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THE INTERACTION OF SLOW π MESONS WITH NUCLEI

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The influence of the interaction of charged mesons with a nucleus on the cross section of the production of mesons on nuclei is considered for the case in which the meson wavelength exceeds the size of the nucleus. A relation is obtained connecting the cross section of photoproduction of mesons with the shifts and widths of mesoatomic levels. Comparison with experimental data permits some conclusions concerning the interaction of mesons with groups of nucleons and the mechanism of the photoproduction of mesons on nuclei.

1. INTRODUCTION

THE interaction of the meson field with groups of nucleons is of decided interest. Information concerning this interaction is obtained from the analysis of data on the scattering and the production of mesons on nuclei.[1] However, these works are usually concerned with the interaction of rather energetic mesons. The interpretation of the data is then strongly dependent on the model representations, and in particular on the form of the effective potential of the meson-nucleus interaction. In order to talk about the interaction of the meson field with a group of nucleons, it is reasonable to consider effects in which the meson wavelength is considerably greater than the size of the nucleon group. One such effect is radiation during mesoatomic transitions.[2] Along this line are the great advances in the measurement of the shifts and the evaluation of the widths of the mesoatomic levels, which may be interpreted in terms of the specifically nuclear interaction between the meson and the nucleus.

The purpose of the present work is the investigation of effects which may yield similar information. We have in mind the scattering and creation on nuclei of mesons whose wavelengths are greater than the nuclear dimensions. These effects have their own peculiarities. The most important role is played by the meson-nucleus Coulomb interaction. It is true that these effects are less "pure" than radiation during mesoatomic transitions. However, they still have some advantages, namely, the possibility of varying the meson wavelength within known limits, and the possibility of studying the interactions between mesons of various signs and nuclei of all the elements in the periodic system. Moreover, on the basis of the study of these effects, conclusions may be drawn with regard to the mechanism of the creation of mesons on the nucleus. As regards the possibilities for the experimental study of these effects, there is at present already some data on the photoproduction of slow mesons,[3,4] which we will discuss below in comparison with our theory.

2. THE INTERACTION OF π MESONS WITH NUCLEI

The wave function of the relative motion of the meson and the nucleus, outside the nucleus, will be a linear combination of regular and non-regular eigenfunctions of the Hamiltonian H_0, which in the case of charged π mesons contains the Coulomb interaction Ze^2/r in addition to the kinetic energy operator. For a definite angular momentum ℓ, the wave function has the form

$$\varphi_l(kr) = \sqrt{\frac{2}{\pi}}\frac{1}{kr}\{AF_l(kr) + BG_l(kr)\}, \tag{1}$$

where the constants A and B may be determined by using a concrete model for the meson-nucleus interaction potential for $r \leq r_Z$ [$r_Z = (\hbar/\mu c)A^{1/3}$ is the radius of a nucleus with atomic number Z], or they may be expressed by the shifts and widths of the mesoatomic levels by considering the interaction potential v' inside the nucleus to be a perturbation v on the Hamiltonian H_0.

The wave function φ of a meson interacting with a nucleus must satisfy the following integral equation:

$$\varphi^{\pm} = \chi + \frac{1}{E \pm i\varepsilon - H_0} v\varphi^{\pm}. \quad (2)$$

Limiting ourselves to the first order perturabtion theory and writing Eq. (2) in the $|r|, \ell$, m representation, we find

$$\varphi_l^{\pm}(kr) \approx \chi_l(kr) + \int \frac{\int \chi_l^*(k'r')\, v(r')\, \chi_l(k'r')\, r'^2 dr'}{E_k \pm i\varepsilon - E_{k'}} \chi_l(k'r)\, k'^2 dk', \quad (3)$$

where ϵ is an infinitely small quantity and $\chi_\ell(kr) = \sqrt{2/\pi} F_\ell(kr)/kr$ is the regular eigenfunction of the operator H_0 with the eigenvalue E_k. For charged π mesons, $\chi_\ell(kr)$ is expressed by a hypergeometric function and as $r \to \infty$ has the form

$$\chi_l(kr)_{r\to\infty} = \sqrt{\frac{2}{\pi}}\frac{1}{kr}\sin\left(kr - \alpha\ln 2kr - \frac{l\pi}{2} + \eta_l\right).$$

