metric data; this turns out to be 9.296 ± 0.014 Mev. According to the measurements by Kinsey and Bartholomew[11] for the reaction $Fe^{54}(n, \gamma)\ Fe^{55}$, this neutron binding energy comes to 9.298 ± 0.007 Mev. The discrepancy comes to merely 12 ± 16 Kev. If one uses the energy of the β decay of Mn^{56} from King's table, the same discrepancy, as shown in Ref. 1 becomes much worse — 60 ± 30 Kev. Calculation of the atomic masses of isotopes of manganese and iron in different ways, with the use of improved values of the energy of the β decay of Mn^{56}, allows one to replace the masses given in the work of Quisenberry et al[1] by the following more reliable weighted mean values: Mn^{55} = 54.955512 ± 8, Mn^{56} = 55.956700 ± 8, Fe^{55} = 54.955761 ± 8.

At the present time the experimental data on hand is very extensive. To calculate good values of atomic masses it is necessary to have as much of the data as possible, to compare and estimate their true accuracy, and then to select all the reliable values for the actual calculation. Incomplete use of all reliable experimental data, as seen from the example cited, often leads to questionable values of mass differences.

Correction to King's Table

Decay	Decay Data					Weighted mean value of the total energy of the decay	
	Form	Mev	Error	Method	Reference	Mev	Error
$Mn^{56} \rightarrow Fe$	β^-	2.88	1	M.s.	[4]	3.710	11
		2.86	5	M.s.	[5]		
		2.81	3*	M.s.	[6]		
		2.82	8	Scin.	[7]		
	γ	0.866	20*	M.s.	[8]		
		0.845	15	M.s.	[5]		
		0.822	8*	M.s.	[6]		
		0.845	10*	Scin.	[9]		
		0.845	10*	Scin.	[10]		

Remark. The errors are presented in units of the last significant figure of the energy. An asterisk next to the value for the error denotes that its value was determined by the compiler of the table, either because the experimenter did not quote it in the cited work, or because in the compiler's opinion the error is undervalued. The abbreviations for the measurement method are: M.s. — magnetic spectrometer, Scin. — scintillation recorder.

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CERTAIN SOLUTIONS OF THE EQUATIONS OF PLASMA HYDRODYNAMICS

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In the present communication we shall consider certain exact solutions of the hydrodynamic equations of cold plasma in the presence of an external magnetic field, and also in its absence. For the sake of simplicity we shall regard the ions as being at rest, but this restriction is not a fundamental one and may be easily removed.

In the case of one-dimensional flow in a plasma the solution of equations

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{e}{m}E; \quad \frac{\partial E}{\partial x} = -4\pi e(n - n_+); \quad \frac{\partial n}{\partial t} + \frac{\partial (nv)}{\partial x} = 0 \tag{1}$$

(v is the speed, n the electron density, n_+ the ion density, E the electric field) may be sought in the form

$$v = e^{-t/t_0} v_0(z); \quad \frac{e}{m} x_0 E = e^{-t/t_0} \varepsilon_0(z); \quad \frac{n}{n_+} = n_0(z); \quad z = \frac{x}{x_0} e^{t/t_0}, \tag{2}$$

where x_0, t_0 are arbitrary constants. In order to elucidate the nature of this solution we make the substitution $y = \ln z$ and insert (2) into (1) after suppressing the index 0 of the functions v_0, ϵ_0, n_0:

$$V_0(v - v') + vv'e^{-y} = -\varepsilon; \quad \varepsilon' e^{-y} = (\omega_0 x_0)^2 (1 - n); \quad -V_0 n' + e^{-y}(nv)' = 0; \quad \omega_0^2 = 4\pi n_+ e^2/m; \quad V_0 = -x_0/t_0. \tag{3}$$

