and the integration and summation should be carried out over the surfaces S^{ℓ}.

V being known, we can find the required probability (5) by means of relation (9) which remains valid.

All the probabilities found above may be useful, for example, in investigations of the cosmic radiation by means of coincidence counters or cloud chambers.[3]

It should be noted that the probabilities found are analogous to the probabilities of certain configurations of molecules in a gaseous medium. In the case studied above, however, the problem is greatly simplified since all the probabilities can be expressed by means of the distribution function (1), while there are no similar expressions for the correlation function in gasses.[4]

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POLARIZATION OF NUCLEONS ELASTICALLY SCATTERED AGAINST TARGET PARTICLES OF SPIN 1

G. M. BUDIANSKII

Academy of Sciences, U.S.S.R.

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The average values of the spin operators for a system of particles having spin 1 and $^1/_2$ are calculated. The transition matrix M is given explicitely. Consirderation is given to the case of small energies, when one can restrict oneself to S- and P-waves. Expressions are obtained for the cross-section, polarization, and correlation function. Relationships are established between the parameters of the transition matrix and the experimentally observed values. A group of experiments is suggested which could enable one to determine, through triple-scattering, the amplitude of the scattered wave and to carry out a phase-shift analysis.

THE theory of reactions involving polarized nucleons has been recently developed in a series of articles.[1] The polarization arising in nucleon-nucleon collision is due to spin-orbit interaction, and its measurement provides additional information about the coefficients of the amplitude for nucleon-nucleon scattering. A group of experiments is indicated which would allow one to determine the nucleon-nucleon scattering amplitude and to carry out a phase-shift analysis.

The present article is concerned with the elastic scattering of nucleons against a target made up of spin 1 particles.

The state of the system is described as usual through the Neuman density matrix ρ in the combined spin space of the system of two particles, or through the density matrix for two independent beams of free particles with spin σ' and s'. Since the density matrix is a six-rowed Hermitian matrix, it is defined through a linear combination of 36 real quantities for which one may take the average values of a complete set of independent Hermitian operators in spin space (their number equals the square of the dimensionality of the spin space). The complete set of operators satisfies the equations

$$\mathrm{Sp}\, S^{\mu} S^{\nu} = (2s_a + 1)(2s_b + 1)\,\delta_{\mu\nu}. \tag{1}$$

The average value of any operator is defined as

$$\langle S \rangle_{\mathrm{inc}} = \mathrm{Sp}\,(\rho_{\mathrm{inc}} S) \tag{2}$$

with the condition that

$$\mathrm{Sp}\,\rho_{\mathrm{inc}} = 1.$$

An arbitrary matrix may be linearly expressed in terms of the operators S^{μ}. Expressing the density matrix in terms of the average values of the spin operators, we obtain

$$\rho_{\mathrm{inc}} = [(2s_a + 1)(2s_b + 1)]^{-1} \sum_{\mu} \langle S^{\mu} \rangle S^{\mu}. \tag{3}$$

In this way, the average value of the matrix S^{μ} determines ρ, and, at the same time, the spin state of the system.

If we know the transition matrix M which transforms each pure state of a mixture described by some matrix ρ_{inc}, into some corresponding state of the mixture described by ρ_{scat}, we then have for the scattering of a beam

$$\rho_{\mathrm{scat}} = M \rho_{\mathrm{inc}} M^{*}. \tag{4}$$

The average value of any operator S_1 in the spin space of particles is given, after collision, by the expression

$$\langle S_1 \rangle_{\mathrm{scat}} = \mathrm{Sp}\,(\rho_{\mathrm{scat}} S_1) \,/\, \mathrm{Sp}\,\rho_{\mathrm{scat}}. \tag{5}$$

The following expressions are then found for the differential reaction cross section and the average value of the operator S_1:

$$Q = [(2s_a + 1)(2s_b + 1)]^{-1} \sum_{\mu} \langle S^{\mu} \rangle_{\mathrm{inc}} \mathrm{Sp}\,(M S^{\mu} M^{*}), \tag{6}$$

$$\langle S_1 \rangle_{\mathrm{scat}} = \sum_{\mu} \langle S^{\mu} \rangle_{\mathrm{inc}}\, \mathrm{Sp}\,(M S^{\mu} M^{*} S_1) \Big/ \sum_{\nu} \langle S^{\nu} \rangle_{\mathrm{inc}} \mathrm{Sp}\,(M S^{\nu} M^{*}). \tag{7}$$

THE AVERAGE VALUES OF THE SPIN OPERATORS

Applying the well-known Clebsch-Gordan formulas, the spin-quartet wave function may be written in the form

$$\begin{gathered}\chi(^3/_2, {}^3/_2) = \chi(^1/_2, {}^1/_2)\,\chi(1, 1), \quad \chi(^3/_2, {}^1/_2) = \sqrt{^2/_3}\,\chi(^1/_2, {}^1/_2)\,\chi(1,0) + \sqrt{^1/_3}\,\chi(^1/_2, -{}^1/_2)\,\chi(1, 1),\\ \chi(^3/_2, -{}^1/_2) = \sqrt{^1/_3}\,\chi(^1/_2, {}^1/_2)\,\chi(1, -1) + \sqrt{^2/_3}\,\chi(^1/_2, -{}^1/_2)\,\chi(1, 0), \quad \chi(^3/_2, -{}^3/_2) = \chi(^1/_2, -{}^1/_2)\,\chi(1, -1),\end{gathered} \tag{8}$$

and the doublet function becomes

$$\begin{gathered}\chi(^1/_2, {}^1/_2) = \sqrt{^1/_3}\,\chi(^1/_2, {}^1/_2)\,\chi(1, 0) - \sqrt{^2/_3}\,\chi(^1/_2, -{}^1/_2)\,\chi(1, 1),\\ \chi(^1/_2, -{}^1/_2) = \sqrt{^2/_3}\,\chi(^1/_2, {}^1/_2)\,\chi(1, -1) - \sqrt{^1/_3}\,\chi(^1/_2, -{}^1/_2)\,\chi(1,0).\end{gathered} \tag{9}$$

Operating on these functions with the operators σ_x, σ_y, σ_z, S_x S_y and S_z, and applying the orthonormality of the functions $\chi\,(s,\, m_s)$, one obtains

$$\sigma_x = \begin{pmatrix} 0 & \sqrt{1/3} & 0 & 0 & -\sqrt{2/3} & 0 \\ \sqrt{1/3} & 0 & 2/3 & 0 & 0 & -\sqrt{2}/3 \\ 0 & 2/3 & 0 & \sqrt{1/3} & \sqrt{2}/3 & 0 \\ 0 & 0 & \sqrt{1/3} & 0 & 0 & \sqrt{2/3} \\ -\sqrt{2/3} & 0 & \sqrt{2}/3 & 0 & 0 & -1/3 \\ 0 & -\sqrt{2}/3 & 0 & \sqrt{2/3} & -1/3 & 0 \end{pmatrix}, \quad \sigma_y = -\begin{pmatrix} 0 & -\sqrt{1/3} & 0 & 0 & \sqrt{2/3} & 0 \\ \sqrt{1/3} & 0 & -2/3 & 0 & 0 & \sqrt{2}/3 \\ 0 & 2/3 & 0 & -\sqrt{1/3} & \sqrt{2}/3 & 0 \\ 0 & 0 & \sqrt{1/3} & 0 & 0 & \sqrt{2/3} \\ -\sqrt{2/3} & 0 & -\sqrt{2}/3 & 0 & 0 & 1/3 \\ 0 & -\sqrt{2}/3 & 0 & -\sqrt{2/3} & -1/3 & 0 \end{pmatrix},$$