Integrating Eq. (3) with $r \to \infty$ and letting ϵ approach zero, we obtain the following asymptotic form for the function $\varphi_\ell^{\pm}(kr)$:

$$\varphi_l^{\pm}(kr)_{r\to\infty} = \sqrt{\frac{2}{\pi}}\frac{1}{kr}\left\{\left(1 \pm iI_l\right)\sin\left(kr - \alpha\ln 2kr - \frac{l\pi}{2} + \eta_l\right) + I_l\cos\left(kr - \alpha\ln 2kr - \frac{l\pi}{2} + \eta_l\right)\right\}, \quad (4)$$

where

$$I_l = -\frac{\pi\mu k}{\hbar^2}\int_0^\infty |\chi_l(kr)|^2 v(r)\, r^2 dr = -\frac{2\mu}{k\hbar^2}\int_0^\infty |F_l(kr)|^2 v(r)\, dr. \quad (5)$$

Remembering that in Eq. (1)

$$G_l(kr)_{r\to\infty} = \cos\left(kr - \alpha\ln 2kr - \frac{l\pi}{2} + \eta_l\right),$$

and comparing the asymptotic forms of Eqs. (1) and (4), we find that $A = 1 \pm iI_\ell$ and $B = I_\ell$. If I_ℓ is small, then

$$\varphi_l^{\pm}(kr) \approx \sqrt{\frac{2}{\pi}}\frac{1}{kr} e^{\pm iI_l}\{F_l(kr) + I_l G_l(kr)\}. \quad (6)$$

In the remainder of this section we will limit ourselves to the case $\ell = 0$ and will consider the interactions of π^- mesons with light nuclei ($Z < 15$), for which $r_Z \ll R$, where $2R = a$ is the radius of the lowest mesonic orbit: $R = \hbar^2/2\mu Ze^2$. We will express I_0 through the shift and width ΔE of the 1S level of the mesoatom.* In the first order of perturbation theory we have

$$\Delta E \approx \frac{1}{2R^3}\int v(r)\, r^2 dr. \quad (7)$$

Since we are considering slow mesons ($kr \ll 1$), the functions $F_0(kr)$ and $G_0(kr)$ in a region of the order of the dimensions of the nucleus will have the form

$$F_0(kr) \approx C_0 kr, \quad G_0(kr) \approx \frac{1}{C_0}\left\{1 - \frac{r}{R}\left[\ln\frac{r}{R} + 2\gamma - 1 + h(\alpha)\right]\right\};$$
$$h(\alpha) = \alpha^2\sum_{n=1}^{\infty}\frac{1}{n(n^2+\alpha^2)} - \ln|\alpha| - \gamma; \quad C_0^2 = \frac{2\pi|\alpha|}{1 - e^{-2\pi|\alpha|}}; \quad \alpha = -\frac{Ze^2}{\hbar c}\sqrt{\frac{\mu c^2}{2E_\pi}}; \quad \gamma = 0.5772\ldots \quad (8)$$

Now, using Eq. (5), we obtain

$$I_0 = -\frac{2\mu C_0^2 k}{\hbar^2}\int v(r)\, r^2 dr = -\frac{4\mu C_0^2 kR^3}{\hbar^2}\Delta E. \quad (9)$$

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*ΔE is a complex quantity. Im(ΔE) (width of mesoatomic level) is connected with the interaction that leads to meson absorption. The level shift Re(ΔE) is due to nuclear as well as to non-nuclear interactions. The part of Re(ΔE) which arises from a specifically nuclear interaction has been measured experimentally.[2]

