in the incident wave $\overline{\delta A}$. The law of refraction is formulated as the "law of tangents"

$$\tan\psi/\tan\bar{\psi} = v/\bar{v},\quad \delta p\{(1 + M\cos\theta) - G(1 + 2M\cos\theta + M^2)\} = \delta\bar{p}\{\bar{M}(\bar{M} + \cos\bar{\theta}) - G(1 + 2\bar{M}\cos\bar{\theta} + \bar{M}^2)\}, \tag{6}$$

$$G = l\frac{1 - ([v]/v)\sin^2\psi}{1 - 2l([v]/v)\sin^2\psi},\quad l = \frac{c_p}{\beta[p]}, \tag{A}$$

c_p and β are the heat capacity and the coefficient of thermal expansion at constant pressure. For normal incidence we obtain

$$\delta p(1 + M)\{1 - l(1 + M)\} = \delta\bar{p}(\bar{M} + 1)\{\bar{M} - l(\bar{M} + 1)\}. \tag{7}$$

For an ideal gas (7) reduces to the formulas given in Refs. 1 and 5.

It is planned to describe the work in greater detail in the Acoustic Journal.

*We follow the notation of Refs. 4 and 5.

[1] D. I. Blokhintsev, Dokl. Akad. Nauk SSSR 47, 22 (1945).
[2] J. M. Burgers, Proc. Ned Akad. Wet. 49, 273 (1946).
[3] J. Brillouin, Acustica 5, 149 (1955).
[4] S. P. D'iakov, J. Exptl. Theoret. Phys. (U.S.S.R.) 27, 288 (1954).
[5] L. D. Landau and E. M. Lifshitz, Механика сплошных сред (Mechanics of Continuous Media), Gostekhizdat (1953) p. 409.

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313