$$S_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & \sqrt{2/3} & 0 & 0 & \sqrt{1/3} & 0 \\ \sqrt{2/3} & 0 & 2\sqrt{2}/3 & 0 & 0 & 1/3 \\ 0 & 2\sqrt{2}/3 & 0 & \sqrt{2/3} & -1/3 & 0 \\ 0 & 0 & \sqrt{2/3} & 0 & 0 & -\sqrt{1/3} \\ \sqrt{1/3} & 0 & -1/3 & 0 & 0 & 2\sqrt{2}/3 \\ 0 & 1/3 & 0 & -\sqrt{1/3} & 2\sqrt{2}/3 & 0 \end{pmatrix}, \quad S_z = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & -\sqrt{2}/3 & 0 \\ 0 & 0 & -1/3 & 0 & 0 & -\sqrt{2}/3 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -\sqrt{2}/3 & 0 & 0 & 2/3 & 0 \\ 0 & 0 & -\sqrt{2}/3 & 0 & 0 & -2/3 \end{pmatrix}.$$

By means of a suitable transformation, one may go from this representation to a 2-particle representation which may be more easily obtained directly as a direct product of the matrices σ' and S' in the spin-spaces corresponding to a nucleon and a deuteron.

We shall denote unit vectors in the direction of motion of the incident and scattered beams in the center-of-mass system respectively by $\mathbf{k}_i$ and $\mathbf{k}_f$, and for convenience in future expansions, we shall introduce as usual the orthogonal vectors

$$\mathbf{N} = [\mathbf{k}_i \times \mathbf{k}_f]/|[\mathbf{k}_i \times \mathbf{k}_f]|, \quad \mathbf{P} = (\mathbf{k}_i + \mathbf{k}_f)/|\mathbf{k}_i + \mathbf{k}_f|, \quad \mathbf{K} = (\mathbf{k}_i - \mathbf{k}_f)/|\mathbf{k}_i - \mathbf{k}_f|. \tag{10}$$

Transforming into this system by means of Euler's formulas,[3] we obtain formulas for σ_N, σ_p, σ_K, S_N, S_p, S_K expressed in terms of the angles ϑ and φ; we shall not list them here, as they are rather cumbersome.

THE TRANSITION MATRIX

The most general form of the matrix M, which determines the amplitude of a scattered wave of given spin and momentum as a function of the spin and momentum of the incident wave, may be obtained by imposing upon it the conditions of invariance with respect to space rotation and time reversal. This matrix must be a scalar which is obtained from a combination of 36 linearly independent matrices in spin space and functions of the momenta K, N, P. The effect of time reversal leads to the transformation

$$\sigma' = -\sigma; \quad \mathbf{k}'_i = -\mathbf{k}_f; \quad \mathbf{k}'_f = -\mathbf{k}_i; \quad \mathbf{K}' = \mathbf{K}; \quad \mathbf{N}' = -\mathbf{N}; \quad \mathbf{P}' = -\mathbf{P}.$$

The expressions $[\sigma \times \mathbf{S}]\mathbf{N}$ and $(\sigma\mathbf{K})(\mathbf{SP}) + (\sigma\mathbf{P})(\mathbf{SK})$ change sign with time reversal and therefore must be excluded from consideration. In writing out the matrix M, one may limit oneself to terms of second order in S, as higher order terms in the spin operator S may be given as functions of these. Thus, for example

$$S_i S_j S_k = \frac{1}{2}[\delta_{ij} S_k + \delta_{jk} S_i] + \frac{i}{2}[S_{i\times j,k} + S_{j\times k,i} - S_{k\times i,j}] + \frac{i}{3}\varepsilon_{ijk},$$

$$S_{ij} S_{kl} = \frac{1}{3}\left[\delta_{ij} S_{kl} + \delta_{kl} S_{ij} + \frac{1}{2}\delta_{ik}\delta_{jl} + \frac{1}{2}\delta_{jk}\delta_{il} - \frac{1}{3}\delta_{ij}\delta_{kl}\right]$$

$$-\frac{1}{4}\left[\delta_{ik}\left(S_{jl} - \frac{i}{2} S_{j\times l}\right) + \delta_{il}\left(S_{jk} - \frac{i}{2} S_{j\times k}\right) + \delta_{jk}\left(S_{il} - \frac{i}{2} S_{i\times l}\right) + \delta_{jl}\left(S_{ik} - \frac{i}{2} S_{i\times k}\right)\right],$$

where

$$S_{j\times k} = \varepsilon_{ijk} S_i, \quad S_{j\times k,l} = \varepsilon_{ijk} S_{il}.$$

In this fashion one obtains the most general form of the scattering matrix for particles of spin 1/2 against particles of spin 1:

$$M = A + BS_N + C[(S_PS_P - {}^2/_3\delta_{ij}) + (S_KS_K - {}^2/_3\delta_{ij})] + \{A_1 + B_1S_N + C_1[(S_PS_P - {}^2/_3\delta_{ij}) + (S_KS_K - {}^2/_3\delta_{ij})]\}\sigma_N + D(\sigma_PS_P + \sigma_KS_K) + E(\sigma_PS_P - \sigma_KS_K) + {}^1/_2F(\sigma_PS_NS_P + \sigma_KS_NS_K) + {}^1/_2G(\sigma_PS_PS_N + \sigma_KS_KS_N) + {}^1/_2H(\sigma_PS_NS_P - \sigma_KS_NS_K) + {}^1/_2K(\sigma_PS_PS_N - \sigma_KS_KS_N), \tag{11}$$

where the coefficients A, B, C etc. appear as functions of $\mathbf{k}_i^2$ and $\mathbf{k}_i\mathbf{k}_f$, i.e., as functions of the energy and the cosine of the scattering angle in the center-of-mass system.

