In lead ($Z = 82$) the ratio $\overline{\cos^2 \vartheta_N}/\overline{\cos^2 \vartheta_{NL}}$ equals 0.99 and 0.98 for $E = 3 \times 10^7$ ev and 1.5×10^7 ev respectively.

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EFFECT OF DAMPING ON POLARIZATION OF DIRAC PARTICLES IN SCATTERING

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THE elastic scattering of both Dirac and spinless particles by a fixed force center was investigated in Refs. 1 to 4 using damping theory. We calculate below, using damping theory, the polarization resulting from elastic scattering of Dirac particles.

The fundamental integral equation of damping theory which determines the scattering amplitude $f'_{s'} \equiv f'_{s'}(\mathbf{k}')$ and is relevant in a discussion of polarization phenomena has the following form (we use the notation of Ref. 3):

$$(f'_{s'} - b'^{+}_{s'} b_s f_s) V_{\mathbf{k}'\mathbf{k}} = \frac{kK}{8\pi^2 c\hbar i} \sum_{s''} \oint d\Omega'' V_{\mathbf{k}'\mathbf{k}''} V_{\mathbf{k}''\mathbf{k}} b'^{+}_{s'} b''_{s''} f''_{s''}, \tag{1}$$

Here $E = c\hbar K$ is the total energy of the particle and $V_{\mathbf{k}'\mathbf{k}''}$ is the Fourier component of the potential $V(\mathbf{r})$.

We shall restrict ourselves to calculating the polarization resulting from elastic scattering of Dirac particles by a delta-function potential $V(\mathbf{r}) = V_0 \delta(\mathbf{r})$, $V_{\mathbf{k}'\mathbf{k}''} = V_0$. In that case we have from formulas (5) of Ref. 3:

$$b'^{+}_{s'} b''_{s''} = \sum_{j=1,2} h^j_{s's''}, \tag{2}$$

where

$$h^1_{s's''} = \frac{1}{2}\left(1 + \frac{k_0}{K}\right)[\cos\theta'_{s'} \cos\theta''_{s''} + e^{i(\varphi''-\varphi')} \sin\theta'_{s'} \sin\theta''_{s''}] \qquad h^2_{s's''} = \frac{1}{2}\left(1 - \frac{k_0}{K}\right) s's''[\cos\theta'_{s'} \cos\theta''_{s''} + e^{i(\varphi''-\varphi')} \sin\theta'_{s'} \sin\theta''_{s''}].$$

We seek a solution of integral equation (1) of the form ($s' = 1, -1$):

$$f'_{s'} = \sum_{j=1,2} \varepsilon_j h^j_{s's} f_s. \tag{3}$$

Taking into account the orthogonality condition for $h^j_{s's''}$ [see Eq. (31) of Ref. 3] we obtain for ϵ_1 and ϵ_2 from Eqs. (1)–(3):

$$\varepsilon_{1,2} = \frac{1}{1 + i\delta_{1,2}}, \quad \delta_{1,2} = \frac{V_0}{4\pi c\hbar} k(K \pm k_0). \tag{4}$$

From Eqs. (3) and (4) we obtain for the amplitudes $f'_{s'}$ of the first scattering

$$\begin{aligned} f'_1 &= \frac{1}{2}[a_1\varepsilon_1 + a_2\varepsilon_2] \cos\frac{\theta'}{2} f_1 - \frac{1}{2}[a_1\varepsilon_1 - a_2\varepsilon_2] e^{-i\varphi'} \sin\frac{\theta'}{2} f_{-1}, \\ f'_{-1} &= \frac{1}{2}[a_1\varepsilon_1 - a_2\varepsilon_2] \sin\frac{\theta'}{2} f_1 + \frac{1}{2}[a_1\varepsilon_1 + a_2\varepsilon_2] e^{-i\varphi'} \cos\frac{\theta'}{2} f_{-1}, \end{aligned} \tag{5}$$

where $a_1 = 1 + k_0/K$, $a_2 = 1 - k_0/K$, and θ', φ' are the angles corresponding to the first scattering.

