in agreement with our previous estimate.[4] For deuterium we obtain a correction of 0.74 + 0.117 = 0.86 Mc/sec plus $\delta_n = (\alpha/3)|\psi(0)|^2 \langle r_n^2 \rangle$ Mc/sec, which also improves the agreement with experiment.

The isotopic volume effect, $\Delta E_D - \Delta E_H$, is equal to 0.74 Mc/sec + δ_n, i.e., together with the mass effect it amounts to 1.33 Mc/sec + δ_n, which is in good agreement with the measurements[6] of Lamb (1.23 ± 0.20 Mc/sec). Before drawing final conclusions about the magnitude of the Lamb shift, however, it would be desirable to ascertain to what extent possible corrections, e.g., higher-order quantum-electrodynamic terms, might modify these results.

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[1] E. E. Chambers and R. Hofstadter, Phys. Rev. 103, 1454 (1956).
[2] Melkonian, Rustad, and Havens, Bull. Am. Phys. Soc. Ser. II, 1, 62 (1956); Hughes et al., Phys. Rev. 90, 407 (1953).
[3] J. McIntyre, Phys. Rev. 103, 1464 (1956).
[4] D. D. Ivanenko and N. N. Kolesnikov, Dokl. Akad. Nauk SSSR 91, 47 (1953).
[5] E. Salpeter, Phys. Rev. 89, 92 (1953).
[6] Triebwasser, Dayhoff, and Lamb, Phys. Rev. 89, 98 (1953).
[7] J. Blatt and V. Weisskopf, Theoretical Nuclear Physics, New York, 1952.
[8] J. Blatt and J. D. Jackson, Phys. Rev. 76, 18 (1949).

Translated by J. Heberle
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APPLICATION OF CHARGE INVARIANCE TO POLARIZATION PHENOMENA

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As is well known, the hypothesis of charge invariance leads to relations between experimentally observable quantities. Up to the present time, however, the only relations that have been derived connect the cross-sections of different processes. In connection with experiments on change of polarization it is also of interest to examine the relations involving the polarization which follow from charge invariance. We shall consider an extremely simple method for finding such relations.

Suppose that the isotopic spin is conserved in the interactions that cause the process $a + A \rightarrow b + B$. Let us denote the respective isotopic spins of the particles by j_a, j_A, j_b, j_B, and the values of a particular component by m_a, m_A, m_b, m_B. The amplitude describing the transition $m_a, m_A \rightarrow m_b, m_B$ can be written in the form

$$R_{m_a m_A;\, m_b m_B} = \sum_j (j_a j_A m_a m_A \mid j_a j_A j m)\, R_j\, (j_b j_B m_b m_B \mid j_b j_B j m). \quad (1)$$

by the use of the Clebsch-Gordan coefficients. Here R_j is the amplitude in a state with a definite total isotopic spin; it depends on the angles, the spins, and the energy.

Let T_k be operators acting on the spin variables of particles b and B and forming a complete set of matrices (for example, 1 and the Pauli matrices σ_x, σ_y, σ_z in the case of a spinless particle and a particle with spin ½). The experimentally observable quantities are average values $\langle T_k \rangle$. From Eq. (1) we have:

$$(\langle T_k \rangle \sigma)_{m_a m_A;\, m_b m_B} = \sum_{j, j_1} (j_a j_A m_a m_A \mid j_a j_A j m)(j_a j_A m_a m_A \mid j_a j_A j_1 m)\, R_j^+ T_k R_{j_1} (j_b j_B m_b m_B \mid j_b j_B j m)(j_b j_B m_b m_B \mid j_b j_B j_1 m), \quad (2)$$

where σ is the differential cross-section. To obtain the relations we sum Eq. (2) over the isotopic spin components, keeping one of them fixed, for example m_b. Using the orthogonality and symmetry of the Clebsch–Gordan coefficients[1] we get

$$\sum_{m_a, m_A, m_B} (\langle T \rangle \sigma)_{m_a m_A;\, m_b m_B} = \sum_j \frac{(2j+1)}{(2j_b+1)} R_j^+ T_k R_j. \tag{3}$$

The sum on the left side does not depend on m_b. Consequently, equating the values of this sum for various values of m_b, we get relations between the observable quantities. It is obvious that any one of the components can be held fixed. It is clear from the proof that in the case in which the number of particles changes, relations between the observable quantities are obtained in just the same way. For $T_k = 1$ we get relations between the differential cross-sections, and the conclusion so obtained provides a foundation for the rule formulated by Shmushkevich.[2]

Let us consider the scattering of π-mesons by nucleons. Equating the sums (2) for π^+ and π^0-mesons we get besides Heitler's relations for the differential cross-sections, the relation

$$(P\sigma)_{\pi^+p;\,\pi^+p} + (P\sigma)_{\pi^-p;\,\pi^-p} = 2\,(P\sigma)_{\pi^0p;\,\pi^0p} + (P\sigma)_{\pi^-p;\,\pi^0n}, \tag{4}$$

which involes the polarization P of the recoil nucleon.

For the production of a π-meson in collision of nucleons with formation of a deuteron we have:

$$\langle T_k \rangle_{pp;\,\pi^+d} = \langle T_k \rangle_{np;\,\pi^0d}\,. \tag{5}$$

T_k characterizes the polarization of the deuteron (3 components of a vector and 5 components of a tensor). If the nucleons in the final state are free, we get

$$(\langle T_k\rangle \sigma)_{pp;\,\pi^+np} + (\langle T_k\rangle \sigma)_{pp;\,\pi^+pn} + (\langle T_k\rangle \sigma)_{np;\,\pi^+nn} + (\langle T_k\rangle \sigma)_{np;\,\pi^-pp}$$
$$= 2\,(\langle T_k\rangle\sigma)_{pp;\,\pi^0pp} + 2(\langle T_k\rangle \sigma)_{np;\,\pi^0np} + 2(\langle T_k\rangle \sigma)_{np;\,\pi^0pn}.$$

In conclusion we remark that the relations (3) are very convenient for expressing the cross-sections and polarizations in states with definite total isotopic spin in terms of the experimentally observable cross-sections and polarizations.

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[1] J. Blatt and V. Weisskopf, Theoretical Nuclear Physics, p. 791 (New York, 1952).
[2] I. Shmushkevich, Dokl. Akad. Nauk SSSR **103**, 235 (1955).

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THERMODYNAMIC FUNCTIONS OF SUPERFLUID HELIUM FILMS

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IT is well known that a calculation of the thermodynamic functions of superfluid helium with an excitation spectrum of the phonon-roton type proposed by Landau gives excellent agreement with experimental data.[1] In liquid helium which has a free surface, however, there exists one more branch of the energy spectrum associated with the presence of surface waves. Its contribution to the thermodynamic functions is proportional to the area of the free surface and consequently it can play a role only for very thin films.

The spectrum of the surface oscillations has the form[2]