In order to obtain the matrix in explicit form, we apply the method of Blatt and Bienderharn.[4] The amplitude of the scattered wave is given as the sum of two terms:

$$f(\vartheta) = f_1(\vartheta) + f_2(\vartheta).$$

The first of these

$$f_1(\vartheta) = (-\lambda\eta/2\zeta^2)\exp[-i\eta\ln\zeta^2] \tag{12}$$

depends on Coulomb scattering, while the second

$$f(s') = i\lambda\Phi_{s'}\sum_{m_{s'}}\chi(s', m_{s'})\sum_{Jll'} i^{l-l'}\pi^{1/2}(2l+1)^{1/2}(l, s, 0, m_s \mid l, s, J, m_s) \times (l', s', m_s - m_{s'}, m_{s'} \mid l, s', J, m_s)(\delta_{s's}\delta_{l'l} - S^J_{s'l';sl})Y_{l', m_s - m_{s'}}(\vartheta, \varphi) = \sum_{m_{s'}}\chi(s', m_{s'})\sum_{m_s} M_{m_{s'}m_s}a_{m_s} \tag{13}$$

gives the difference between the amplitude of a wave scattered against a charged sphere and $f_1(\vartheta)$. In the above expression we have used certain quantities defined as follows: $\eta = e^2/hv$; $\zeta = \sin(\vartheta/2)$; $\Phi_{s'}$ is the product of the wave functions of the final nucleus and the scattered particle; the quantities a_{m_s} obey the condition

$$a_{m_{s'}}a_{m_s} = \delta(m_s - m_{s'})/(2s+1). \quad S^J_{s', l'; s, l} = \delta_{s', s}\delta_{l', l}\exp[2i(\xi_l - \eta\ln 2kr)]$$

are the elements of the scattering matrix, r is the radius of the screened Coulomb field in a given channel and

$$\xi_l = \psi_l + \Phi_l + \sigma_0,$$

where $\psi_l = \sigma_l - \sigma_0$ is the Coulomb phase shift:

$$\exp(2i\psi_l) = (l + i\eta)\ldots(1 + i\eta)/(l - i\eta)\ldots(1 - i\eta),$$

σ_0 is the S-wave phase shift; $\Phi_l = \xi_l - \sigma_l$ describes a further phase shift occurring during scattering against a charged sphere of radius R as compared with the shift which occurs in point charge scattering.

In order to carry out a phase-shift analysis, it is necessary to determine the dependence of M upon the scattering angle and the phase shift. The dependence of the elements of the matrix M upon φ is evident from Eq. (13), and comparing them with those matrices of which it consists, we obtain

$$M = \lambda\begin{Bmatrix} a & ce^{-i\varphi} & ge^{-2i\varphi} & he^{-3i\varphi} & 0 & 0 \\ be^{i\varphi} & d & fe^{-i\varphi} & ge^{-2i\varphi} & 0 & 0 \\ ge^{2i\varphi} & -fe^{i\varphi} & d & -be^{-i\varphi} & 0 & 0 \\ -he^{3i\varphi} & ge^{2i\varphi} & -ce^{i\varphi} & a & 0 & 0 \\ 0 & 0 & & 0 & \mu & -\nu e^{-i\varphi} \\ 0 & 0 & 0 & 0 & \nu e^{i\varphi} & \mu \end{Bmatrix}. \tag{14}$$

the matrix elements a, b, c represent series of Legendre polynomials $P_l^m(\cos\vartheta)$, and the coefficients in these series depend on the phase shifts.

The elements of this matrix obey the additional condition

$$\frac{1}{\cos 2\vartheta}\left(a - d - \frac{2}{\sqrt{3}}g\right) = \frac{2}{\sqrt{3}}\frac{1}{\sin 2\vartheta}(b + c), \tag{15}$$

which appears as a result ot the invariance of M with respect to time reversal. As mentioned above, the matrix **σK·SP + σP·SK** is one of the matrices belonging to the complete set in terms of which M may be expanded; thus it can only enter in the expansion when Sp M (**σK·SP + σP·SK**) is identically zero.

In the particular can when the energy is sufficiently small to permit the use of S- and P-waves only, the scattering amplitude may be written in the form:

$$\psi(s, s') = \frac{e^{ikr}}{2ikr} \sum_{m_s m_{s'}} \sum_{L, J} [4\pi(2L+1)]^{1/2} (L, s, 0, m_s \mid L, s, J, m_s)$$
$$\times (L, s, m_s - m_{s'}, m_{s'} \mid L, s', J, m_s) [1 - \exp(2i\xi_l)] Y_{L,\, m_s - m_{s'}} (\vartheta, \varphi). \tag{16}$$

For doublet transitions we obtain

$$M_{1/2,\, 1/2} = -\lambda \{\gamma^{1/2}_{0,1/2} + \cos\vartheta e^{-2i\psi} (\gamma^{1/2}_{1,1/2} + 2\gamma^{3/2}_{1,1/2})\}, \quad M_{-1/2,\, 1/2} = \lambda \sin\vartheta \exp[i(\varphi - 2\psi)] (\gamma^{1/2}_{1,1/2} - \gamma^{3/2}_{1,1/2}),$$
$$M_{1/2,\, -1/2} = \lambda \sin\vartheta \exp[-i(\varphi + 2\psi)] (-\gamma^{1/2}_{1,1/2} + \gamma^{3/2}_{1,1/2}), \quad M_{-1/2,\, -1/2} = \lambda \{\gamma^{1/2}_{0,1/2} + \cos\vartheta \exp(-2i\psi) (\gamma^{1/2}_{1,1/2} + 2\gamma^{3/2}_{1,1/2}), \tag{17}$$

where $\gamma^{J}_{l,s} = \exp(-\Phi^{J}_{l,s}) \sin \Phi^{J}_{l,s}$,

and for quartets

$$M_{3/2,\, 3/2} = -\lambda \left\{\gamma^{3/2}_{0,3/2} + \cos\vartheta \exp(-2i\psi) \left(\frac{9}{5}\gamma^{3/2}_{1,3/2} + \frac{6}{5}\gamma^{5/2}_{1,3/2}\right)\right\}, \quad M_{1/2,\, 3/2} = \frac{3\sqrt{3}}{5} \lambda \exp[i(\varphi - 2\psi)] \sin\vartheta (\gamma^{3/2}_{1,3/2} - \gamma^{5/2}_{1,3/2}),$$
$$M_{-1/2,\, 3/2} = 0, \qquad M_{-3/2,\, 3/2} = 0; \quad M_{3/2,\, 1/2} = \lambda \sin\vartheta \exp[-i(\varphi + 2\psi)] \left(-\frac{\sqrt{3}}{2}\gamma^{1/2}_{1,3/2} + \frac{\sqrt{3}}{5}\gamma^{3/2}_{1,3/2} + \frac{3\sqrt{3}}{10}\gamma^{5/2}_{1,3/2}\right),$$
$$M_{1/2,\, 1/2} = -\lambda \left\{\gamma^{3/2}_{0,3/2} + \cos\vartheta \exp(-2i\psi) \left(\gamma^{1/2}_{1,3/2} + \frac{1}{5}\gamma^{3/2}_{1,3/2} + \frac{9}{5}\gamma^{5/2}_{1,3/2}\right)\right\},$$
$$M_{-1/2,\, 1/2} = \lambda \sin\vartheta \exp[i(\varphi - 2\psi)] \left(\frac{1}{2}\gamma^{1/2}_{1,3/2} + \frac{2}{5}\gamma^{3/2}_{1,3/2} - \frac{9}{10}\gamma^{5/2}_{1,3/2}\right), \quad M_{-3/2,\, 1/2} = 0; \quad M_{3/2,\, -1/2} = 0, \tag{18}$$
$$M_{1/2,\, -1/2} = -\lambda \sin\vartheta \exp[-i(\varphi + 2\psi)] \left(\frac{1}{2}\gamma^{1/2}_{1,3/2} + \frac{2}{5}\gamma^{3/2}_{1,3/2} - \frac{9}{10}\gamma^{2}_{1,3/2}\right), \quad M_{-1/2,\, -1/2} = -\lambda \{\gamma^{3/2}_{0,3/2} + \cos\vartheta e^{-2i\psi}$$
$$\times \left(\gamma^{1/2}_{1,3/2} + \frac{1}{5}\gamma^{3/2}_{1,3/2} + \frac{9}{5}\gamma^{5/2}_{1,3/2}\right), \quad M_{-3/2,\, -1/2} = \lambda \exp[i(\varphi - 2\psi)] \sin\vartheta \left(\frac{\sqrt{3}}{2}\gamma^{1/2}_{1,3/2} - \frac{\sqrt{3}}{5}\gamma^{3/2}_{1,3/2} - \frac{3\sqrt{3}}{10}\gamma^{5/2}_{1,3/2}\right);$$
$$M_{3/2,\, -3/2} = 0, \quad M_{1/2,\, -3/2} = 0, \quad M_{-1/2,\, -3/2} = -\frac{3\sqrt{3}}{5} \lambda \sin\vartheta \exp[-i(\varphi + 2\psi)] (\gamma^{3/2}_{1,3/2} - \gamma^{5/2}_{1,3/2}),$$
$$M_{-3/2,\, -3/2} = \lambda \left\{\gamma^{3/2}_{0,3/2} + \cos\vartheta e^{-2i\psi} \left(\frac{9}{5}\gamma^{3/2}_{1,3/2} + \frac{6}{5}\gamma^{5/2}_{1,3/2}\right)\right\}.$$