The expression for the second scattering amplitude $f''_{s''}(f''_1, f''_{-1})$ is obtained by replacing θ', φ' and f_1, f_{-1} on the right side of Eq. (5) by θ'', φ'' and f'_1, f'_{-1}. The effective differential cross section for the second scattering is given in terms of $f'_{s'}$ and $f'_{s''}$ by the relation

$$d\sigma = \frac{1}{N}\frac{\partial}{\partial t}\sum_{\mathbf{k}'',s''} C''^{+}_{s''} C''_{s''} = \frac{K^2}{4\pi^2 c^2\hbar^2}|V_{\mathbf{k}''\mathbf{k}'}|^2 \frac{f''^{+}_1 f''_1 + f''^{+}_{-1} f''_{-1}}{f'^{+}_1 f'_1 + f'^{+}_{-1} f'_{-1}} d\Omega'', \tag{6}$$

where

$$d\Omega'' = \sin\theta'' d\theta'' d\varphi'', \quad k' = k'' = k.$$

From here, using Eqs. (4) and (5), we obtain the following expression for the effective differential cross section for the double scattering of an initially unpolarized beam of Dirac particles ($f_1^+ f_1 + f_{-1}^+ f_{-1} = 1$, $f_1^+ f_1 - f_{-1}^+ f_{-1} = 0$, $f_1^+ f_{-1} = f_{-1}^+ f_1 = 0$) by a delta-function potential:

$$\sigma(\theta'', \varphi'') = \sigma_0(\theta'')[1 + \delta(\theta', \theta'')\cos\varphi''], \tag{7}$$

where

$$\delta(\theta', \theta'') = P(\theta') P(\theta'') = \left(\frac{V_0^2 K^2}{4\pi^2 c^2 \hbar^2}\right)^2 \frac{k^4(\delta_2 - \delta_1)^2 \sin\theta' \sin\theta''}{4K^4(1+\delta_1^2)^2(1+\delta_2^2)^2 \sigma_0(\theta')\sigma_0(\theta'')}, \quad \cos\varphi'' = \mathbf{n}'\mathbf{n}''. \tag{8}$$

$P(\theta')$ and $P(\theta'')$ are the polarizations acquired by an initially unpolarized beam separately in the first and second scattering, $\mathbf{n}'$ and $\mathbf{n}''$ are unit vectors in the two polarization directions, and $\sigma_0(\theta')$ is the differential scattering cross section, with damping taken into account, of an initially unpolarized beam of Dirac particles indicent on a delta-function potential

$$\sigma_0(\theta') = \frac{V_0^2 K^2}{16\pi^2 c^2 \hbar^2}\left[\frac{a_1^2}{1+\delta_1^2} + \frac{a_2^2}{1+\delta_2^2} + 2\frac{k^2}{K^2}\cdot\frac{1+\delta_1\delta_2}{(1+\delta_1^2)(1+\delta_2^2)}\cdot\cos\theta'\right].$$

It follows from Eq. (8) that no polarization is obtained in the first approximation of perturbation theory ($\delta_1 = \delta_2 \cong 0$). In the high-energy region $k \gg (4\pi c\hbar/V_0)^{1/2}$ ($\delta_1^2 \gg 1$, $\delta_2^2 \gg 1$) we obtain for the degree of polarization in the case of two identical scatterings [$\theta'' = \theta'$, $P(\theta'') = P(\theta')$]

$$\delta(\theta', \theta') = [P(\theta')]^2 = \left(\frac{4\pi c\hbar}{V_0}\right)^2 \frac{4k_0^2}{k^6}\tan^2\frac{\theta'}{2}. \tag{9}$$

As can be seen from Eq. (7) the assymmetry in the second scattering is due only to the phase polarization ($f_1'^{+} f'_{-1} \neq 0$) resulting from the first scattering. In the case when the incident beam possesses only amplitude polarization, $f_1^+ f_1 - f_{-1}^+ f_{-1} \neq 0$, $f_1^+ f_{-1} = f_{-1}^+ f_1 = 0$, the first Born approximation gives for the effective scattering cross section

$$d\sigma_{ss'} = \frac{K^2}{4\pi^2 c^2 \hbar^2}|V_{\mathbf{k}'\mathbf{k}}|^2 \frac{1}{2}\sum_{s=\pm 1}\left[(1+ss')\cos^2\frac{\theta'}{2} + \frac{k_0^2}{K^2}(1-ss')\sin^2\frac{\theta'}{2}\right] f_s^+ f_s\, d\Omega'. \tag{10}$$

Thus the ratio of the non-spin-flip cross section ($ss' = 1$) to the spin-flip cross section ($ss' = -1$) is equal to

$$\frac{d\sigma^{\uparrow\uparrow}}{d\sigma^{\uparrow\downarrow}} = \frac{K^2}{k_0^2}\cot^2\frac{\theta'}{2}.$$

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