3. PRODUCTION OF SLOW MESONS

If the interaction leading to meson production is either weak (for example, photoproduction) or very strong (production during collisions of nucleons with nuclei), the production matrix element may be written in the form[5]

$$T \approx (\varphi^{-}, U\psi^{0+}), \tag{10}$$

where φ^{-} is the wave function of the final state, U is the interaction leading to meson production, and the wave function ψ^{0+} is independent of the interaction in the final state. Summing and integrating over all variables in this expression, with the exception of the meson-nucleus relative coordinate, we find that the corss section for meson production is

$$\sigma = W \left| \int \varphi^{-}(kr)\, T(\mathbf{r})\, d\mathbf{r} \right|^{2}. \tag{11}$$

where W is the usual factor connecting the square of the transition amplitude with the transverse cross section, and $T(\mathbf{r})$ differs from zero in the region where meson production occurs, i.e., in a region on the order of the nuclear dimensions.

Using the theorem of the mean, we take the wave function $\varphi^{-}(kr)$ at some mean point of the interval $0 \leq r_m \leq r_Z$ out in Eq. 11:

$$\sigma = W \left| \frac{\varphi^{-}(kr_m)}{\varphi^{0}(kr_m)} \right|^{2} \left| \int \varphi^{0}(kr)\, T(\mathbf{r})\, d\mathbf{r} \right|^{2} = \left| \frac{\varphi^{-}(kr_m)}{\varphi^{0}(kr_m)} \right|^{2} \sigma^{0}, \tag{12}$$

where σ^0 is the production cross section without taking account of the interaction in the final state, and $\varphi^0(kr)$ is the wave function for free motion.

Let us suppose that $r_m = r_Z$. Then the function in Eq. (1) may be used as $\varphi^{-}(kr)$, if production occurs in a state with definite momentum ℓ. In the case of S-production of slow π^{-} mesons, this function is

$$\varphi_0^{-}(kr) = \sqrt{\frac{2}{\pi}}\, \frac{1}{kr} \{A F_0(kr) + B G_0(kr)\}. \tag{13}$$

For light nuclei, the functions $F_0(kr)$ and $G_0(kr)$ have the form of Eq. (8). The factor $|\varphi_0^{-}(kr_Z)/\varphi_0^{0}(kr_Z)|^2$ is expressed in this case ($\varphi_0^0(kr) = \sqrt{2/\pi}\, \sin kr/kr$) by the width and shift of the mesoatomic level. If data were available at present regarding the cross section of the production of slow mesons on light nuclei as a function of Z, and also regarding the widths and shifts of the mesoatomic levels, the above relations could be used to draw conclusions regarding the mechanism of meson production on nuclei. The functions $F_0(kr)$ and $G_0(kr)$ presented above are not suitable for heavier nuclei ($Z > 15$), because in this case the radius of the Bohr orbit becomes comparable to the radius of the nucleus. For the case of a Coulomb field of repulsion, expansions of $F_\ell(kr)$ and $G_\ell(kr)$ with respect to kr are known[6] which do not use the smallness of r/R. These expansions are easily obtained for an attractive field by changing the sign of R and going to Bessel functions with imaginary arguments. Then for $\ell = 0$, we have

$$F_0(kr) = C_0 kr \left\{ \left(\frac{r}{R}\right)^{1/2} J_1\left[2\left(\frac{r}{R}\right)^{1/2}\right] - \frac{(kr)^2}{3}\left(\frac{r}{R}\right)^{-1} J_2\left[2\left(\frac{r}{R}\right)^{1/2}\right] + \cdots \right\}, \quad G_0(kr) = \frac{1}{C_0}\left\{ -\left(\frac{r}{R}\right)^{1/2} \pi Y_1\left[2\left(\frac{r}{R}\right)^{1/2}\right] \right.$$
$$\left. - h(\alpha)\left(\frac{r}{R}\right)^{1/2} J_1\left[2\left(\frac{r}{R}\right)^{1/2}\right] + \frac{(kr)^2}{3}\left[\left(\frac{r}{R}\right)^{-3/2} J_1\left[2\left(\frac{r}{R}\right)^{1/2}\right] + \pi Y_2\left[2\left(\frac{r}{R}\right)^{1/2}\right] + h(\alpha) J_2\left[2\left(\frac{r}{R}\right)^{1/2}\right]\right] - \cdots \right\}, \tag{14}$$