DETERMINATION OF THE PARAMETERS OF THE TRANSITION MATRIX FROM EXPERIMENTAL DATA

Expressions (6) and (7) apply to particles of arbitrary spin and beams of arbitrary polarization; we shall apply them to the concrete case of scattering of particles having spin 1/2 against an unpolarized target of particles with spin 1.

In this case, the only non-zero matrices before collision are 1 × 1 and σ × 1, and these formulas become

$$Q^{P} = \frac{1}{6} \{\mathrm{Sp}(MM^*) + \mathrm{Sp}(M\sigma M^*) \mathbf{P}_{\mathrm{inc}}\}, \quad \mathbf{P}^{P} = \langle \sigma \times 1 \rangle_{\mathrm{scat}} = \frac{\mathrm{Sp}(M^*\sigma M) + \mathrm{Sp}(\sigma M \sigma \mathbf{P}_{\mathrm{inc}} M^*)}{\mathrm{Sp}(MM^*) + \mathrm{Sp}(M\sigma M^*) \mathbf{P}_{\mathrm{inc}}}. \tag{19}$$

We write here **σ** for **σ × 1**, and denote $\langle \sigma \times 1 \rangle_{\mathrm{inc}}$ by $\mathbf{P}_{\mathrm{inc}}$. When scattering an unpolarized beam of nucleons, the expressions for the differential cross-section and the polarization simplify to

$$Q^{N} = \frac{1}{6} \mathrm{Sp}\, MM^*, \quad Q^{N} P^{N} = \frac{1}{6} \mathrm{Sp}\, MM^* \sigma_N \tag{20}$$

Substituting Eq. (14) for the transition matrix M, and expressing σ_N in terms of the Eulerian angles, we obtain explicit forms for the cross-section and the polarization:

$$Q^{N} = \frac{1}{3} \lambda^2 \{|a|^2 + |b|^2 + |c|^2 + |d|^2 + |f|^2 + 2|g|^2 + |h|^2 + |\mu|^2 + |\nu|^2\},$$
$$Q^{N} P^{N} = \frac{2}{3} \lambda^2 \,\mathrm{Im} \left\{\sqrt{\frac{1}{3}} (ba^* + dc^* + fg^* + gh^*) + \frac{2}{3} (bg^* + df^*) + \frac{1}{3} \mu\nu^*\right\}. \tag{21}$$

In the particular case of small energies, limiting ourselves to S- and P-waves, we obtain for the scattering of a proton against a deuteron:

$$Q^{\mathrm{N}} = \lambda^2 \Big\{ \Big(\frac{\eta}{2}\frac{1}{\zeta^2}\Big)^2 + \frac{2}{3}\Big(\frac{\eta}{2}\frac{1}{\zeta^2}\Big)[u(\zeta, \Phi^{1/2}_{0,1/2}) + 2u(\zeta, \Phi^{3/2}_{0,3/2})] + \frac{2}{3}\frac{\eta}{2\zeta^2}\cos\vartheta\,[u(\zeta, \Phi^{1/2}_{1,1/2}) + 2u(\zeta, \Phi^{3/2}_{1,1/2}) + u(\zeta, \Phi^{1/2}_{1,3/2}) + 2u(\zeta, \Phi^{3/2}_{1,3/2})$$

$$+ 3u(\zeta, \Phi^{5/2}_{1,3/2})] + \frac{1}{3}\sin^2\Phi^{1/2}_{0,3/2} + \frac{2}{3}\sin^2\Phi^{3/2}_{0,3/2} + \frac{2}{3}\cos\vartheta\,\{\sin\Phi^{1/2}_{0,1/2}\,[u(\Phi^{1/2}_{0,1/2}, \Phi^{1/2}_{1,1/2})$$

$$+ 2u(\Phi^{1/2}_{0,1/2}, \Phi^{3/2}_{1,1/2})] + \sin\Phi^{3/2}_{0,3/2}\,[u(\Phi^{3/2}_{0,3/2}, \Phi^{1/2}_{1,3/2}) + 2u(\Phi^{3/2}_{0,3/2}, \Phi^{3/2}_{1,3/2}) + 3u(\Phi^{3/2}_{0,3/2}, \Phi^{5/2}_{1,3/2})] + \cos^2\vartheta\,[(\sin^2\Phi^{1/2}_{1,1/2} + \sin^2\Phi^{1/2}_{1,3/2})$$

$$+ 2(\sin^2\Phi^{3/2}_{1,1/2} + \sin^2\Phi^{3/2}_{1,3/2}) + 3\sin^2\Phi^{5/2}_{1,3/2} - 2W] + \sin^2\vartheta\cdot W\Big\},$$

where $u(\alpha^{j}_{l,s};\ \beta^{j'}_{l',s'}) = \sin\beta^{j'}_{l',s'}\cos(\alpha^{j}_{l,s} - 2l'\psi - \beta^{j'}_{l',s'})$ and

$$W = \frac{1}{3}\sin^2(\Phi^{3/2}_{1,1/2} - \Phi^{1/2}_{1,1/2}) + \frac{1}{30}\sin^2(\Phi^{1/2}_{1,3/2} - \Phi^{3/2}_{1,3/2}) + \frac{3}{10}\sin^2(\Phi^{1/2}_{1,3/2} - \Phi^{5/2}_{1,3/2}) + \frac{21}{50}\sin^2(\Phi^{3/2}_{1,3/2} - \Phi^{5/2}_{1,3/2}); \qquad (22)$$

$$Q^{\mathrm{N}}P^{\mathrm{N}} = \frac{\lambda^2}{3}\frac{\eta}{2\zeta^2}\sin\vartheta\,\{5z(\Phi^{5/2}_{1,3/2}, \Phi^{1/2}_{1,3/2}\,\zeta) + 4z(\Phi^{5/2}_{1,3/2}, \Phi^{3/2}_{1,3/2}, \zeta) + 2z(\Phi^{3/2}_{1,1/2}, \Phi^{1/2}_{1,1/2}\,\zeta)\} + \frac{\lambda^2}{3}\sin\vartheta\,\{[5z(\Phi^{1/2}_{1,3/2}, \Phi^{1/2}_{1,3/2}, \Phi^{3/2}_{0,3/2})$$

