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WAVE EQUATION FOR A FREE NUCLEON

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It is well known that so far no nucleon wave equation, even for the free nucleon, has been formulated. This is specifically due to the fact that the mass of the proton differs from that of the neutron. In this connection it is of interest to remark that the linearization of a second-degree wave equation leads in general to the equation

$$[\gamma^{\nu}\partial/\partial x_{\nu} - (m_0 c/\hbar)\exp(a\gamma^5)]\psi = 0. \quad (1)$$

Here m_0 and a are constants. Unfortunately, the mass operator $M = m_0 \exp(a\gamma^5)$ does not commute with the Hamiltonian that follows from (1). We therefore generalize (1) in the following manner:*

$$[\Gamma^{\nu}\partial/\partial x_{\nu} - (m_0 c/\hbar) I \exp(2aT_3)]\psi = 0. \quad (2)$$

The commutation relations for the operators Γ^{ν}, I, and T_3 are determined by the following postulates: (a) the Hamiltonian, the spin-projection operator, and the third component of the isotopic spin, T_3, form a system of commuting operators; (b) the relation between momentum and energy has the usual form.

This gives

$$[\Gamma^{\nu}, \Gamma^{\sigma}]_+ = 2\delta_{\nu\sigma}, \quad [\Gamma^{\nu}, T_3]_+ = 0, \quad [T_3, I]_+ = 0, \quad [\Gamma^{\nu}, I]_- = 0, \quad T_3T_3 = 1, \quad II = 1. \quad (3)$$

As is well known, the components of the isotopic spin have to satisfy the commutation relations

$$[T_2, T_3]_- = iT_1, \quad [T_3, T_1]_- = iT_2, \quad [T_1, T_2]_- = iT_3. \quad (4)$$

The operators Γ^{ν}, T_k, and I have irreducible eight-by-eight representations:

$$\Gamma^{\nu} = 1^{\mathrm{II}} \times \gamma^{\nu}, \quad T_k = \sigma_k \times \gamma^5, \quad I = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \times 1^{\mathrm{IV}}. \quad (5)$$

Here σ_k are two-by-two spin matrices; 1^{II} and 1^{IV} are two-by-two and four-by-four unit matrices respectively. The sign × denotes the direct product of the matrices.

It follows from (2) that in the nonrelativistic limit the components of the wave function are characterized by the following quantum numbers.

One sees from the table that the eigenvalue $t_3 = -\frac{1}{2}$ corresponds to a particle mass $m_1 = m_0 e^{-a}$ and $t_3 = \frac{1}{2}$ corresponds to a mass $m_2 = m_0 e^{a}$. Taking $m_0 = 1837.38\ m_e$ (the average of the proton and neutron masses) and $a = 3\alpha/10\pi$, then m_1 and m_2 are equal to the proton and neutron mass respectively within the experimental errors for these masses,[1] and (2) can be considered as the wave equation of the nucleon.

Component of the wave function	Sign of the spin projection	Sign of the energy	t_3	Particle rest mass
ψ_1	+	+	$-1/2$	m_0e^{-a}
ψ_2	—	+	$-1/2$	m_0e^{-a}
ψ_3	+	+	$1/2$	m_0e^{a}
ψ_4	—	+	$1/2$	m_0e^{a}
ψ_5	+	—	$1/2$	m_0e^{a}
ψ_6	—	—	$1/2$	m_0e^{a}
ψ_7	+	—	$-1/2$	m_0e^{-a}
ψ_8	—	—	$-1/2$	m_0e^{-a}

It is easy to show that one can obtain from (2) two current vectors:

$$s_{\nu}^{(1)} = ic\varkappa\psi^{*}\Gamma^4\Gamma^{\nu}\psi, \quad (6)$$

$$s_{\nu}^{(2)} = -ic\varepsilon\psi^{*}(T_3 - 1/2)\Gamma^4\Gamma^{\nu}\psi. \quad (7)$$

The first can be interpreted as the specific nucleonic current (κ is the nucleonic charge) and the second as the electric current. From (7) we obtain the nucleon electric-charge operator

$$Q = -\varepsilon(T_3 - {}^1/_2), \tag{8}$$

which commutes with the Hamiltonian and with the spin operator. The eigenvalues of Q are equal to zero for the neutron states and ϵ for the proton states.

It should be mentioned that (2) is not invariant under improper Lorentz transformations. It follows from the pseudoscalar character of γ^5 that a reflection of the space coordinates changes a proton into a neutron and vice versa. This means that (2) is invariant under simultaneous reflection of space and isospin coordinates.

Equation (2) can also be used to describe the doublet of the Ξ hyperons. One then has to take $m_0 = 2588\ m_e$ while a has the same value as for the nucleons. This yields a mass of 2586 m_e for the Ξ^- and 2590 m_e for the Ξ^0.

It is interesting to note that one can also find for the operators Γ^ν, T_k, and I a twelve by twelve irreducible representation which allows a description of the triplet of Σ hyperons.

*A more detailed derivation of (2) and a generalization for the case of weak interactions will be published in the Transactions (Trudy) of the Institute for Physics and Astronomy of the Academy of Sciences, Estonian S.S.R.

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CAPTURE OF ELECTRONS IN BETATRONS

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THE process of electron capture into betatron orbits has been studied by many authors, with various accelerators, and sufficient experimental material has consequently been compiled. A number of theoretical papers have been devoted to the subject of electron capture.[1-5] Although they were not fully successful in giving a complete explanation of the capture process, they still allow an estimate of the contributions of the different capture mechanisms and a comparison with experiment. However, no attention has been paid so far to the following process, which can contribute to the capture of electrons into betatron orbits. Consider the motion the electrons in a coordinate system moving along the equilibrium orbit with a velocity equal to the azimuthal injection velocity of the electrons. In this coordinate system the electrons will move towards each other. Since the electron velocities corresponding to radial motion are small, the collision probability will be sufficiently large. As a result of multiple scattering, electrons starting out with equal oscillation amplitudes will later acquire a gaussian amplitude distribution. Thus, the conditions for strong damping of the oscillations will statistically be fulfilled for a certain fraction of the injected electrons.

A rough estimate of this effect can be made in the following manner. If an electron makes an elastic collision at the time when its velocity is at maximum, then the amplitude of oscillation will decrease as

$$\rho_0' = \rho_0 \cos\psi, \tag{1}$$

where ψ is the scattering angle. The probability of single scattering at an angle ψ in traversing a distance dx through an electron gas of density N_0 is given by

$$P(\psi)\,d\psi = \pi \frac{e^4}{W_\rho^2} \frac{\cos(\psi/2)}{\sin^3(\psi/2)}\, d\psi N_0\, dx, \tag{2}$$