where J_1, J_2, Y_1 and Y_2 are Bessel functions of the first and second order, of the first and second kinds, respectively. The factor $|\varphi_0^{-}(kr_Z)/\varphi_0^{0}(kr_Z)|^2$ is determined for all nuclei by the constants A and B, which may be determined by joining the logarithmic derivatives at the boundary of the nucleus.

4. MODEL OF THE MESON-NUCLEUS INTERACTION POTENTIAL FOR $\ell = 0$

Since we are considering the interactions of mesons with wavelengths much greater than the dimensions of the region in which nuclear forces act, we may expect that our conclusions will depend only weakly on the model, i.e., on the form of the potential. We will therefore consider it to be constant: $v' = V_0 + iK + V'$, where $V_0 + iK$ is the specifically nuclear interaction between the π meson and the nucleus, and V' contains all the non-nuclear interactions, among which we will consider only the electrostatic potential, since the calculated results do not depend critically on the magnitude of the real part of the potential. For the same reason, we will consider V' equal to the value of the Coulomb potential at the boundary of the nucleus. For the potential V_0, we will use a value $\sim$ 5 Mev, which corresponds to the magnitude and sign of the shift of the 1s level ($\mathrm{Re}(\Delta E)_{\mathrm{nucl}}$) in mesoatoms.[7-9] The quantity K must have a value of 1 — 2 Mev in order to conform to the observed width of the mesoatomic level.[10-12]

The potential v′ is independent of meson energy for small energies.

5. PHOTOPRODUCTION OF SLOW π MESONS

Using the proposed model, it was possible to interpret the experimental data[3,4] on the photoproduction of very slow π^- mesons (with mean energy of 2 Mev) on nuclei of various elements (see also Ref. 13). Since in this case, the mesons are produced in the form of an S-wave, the production cross section based on Eqs. (12) and (13) is*

$$\sigma_Z = \left|\frac{\sin kr_Z}{kr_Z}\right|^{-2} \sigma^0 \{[F_0(kr_Z)\, b_Z r_Z - r_Z F_0'(kr_Z)]^2 + [r_Z G_0'(kr_Z) - G_0(kr_Z)\, b_Z r_Z]^2\}^{-1};$$

$$b = \frac{\beta \sinh 2\beta r + \gamma \sin 2\gamma r}{\cosh 2\beta r - \cos 2\gamma r} + i\,\frac{\gamma \sinh 2\beta r - \beta \sin 2\gamma r}{\cosh 2\beta r - \cos 2\gamma r}; \qquad (15)$$

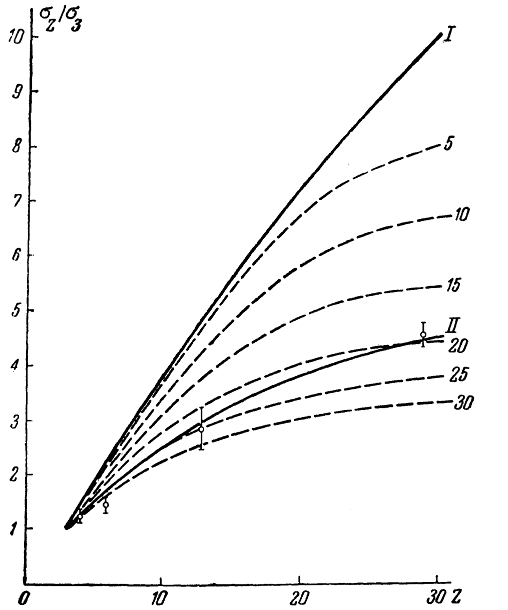
$$\beta = \left(\frac{\mu}{\hbar^2}\right)^{1/2}\left[\sqrt{(V_0+V'-E_\pi)^2+K^2} + (V_0+V'-E_\pi)\right]^{1/2}; \quad \gamma = \left(\frac{\mu}{\hbar^2}\right)^{1/2}\left[\sqrt{(V_0+V'-E_\pi)^2+K^2} - (V_0+V'-E_\pi)\right]^{1/2}.$$