$$+ 4z(\Phi^{5/2}_{1,3/2}, \Phi^{3/2}_{1,3/2}, \Phi^{3/2}_{0,3/2})]\sin\Phi^{3/2}_{0,3/2} + 2z(\Phi^{3/2}_{1,1/2}, \Phi^{1/2}_{1,1/2}, \Phi^{1/2}_{0,1/2})\sin\Phi^{1/2}_{0,1/2}\} + \frac{\lambda^2}{5}\sin\vartheta\cos\vartheta\sin(\Phi^{1/2}_{1,3/2}, - \Phi^{3/2}_{1,3/2})\sin(\Phi^{1/2}_{1,3/2} - \Phi^{5/2}_{1,3/2})$$

$$\times \sin(\Phi^{3/2}_{1,1/2} - \Phi^{5/2}_{1,3/2}) - \lambda^2\sin\vartheta\cos\vartheta\,[5Y(\Phi^{1/2}_{1,3/2}, \Phi^{5/2}_{1,3/2}) + 4Y(\Phi^{3/2}_{1,3/2}, \Phi^{5/2}_{1,3/2}) + 2Y(\Phi^{1/2}_{1,1/2}\Phi^{3/2}_{1,1/2})],$$

where $Y(\alpha, \beta) = \sin(\alpha - \beta)\sin\alpha\cdot\sin\alpha$, and $z(\alpha, \beta, \gamma) = \sin(\alpha - \beta)\sin(2\psi + \alpha + \beta - \gamma)$.

In addition to the cross-section and polarization, one may also obtain an experimental value for the correlation function $\mathbf{C(p, q)} = \langle \boldsymbol{\sigma}\mathbf{p}\cdot\mathbf{Sq}\rangle_{\mathrm{scatt}}$, by measuring simultaneously the polarization of the scattered beam of particles $(\mathbf{k}_f)$ along a direction $\mathbf{p}$ and that of the beam of recoiling particle $(-\mathbf{k}_f)$ along the direction $\mathbf{q}$.

If the incident beam is unpolarized, one finds

$$Q^{\mathrm{N}}C^{\mathrm{N}}(\mathbf{N},\mathbf{N}) = \frac{1}{6}\mathrm{Sp}\,MM^*(\boldsymbol{\sigma}\mathbf{N})(\mathbf{SN}) = \frac{2}{3}\lambda^2\Big\{\frac{1}{3}[|b|^2 + |d|^2 + |f|^2 + |g|^2 - |\mu|^2 - |\nu|^2] + \frac{1}{\sqrt{3}}\mathrm{Re}\,[c^*f + b^*h - a^*g - d^*g]\Big\}, \qquad (23)$$

$$Q^{\mathrm{N}}C^{\mathrm{N}}(\mathbf{m},\mathbf{P}) = \frac{1}{6}\mathrm{Sp}\,MM^*(\boldsymbol{\sigma}\mathbf{m})(\mathbf{SP}) = \frac{\lambda^2}{3}\{\sin(\vartheta + \beta)\cos\vartheta\,[|a|^2 + |c|^2 + |h|^2 - \frac{1}{3}(|b|^2 + |d|^2 + |f|^2)]$$

$$- \sin\vartheta\cos(\vartheta + \beta)\Big[\frac{2}{3}(|b|^2 + |d|^2 + |f|^2)\Big] + \frac{2}{\sqrt{3}}\sin\vartheta\cos(\vartheta + \beta)\,\mathrm{Re}\,(bh^* + cf^* - ag^* - dg^*)$$

$$- \frac{2}{\sqrt{3}}\cos(2\vartheta + \beta)\,\mathrm{Re}\,(ab^* + cd^* + fg^* + gh^*)\},$$

β is the angle between $\mathbf{m}$ and $\mathbf{K}$, and equals $\pi/2$ when $\mathbf{m}$ coincides with $\mathbf{P}$. By carrying out triple scattering experiments, one may obtain additional data, for example how the direction and magnitude of polarization changes after a second scattering. In order to describe the geometry of these experiments, we introduce a unit vector $\mathbf{n}$ in the direction $[\mathbf{N} \times \mathbf{k}_i]$, lying in the plane of the second scattering. It may be seen from Eq. (19) that the cross section and polarization for second scattering may be written in the form

$$Q^{\mathrm{P}} = Q^{\mathrm{N}}(1 + \mathbf{P}^{\mathrm{N}}\mathbf{P}_{\mathrm{inc}}) = Q^{\mathrm{N}}(1 + A), \quad \mathbf{P}^{\mathrm{P}} = \mathbf{P}^{\mathrm{N}} + \tau\mathbf{P}_{\mathrm{inc}}/(1 + \mathbf{P}^{\mathrm{N}}\mathbf{P}_{\mathrm{inc}}), \qquad (24)$$

where $A = \mathbf{P}^{\mathrm{N}}\mathbf{P}_{\mathrm{inc}}$ represents the scattering asymmetry, and τ is a second rank tensor having components

$$\tau_{ik} = \mathrm{Sp}\,(\sigma_i M \sigma_k M^*)/\mathrm{Sp}\,(MM^*). \qquad (25)$$

Straightforward triple scattering experiments yield

$$Q^{\mathrm{N}}(\mathbf{N},\tau,\mathbf{N}) = \frac{1}{6}\mathrm{Sp}\,M\sigma_N M^*\sigma_N = \frac{\lambda^2}{3}\Big\{\frac{2}{3}\mathrm{Re}\,[a^*(d - 2\mu)] + \frac{4}{3\sqrt{3}}\mathrm{Re}\,[(f - \nu - b)c^*] + \frac{8}{3\sqrt{3}}\mathrm{Re}\,[g^*(-d + \mu)]$$

$$+ \frac{4}{3\sqrt{3}}[b^*(f - \nu)]\Big\} \qquad Q^{\mathrm{N}}(\mathbf{m},\tau,\mathbf{n}) = \frac{\lambda^2}{3}\sin 2\vartheta\sin\vartheta\cos\beta\Big\{|a|^2 - |h|^2 + \frac{1}{3}(|b|^2 + |c|^2 - |d|^2 + |f|^2) - \frac{4}{3}|g|^2\Big\}$$

$$- \frac{2}{3}\lambda^2\sin 2\vartheta\cos\vartheta\cos\beta\Big\{\mathrm{Re}\Big[\sqrt{\frac{1}{3}}(a^*b + a^*c) + \frac{2}{3}(c^*g + b^*g) + \frac{1}{3\sqrt{3}}(c^*d + b^*d) - \frac{4}{3\sqrt{3}}(b^*\mu + c^*\mu)\Big]\Big\}$$