If y is now expanded into a series about some point x_1, t_1 ($x = x_1 + \xi$; $t = t_1 + \tau$) and the term $V_0 v$ is neglected the system (3) takes on the form:

$$(v - V)v' = -\varepsilon_1; \quad \varepsilon_1' = (\omega_0 x_1)^2 (1 - n); \quad -Vn' + (nv)' = 0; \quad \varepsilon_1 = x_1 \varepsilon / x_0; \quad V = -x_1/t_0. \tag{4}$$

It describes a wave propagating with a speed V; the form of the wave

$$c_1 + \sin^{-1}\frac{n_+/n - 1}{c_2} - \sqrt{c_2^2 - \left[\left(\frac{n_+}{n}\right)^2 - 1\right]^2} = \omega_0\left(\frac{x}{V} - t\right) \tag{5}$$

agrees with that found by Akhiezer and Liubarskii[1] ($c_{1,2}$ are constants of integration).

In the presence of an external homogeneous and constant magnetic field H (directed along the z axis) the equations assume the form (all quantities are independent of φ, z):

$$\begin{aligned} &\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} - \frac{v_\varphi^2}{r} = -\frac{e}{m}E - \frac{e}{mc} v_\varphi H; \quad \frac{\partial n}{\partial t} + \frac{1}{r}\frac{\partial n r v_r}{\partial r} = 0; \\ &\frac{\partial v_\varphi}{\partial t} + v_r \frac{\partial v_\varphi}{\partial r} + \frac{v_\varphi v_r}{r} = \frac{e}{mc} v_r H; \quad \frac{1}{r}\frac{\partial r E}{\partial r} = -4\pi e(n - n_+). \end{aligned} \tag{6}$$

In this case the following solution exists

$$v_r = \frac{r}{r_0} v_r^0(\xi); \quad v_\varphi = \frac{r}{r_0} v_\varphi^0(\xi); \quad E = \frac{r}{r_0} E^0(\xi); \quad n = n^0(\xi); \quad \xi = \frac{t}{t_0} + \ln\frac{r}{r_0}, \tag{7}$$

where the functions with the index 0 satisfy the system of ordinary differential equations obtained from (6) by means of the substitution (7).

A similar solution exists also in the case when only v_x, v_y, E_x differ from zero and all the quantities depend on x, t:

$$\begin{aligned} &\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} = -\frac{e}{m}E_x - \frac{e}{mc} v_y H; \quad \frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} = \frac{e}{mc} v_x H; \\ &\frac{\partial n}{\partial t} + \frac{\partial n v_x}{\partial x} = 0; \quad \frac{\partial E_x}{\partial x} = -4\pi e(n - n_+). \end{aligned} \tag{8}$$

In this case

$$E_x = \frac{x}{x_0} E^0\left[\frac{t}{t_0} + \ln\frac{x}{x_0}\right], \quad n = n^0\left[\frac{t}{t_0} + \ln\frac{x}{x_0}\right]$$

($v_{x,y}$ vary in a manner similar to $E_{x,y}$).

The system (8) also has solutions in the form of waves in which all the quantities depend on the difference $x - Vt$; integration of the system of equations obtained from the system (8) in this case gives:

$$c_1 + \delta \sin^{-1}\frac{n_+/n - \delta}{c_2} - \sqrt{c_2^2 - \left[\left(\frac{n_+}{n}\right)^2 - \delta\right]^2} = \sqrt{\omega_0^2 + \Omega^2}\left(\frac{x}{V} - t\right), \quad \delta = (\omega_0^2 - \Omega^2)/(\omega_0^2 + \Omega^2); \quad \Omega = eH/mc. \tag{9}$$

Solution (9) [as well as (5)] becomes harmonic for $|1 - n/n_+| \ll 1$. One can therefore say that these solutions give the form of the wave of finite amplitude which arises when a wave is propagated in a plasma of slowly varying density n_+ in the direction of small n_+.