It follows from Eq. (15) that the finite dimensions of the nucleus have a very strong effect on the influence of the Coulomb field of the nucleus on the photoproduction of mesons. In fact, if the nucleus was a point ($r_2 \to 0$), then $\sigma = C_0^2 \sigma^0$. For slow mesons, C_0^2 is a large quantity, depending sharply on Z. The photoproduction cross section would then increase very rapidly with increasing atomic number, which is not observed experimentally.[3,4] For $\sim$ 2 Mev mesons, for example, the factor C_0^2 leads to a transverse cross section for heavy nuclei 20 — 40 times greater than that for light nuclei, while the ratio σ_Z/σ_L calculated by Eq. (15) is only 2.5 — 5. Experiment gives 4.0 ± 1.0 for this ratio.[3]

Let us assume volume photoproduction of mesons,

$$\sigma^0 = \mathrm{const}\,(A - Z)\,\sigma^0_{\mathrm{nucl}} \qquad (16)$$

We shall try to explain the experimentally observed Z-dependence of the π^--meson yield by reabsorption, i.e., we shall pick a value for K which yields agreement between the calculated and the observed dependence of σ on Z.



The Z-dependence of the ratio σ_Z/σ_3. The solid curves I and II were calculated for K = 0 from Eq. (15) for the case of volume and surface meson production, respectively. The dotted curves were calculated for volume production, taking account of self-absorption; the corresponding values of K are indicated alongside each curve. The experimental points were taken from Ref. 4.

The experimentally obtained Z-dependence of the ratio σ_Z/σ_3 of the photoproduction cross sections for slow mesons, and also curves calculated from Eq. (15) taking account of Eq. (16), are shown in the figure. As is clear from the figure, the magnitude of the imaginary part of the potential must be taken as K = 15 — 20 Mev in order to explain the experimental data. This value disagrees sharply with the value of K = 1 — 2 Mev obtained from mesoatom data.[10-12]

Thus, assumption (16) does not correspond to reality.

On the other hand, the curve calculated on the assumption of surface photoproduction of mesons with K = 0 (the curve calculated with K = 1.5 Mev practically coincides with this curve) agrees well with experiment.

*The primes on F(kr) and G(kr) denote differentiation with respect to r.

6. CONCLUSIONS

(a) According to the theory presented here, the mechanism of meson reabsorption cannot explain the dependence of the photoproduction cross section on atomic number. Some mechanism must be introduced to forbid meson production inside the nucleus. This result was obtained on the basis of a concrete model of the meson-nucleus interaction potential. However, since the interaction of mesons with wavelengths considerably greater than the nuclear dimensions are considered, the results do not depend critically on the choice of the model.*

(b) Without referring to a model of the potential, the same result may be obtained on the basis of the material presented in Sec. 2, if data on the photoproduction of mesons of low energy on light nuclei ($Z < 15$) and on the shifts and widths of mesoatomic levels are accumulated.

(c) The finite dimensions of the nucleus have a great influence on the photoproduction of slow mesons by sharply changing the action of the Coulomb field.†

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*Effects connected with the increase of the yield of low-energy mesons because of the rescattering of higher-energy mesons lead only to an increase of the slow meson output with an increase of Z. Thus, these effects will not influence our conclusions.

†The ratio of the yields of positive and negative mesons, which is very sensitive to the effect of the Coulomb interaction, is described well by Eq. (15) (according to the preliminary experimental data of Popova, Semashko, and Iagudina, private communication).