$$-\frac{2}{9}\lambda^2\sin 2\vartheta\sin\vartheta\cos\beta\Big\{\mathrm{Re}\Big[(a^*d+b^*c+f^*h)+2(a^*\mu-d^*\mu+f^*\nu+h^*\nu)+\frac{2}{\sqrt{3}}(-b^*f+b^*\nu+c^*f-c^*\nu)$$
$$+\frac{4}{\sqrt{3}}(d^*g-g^*\mu)\Big]\Big\}+\frac{2}{3}\lambda^2\sin(\vartheta+\beta)\Big\{\mathrm{Re}\Big[\sqrt{\frac{1}{3}}(a^*c+g^*h)+\frac{1}{3\sqrt{3}}(b^*d+g^*f)-\frac{4}{3\sqrt{3}}(b^*\mu+g^*\nu)$$
$$+\frac{1}{3}\Big(2c^*g+\frac{2}{3}d^*f+\frac{2}{3}d^*\nu+\frac{4}{3}f^*\mu-\frac{1}{3}\mu^*\nu\Big)\Big]\Big\}+\frac{2}{3}\lambda^2\cos(\vartheta-\beta)\Big\{\mathrm{Re}\Big[\frac{1}{3}(a^*d+b^*c+f^*h)+\frac{2}{3}(a^*\mu+h^*\nu)+\frac{2}{3\sqrt{3}}\quad (26)$$
$$\times(b^*\nu-b^*f-c^*f-c^*\nu)+\frac{4}{3\sqrt{3}}(d^*g-g^*\mu)+\frac{2}{9}(d^*\mu-f^*\nu)\Big]\Big\}+\frac{\lambda^2}{9}\cos(\vartheta-\beta)\Big\{2|g|^2+\frac{4}{3}(|d|^2-|f|^2)+\frac{1}{3}(|\mu|^2-|\nu|^2)\Big\}.$$

Making use of the recoil beam instead of the scattered beam, we obtain two new expressions

$$Q^N(\mathbf{N},\tau',\mathbf{N})=\frac{1}{6}\mathrm{Sp}\,MS_NM^*\sigma_N=\frac{2}{9}\lambda^2\Big\{|g|^2+\frac{2}{3}|d|^2+|f|^2-|\mu|^2-|\nu|^2+\mathrm{Re}\,[a^*(d-\mu)-b^*(c+f+\nu)$$
$$+c^*\Big(\frac{2}{\sqrt{3}}f+\frac{1}{\sqrt{3}}\nu\Big)-g^*\Big(\frac{4}{\sqrt{3}}d+\frac{2}{\sqrt{3}}\mu\Big)-f^*\Big(h+\frac{1}{3}\nu\Big)+h^*\nu-d^*\mu]\Big\};\quad Q^N(\mathbf{m},\tau',\mathbf{n})=\frac{\lambda^2}{6}\sin 2\vartheta\sin(\vartheta+\beta)\Big[2|a|^2$$
$$+\frac{2}{3}|b|^2+\frac{2}{3}|c|^2-\frac{2}{9}|d|^2+\frac{2}{9}|f|^2-2|g|^2-2|h|^2-\frac{2}{9}|\mu|^2+\frac{2}{9}|\nu|^2\Big]-\frac{\lambda^2}{3}\sin 2\vartheta\cos(\vartheta+\beta)$$
$$\times\mathrm{Re}\Big[2\sqrt{\frac{1}{3}}a^*b+\frac{4}{3}b^*g+\frac{2}{3\sqrt{3}}c^*d+\frac{4}{3\sqrt{3}}c^*\mu-\frac{1}{3\sqrt{3}}f^*g-\frac{2}{3\sqrt{3}}g^*\nu-\sqrt{\frac{1}{3}}g^*h+\frac{2}{9}f^*\mu+\frac{2}{9}d^*\nu-\frac{2}{9}d^*f$$
$$-\frac{2}{9}\mu^*\nu+\sqrt{\frac{1}{3}}a^*c+\frac{1}{3\sqrt{3}}b^*d+\frac{2}{3\sqrt{3}}b^*\mu+\frac{2}{3}c^*g\Big]+\frac{\lambda^2}{3}\Big[\cos(\vartheta-\beta)-\sin 2\vartheta\sin(\vartheta+\beta)\Big]2\mathrm{Re}\Big[\frac{1}{3}(a^*d-a^*\mu\quad (27)$$
$$+f^*h-h^*\nu+b^*c)+\frac{2}{3\sqrt{3}}(c^*f-b^*f+g^*\mu)+\frac{1}{3\sqrt{3}}(c^*\nu-b^*\nu+2d^*g)+\frac{1}{9}(f^*\nu-d^*\mu)\Big]+\frac{\lambda^2}{3}\sin 2\vartheta\sin(\vartheta+\beta)\frac{4}{9}$$
$$\times\mathrm{Re}\,[f^*\nu-d^*\mu]+\frac{\lambda^2}{3}\Big[\sin(\vartheta-\beta)-\frac{1}{2}\sin 2\vartheta\cos(\vartheta+\beta)\Big]2\mathrm{Re}\Big[\sqrt{\frac{1}{3}}(a^*c+g^*h)+\frac{1}{3\sqrt{3}}(b^*d+g^*f+2b^*\mu+2g^*\nu)+\frac{2}{3}c^*g$$
$$+\frac{2}{9}(d^*f+\mu^*\nu-f^*\mu-d^*\nu)\Big]+\frac{\lambda^2}{3}\Big[\cos(\vartheta-\beta)-\frac{1}{2}\sin 2\vartheta\sin(\vartheta+\beta)\Big]\Big[\frac{4}{9}(|d|^2-|f|^2)+\frac{2}{3}|g|^2-\frac{2}{9}(|\mu|^2-|\nu|^2)\Big].$$

Simultaneous measurements of the polarizations of the scattered beam and the "recoil" beam after triple scattering, yield three further relations.

The correlation function for the polarized beam has the form

$$C^P(\mathbf{p},\mathbf{q})=[C^N(\mathbf{p},\mathbf{q})+\mathbf{Z}(\mathbf{p},\mathbf{q})\,\mathbf{P}_{\mathrm{inc}}]/(1+\mathbf{P}^N\mathbf{P}_{\mathrm{inc}}),\qquad (28)$$

where

$$C^N(\mathbf{p},\mathbf{q})+\mathbf{Z}(\mathbf{p},\mathbf{q})\,\mathbf{P}_{\mathrm{inc}}=\mathrm{Sp}\,M(1+\sigma_1\mathbf{P}_{\mathrm{inc}})M^*\sigma_pS_q/\mathrm{Sp}\,MM^*,$$