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WAVE EQUATION FOR A FREE NUCLEON

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It is well known that so far no nucleon wave equation, even for the free nucleon, has been formulated. This is specifically due to the fact that the mass of the proton differs from that of the neutron. In this connection it is of interest to remark that the linearization of a second-degree wave equation leads in general to the equation

$$[\gamma^\nu \partial / \partial x_\nu - (m_0 c / \hbar) \exp (a\gamma^5)] \psi = 0. \quad (1)$$

Here m_0 and a are constants. Unfortunately, the mass operator $M = m_0 \exp(a\gamma^5)$ does not commute with the Hamiltonian that follows from (1). We therefore generalize (1) in the following manner:*

$$[\Gamma^\nu \partial / \partial x_\nu - (m_0 c / \hbar) I \exp (2aT_3)] \psi = 0. \quad (2)$$

The commutation relations for the operators Γ^ν, I, and T_3 are determined by the following postulates: (a) the Hamiltonian, the spin-projection operator, and the third component of the isotopic spin, T_3, form a system of commuting operators; (b) the relation between momentum and energy has the usual form.

This gives

$$[\Gamma^\nu, \Gamma^\sigma]_+ = 2\delta_{\nu\sigma}, \quad [\Gamma^\nu, T_3]_+ = 0, \quad [T_3, I]_+ = 0, \quad [\Gamma^\nu, I]_- = 0, \quad T_3 T_3 = 1, \quad II = 1. \quad (3)$$

As is well known, the components of the isotopic spin have to satisfy the commutation relations

$$[T_2, T_3]_- = iT_1, \quad [T_3, T_1]_- = iT_2, \quad [T_1, T_2]_- = iT_3. \quad (4)$$

The operators Γ^ν, T_k, and I have irreducible eight-by-eight representations:

$$\Gamma^\nu = 1^{\rm II} \times \gamma^\nu, \quad T_k = \sigma_k \times \gamma^5, \quad I = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \times 1^{\rm IV}. \quad (5)$$

Here σ_k are two-by-two spin matrices; $1^{\rm II}$ and $1^{\rm IV}$ are two-by-two and four-by-four unit matrices respectively. The sign × denotes the direct product of the matrices.

It follows from (2) that in the nonrelativistic limit the components of the wave function are characterized by the following quantum numbers.

One sees from the table that the eigenvalue $t_3 = -\frac{1}{2}$ corresponds to a particle mass $m_1 = m_0 e^{-a}$ and $t_3 = \frac{1}{2}$ corresponds to a mass $m_2 = m_0 e^{a}$. Taking $m_0 = 1837.38\, m_e$ (the average of the proton and neutron masses) and $a = 3\alpha/10\pi$, then m_1 and m_2 are equal to the proton and neutron mass respectively within the experimental errors for these masses,[1] and (2) can be considered as the wave equation of the nucleon.

Component of the wave function	Sign of the spin projection	Sign of the energy	t_3	Particle rest mass
ψ_1	+	+	$-1/2$	$m_0 e^{-a}$
ψ_2	—	+	$-1/2$	$m_0 e^{-a}$
ψ_3	+	+	$1/2$	$m_0 e^{a}$
ψ_4	—	+	$1/2$	$m_0 e^{a}$
ψ_5	+	—	$1/2$	$m_0 e^{a}$
ψ_6	—	—	$1/2$	$m_0 e^{a}$
ψ_7	+	—	$-1/2$	$m_0 e^{-a}$
ψ_8	—	—	$-1/2$	$m_0 e^{-a}$

It is easy to show that one can obtain from (2) two current vectors:

$$s_\nu^{(1)} = ic\,\varkappa\psi^* \Gamma^4 \Gamma^\nu \psi, \quad (6)$$

$$s_\nu^{(2)} = -ic\,\varepsilon\psi^* (T_3 - 1/2) \Gamma^4 \Gamma^\nu \psi. \quad (7)$$

The first can be interpreted as the specific nucleonic current (κ is the