and one can find

$$Q^N\mathbf{Z}(\mathbf{m},\mathbf{P})\,\mathbf{N}=\frac{1}{6}\mathrm{Sp}\,M\sigma_NM^*\sigma_mS_p=\frac{2}{3}\lambda^2\,\mathrm{Im}\Big\{\sin(\vartheta+\beta)\cos\vartheta\Big[\sqrt{\frac{1}{3}}(ac+gh^*)-\frac{2}{3\sqrt{3}}(b\mu^*-g\nu^*)+\frac{2}{3}cg^*$$
$$+\frac{2}{9}(d\nu^*-f\mu^*-df^*)-\frac{1}{3\sqrt{3}}(bd^*+fg^*)\Big]+\cos(\vartheta+\beta)\sin\Big[-\frac{1}{3\sqrt{3}}(b\mu^*+c\mu^*)+\frac{1}{3}(cg^*+af^*-a\nu^*-bg^*-dh^*-h\mu^*)$$
$$+\frac{1}{9}(d\nu^*-f\mu^*-4df^*)-\frac{2}{3\sqrt{3}}(bd^*+cd^*)\Big]+\cos(\vartheta+\beta)\cos\vartheta\Big[-\frac{1}{3}(a\mu^*-h\nu^*+ad^*+bc^*+fh^*)+\frac{1}{3\sqrt{3}}(c\nu^*-g\mu^*)$$
$$-\frac{2}{3\sqrt{3}}(cf^*+dg^*)\Big]+\sin(\vartheta+\beta)\sin\vartheta\Big[\frac{1}{3}(-g\mu^*+ad^*+bc^*+fh^*-d\mu^*-f\nu^*+bc^*)+\frac{2}{3\sqrt{3}}(cf^*+dg^*)-\sqrt{\frac{1}{3}}b\nu^*\Big]$$
$$+\cos\beta\Big[-\frac{1}{3}(a\mu^*-h\nu^*)+\frac{1}{3\sqrt{3}}(c\nu^*-g\mu^*)\Big]-\frac{2}{9}\mu\nu^*\sin\beta\Big\},\quad Q^N\mathbf{Z}(\mathbf{N},\mathbf{P})\,\mathbf{n}=\frac{2}{3}\lambda^2\,\mathrm{Im}\Big\{\frac{1}{2}\sin 2\vartheta\sqrt{\frac{1}{3}}(ba^*+b\mu^*+g\nu^*$$
$$-gh^*-ag^*)+\frac{1}{3}(-af^*+a\nu^*+bg^*+cg^*-dh^*-d\nu^*-f\mu^*-h\mu^*-2a\mu^*+bc^*+2d\mu^*+fh^*-2f\nu^*-2h\nu^*)$$
$$+\frac{1}{3\sqrt{3}}(c\mu^*-cd^*-fg^*-g\nu^*-cf^*+dg^*)+\frac{4}{3\sqrt{3}}(c\mu^*-g\nu^*+c\nu^*+g\mu^*)+\frac{2}{3\sqrt{3}}fg^*\Big]+\sin^2\vartheta\Big[\sqrt{\frac{1}{3}}(-ag^*+bh^*$$
$$-b\mu^*-g\nu^*)+\frac{1}{3}(af^*-a\nu^*-bg^*-cg^*+dh^*+d\nu^*+f\mu^*+h\mu^*)+\frac{1}{3\sqrt{3}}(cf^*-dg^*-c\mu^*+g\nu^*)+\frac{2}{3\sqrt{3}}(c\nu^*+g\mu^*$$
$$-cd^*-fg^*)+\frac{2}{3}(d\mu^*-f\nu^*)\Big]+\cos^2\vartheta\Big[\frac{1}{3}(ad^*-bc^*-fh^*)+\frac{2}{3}(a\mu^*+h\nu^*)+\frac{2}{3\sqrt{3}}(cf^*-c\nu^*-dg^*-g\mu^*)$$

$$-\sqrt{\frac{1}{3}}(ab^*+gh^*)-\frac{1}{3\sqrt{3}}(cd^*+fg^*)+\frac{4}{3\sqrt{3}}(c\mu^*-g\nu^*)\Big]\Big\},\quad Q^{\mathrm{N}}\mathbf{Z}(\mathbf{m},\mathbf{N})\mathbf{n}=\frac{\lambda^2}{3}\sin 2\vartheta\sin(\vartheta+\beta)\,\mathrm{Im}\Big\{-\frac{2}{\sqrt{3}}(ab^*+gh^*)$$

$$-\sqrt{\frac{1}{3}}(b\mu^*+c\mu^*)+\frac{1}{3}(-af^*+a\nu^*+bg^*+cg^*-dh^*-h\mu^*)-(d\nu^*+f\mu^*)\Big\}+\frac{\lambda^2}{3}\sin 2\vartheta\cos(\vartheta+\beta)\,\mathrm{Im}\Big\{\sqrt{\frac{1}{3}}(2ag^*$$

$$-2bh^*-b\nu^*-c\nu^*)+\frac{1}{3}(ad^*-a\mu^*-bc^*-h\nu^*-fh^*)-f\nu^*+d\mu^*\Big\}-\lambda^2\Big[\frac{2}{3}\cos(\vartheta-\beta)-\frac{1}{2}\sin 2\vartheta\sin(\vartheta+\beta)\Big]$$

$$\times\mathrm{Im}\Big[\frac{1}{3}(-af^*+a\nu^*+bg^*+cg^*-dh^*+d\nu^*+f\mu^*-h\mu^*)+\frac{1}{3\sqrt{3}}(c\mu^*+2cd^*+2fg^*-4g\nu^*)-\sqrt{\frac{1}{3}}b\mu^*\Big]$$

$$+\lambda^2\Big[\frac{2}{3}\sin(\vartheta-\beta)-\frac{1}{3}\sin 2\vartheta\cos(\vartheta+\beta)\Big]\mathrm{Im}\Big[\frac{1}{3}(-ad^*+a\mu^*+bc^*+d\mu^*$$

$$+fh^*-f\nu^*+h\nu^*)-\frac{1}{3\sqrt{3}}(c\nu^*-2dg^*+2cf^*+4g\mu^*)+\sqrt{\frac{1}{3}}b\nu^*\Big]. \tag{29}$$

These same results can be obtained by using the expression for M given by Eq. (11). In this case, one finds, for example,

$$Q^{\mathrm{N}}=A^2+\frac{2}{3}B^2+\frac{2}{9}C^2+A_1^2+\frac{2}{3}B_1^2+\frac{2}{9}C_1^2+\frac{4}{3}D^2+\frac{4}{3}E^2+\frac{1}{6}F^2+\frac{1}{6}G^2+\frac{1}{6}H^2+\frac{1}{6}K^2,$$

$$Q^{\mathrm{N}}P^{\mathrm{N}}=2\mathrm{Re}\Big\{AA_1^*+\frac{2}{3}BB_1^*+\frac{2}{9}CC_1^*\Big\},$$

$$Q^{\mathrm{N}}(\mathbf{N},\tau,\mathbf{N})=|A|^2+\frac{2}{3}|B|^2+\frac{2}{9}|C|^2+|A_1|^2+\frac{2}{3}|B_1|^2+\frac{2}{9}|C_1|^2 \tag{30}$$

$$-\frac{4}{3}(|D|^2+|E|^2)-\frac{1}{6}(|F|^2+|G|^2+|H|^2+|K|^2).$$

The coefficients A, B, C, ..., are easily obtained from Eq. (11), substituting in it the explicit form of M given in (14) and the average values of the spin operator expressed interms of Eulerian angles; one finds

$$A=\frac{1}{6}\mathrm{Sp}\,M=\frac{\lambda}{3}(a+d+\mu),\qquad A_1=\frac{1}{6}\mathrm{Sp}\,M\sigma\mathbf{N}=$$

$$B=\frac{1}{4}\mathrm{Sp}\,MSN=\qquad =\lambda\frac{i}{3}\Big[\sqrt{\frac{1}{3}}(c-b)+\frac{2}{3}f+\frac{1}{3}\nu\Big],$$

$$=\frac{i}{2}\lambda\Big[\sqrt{\frac{1}{3}}(c-b)+\frac{2}{3}(f-\nu)\Big],\qquad B_1=\frac{\lambda}{3}[(d-\mu)-\sqrt{3}g],$$

$$C=\frac{3}{4}\mathrm{Sp}M\Big(S_PS_P+S_KS_K-\frac{4}{3}\delta_{ij}\Big)=\qquad C_1=\frac{3}{4}\mathrm{Sp}M\sigma\mathbf{N}\Big(S_PS_P+S_KS_K-\frac{4}{3}\delta_{ik}\Big)=$$

$$=\frac{3}{4}\lambda\Big[\frac{1}{3}(a-d)+\frac{2}{\sqrt{3}}g\Big],\qquad =\frac{3}{4}i\lambda\Big[\frac{1}{3\sqrt{3}}(c-b)-\frac{7}{9}f+h-\frac{8}{9}\nu\Big],$$

$$D=\frac{1}{8}\mathrm{Sp}\,M(\sigma_PS_P+\sigma_KS_K)=\frac{\lambda}{4}\Big(a+\frac{1}{3}d+\frac{2}{\sqrt{3}}g-\frac{4}{3}\mu\Big),$$

$$E=\frac{1}{8}\mathrm{Sp}\,M(\sigma_PS_P-\sigma_KS_K)=\frac{\lambda}{4}\Big\{\Big(a-d-\frac{2}{\sqrt{3}}g\Big)\cos 2\vartheta+\frac{2}{\sqrt{3}}(b+c)\sin 2\vartheta\Big\}=\frac{\lambda}{4}\frac{1}{\sin 2\vartheta}\frac{2}{\sqrt{3}}(b+c), \tag{31}$$

$$F=\frac{1}{2}\mathrm{Sp}\,M(\sigma_PS_PS_N+\sigma_KS_KS_N)=\frac{\lambda i}{\sqrt{2}}\Big\{\sqrt{\frac{1}{6}}(c-b)-\frac{1}{3\sqrt{2}}f+\sqrt{\frac{1}{2}}h+\frac{2\sqrt{2}}{3}\nu\Big\},$$

$$G=\frac{1}{2}\mathrm{Sp}\,M(\sigma_PS_NS_P+\sigma_KS_NS_K)=\frac{\lambda i}{\sqrt{2}}\Big\{\sqrt{\frac{1}{6}}(c-b)-\frac{1}{3\sqrt{2}}f+\sqrt{\frac{1}{2}}h+\frac{2\sqrt{2}}{3}\nu\Big\},$$

$$H=\frac{1}{2}\mathrm{Sp}\,M(\sigma_PS_PS_N-\sigma_KS_KS_N)=\frac{\lambda i}{\sqrt{2}}\Big\{\sin 2\vartheta\Big(\sqrt{\frac{1}{2}}a-\sqrt{\frac{2}{3}}g-\sqrt{\frac{1}{2}}d\Big)+\cos 2\vartheta\Big[-\sqrt{\frac{3}{2}}b-\sqrt{\frac{1}{6}}c-\sqrt{\frac{1}{2}}(f+h)\Big]\Big\}.$$

$$K=\frac{1}{2}\mathrm{Sp}\,M(\sigma_PS_NS_P-\sigma_KS_NS_K)=\frac{\lambda i}{\sqrt{2}}\Big\{\sin 2\vartheta\Big(-\sqrt{\frac{1}{2}}a+\sqrt{\frac{2}{3}}g+\sqrt{\frac{1}{2}}d\Big)+\cos 2\vartheta\Big[\sqrt{\frac{3}{2}}c+\sqrt{\frac{1}{6}}b-\sqrt{\frac{1}{2}}(f+h)\Big]\Big\}.$$

With these expressions for the coefficients, one can use Eq. (30) to obtain the previous results for the cross section, polarization and $Q^{\mathrm{N}}(\mathbf{N},\tau,\mathbf{N})$.

Thus, for given values of energy and angle, the results of double and triple scattering experiments can yield 11 independent relations among the parameters of the transition matrix in the case of elastic scattering of neutrons against deuterons. One additional relation can be obtained from experiments on the scattering of protons against deuterons, since due to charge invariance, all the above results also apply to this case (if one only includes the electromagnetic interaction).

As shown in the work of Smorodinskii and others,[5] the transition matrix is determined by as many real functions as there are variables in its most general formulation. Thus, if one carries out triple scattering experiments and obtains the above mentioned experimental data, it should be possible to carry out a phase shift analysis.

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NUCLEAR FORCES AND THE SCATTERING OF π MESONS

IU. V. NOVOZHILOV

Leningrad State University

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The interaction of two fixed nucleons in pseudoscalar meson theory with pseudovector coupling is considered. The principal part of the functional of the two-nucleon state is represented in the form of a product of single-nucleon functionals. Consideration is given only to the states without real mesons and with one real meson. A procedure is developed for reducing two-nucleon renormalized matrix elements to single nucleon elements, which are then calculated by the method of Chew, Low, and Wick. The potential of order e^{-2R} is calculated. It consists of two parts: one part is proportional to f^4 (f is the interaction constant), and the other is a function of the phases of π meson scattering on nucleons.

RECENTLY, Chew and Low[1] and Wick[2] considered the one-nucleon problem from a new point of view. Characteristic of their approach is the attempt to solve the problem without perturbation theory, and thus to deal only with renormalized quantities.

In considering the two-nucleon problem in the region of nonrelativistic energies, it may be assumed that the meson clouds of the interacting nucleons conserve their individuality.

Therefore, we may feel confident that in this energy region, quantities referring to two interacting nucleons will be expressed by single-nucleon quantities, so that the method of Chew, Low, and Wick may

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ERRATA TO VOLUME 6

Page	Line	Reads	Should Read
643	16 from bottom	where $\kappa = \pi a^2 \Omega - \ldots$	where $\kappa = \pi a^2 \Omega\varphi - \ldots$
690	8 from bottom	$\ldots \sin [- \ldots$	$\ldots \sin \vartheta [- \ldots$
	5 from bottom	$\ldots \sin 2\vartheta \sqrt{\frac{1}{3}} \ldots$	$\ldots \sin 2\vartheta \left[\sqrt{\frac{1}{3}} \ldots \right.$
809	9 from top	$\ldots \left(\frac{1}{2 \sinh u} + \ldots \right.$	$\ldots \left(\frac{1}{\sinh u} + \ldots \right.$
973	unnumbered equation	$\ldots C^{S'-\mu'; S\mu}_{n\mu-\mu'} T^{(n)}_{\mu'-\mu'}.$	$\ldots C^{S'-\mu'; S\mu}_{n\mu-\mu'} < S' \Vert T^{(n)} \Vert S^{-1} > \times T^{(n)}_{\mu'-\mu}.$
975	5 from bottom	...of a particle by a nucleus ...	...of a particle in state a by a nucleus ...
992	Eq. (18)	$\ldots \tau_1 \tau_2^{-2}/2\hbar^1 \ldots$	$\ldots \tau_1 \tau_2^{-1}/2\hbar^2 \ldots$