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CLASSIFICATION OF NUCLEAR MAGNETIC MOMENTS

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It is proposed that the region of maximum concentration of the nuclear magnetic moment values be delimited by introducing, along with the Schmidt and Dirac lines, new lines as boundaries of these regions. Empirical data are presented as a proof of the real significance of these lines. It is noted that the quenching of the anomalous part of the nucleon magnetic moment in nuclear matter, which takes place in addition to that from deformation of the nucleus, is, apparently, more important for the formation of nuclear magnetic moments than usually considered.

THE values of magnetic moments are usually divided into two groups—those near the upper and lower Schmidt lines.[1] Together with these, Dirac lines have been proposed.[2] A more detailed classification of the values of μ is both possible and necessary, as follows from the following considerations.

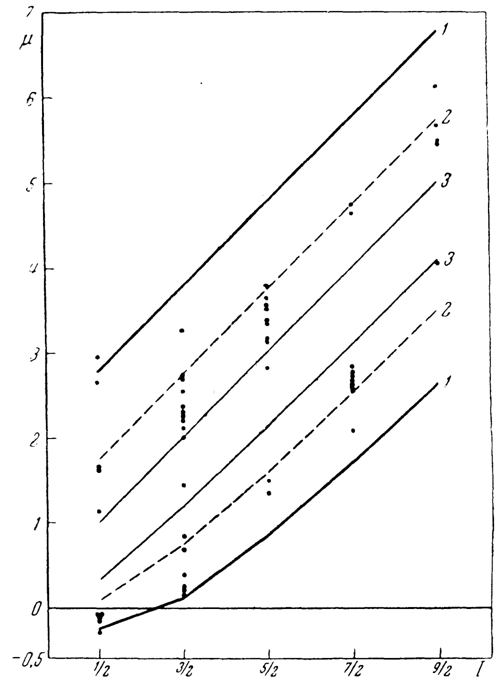

FIG. 1. Classification of magnetic moments of nuclei with odd Z and A. (1) Schmidt line, (2) line a ($\Delta g_s/2 =$ 1.04 n.m.), (3) Dirac line.

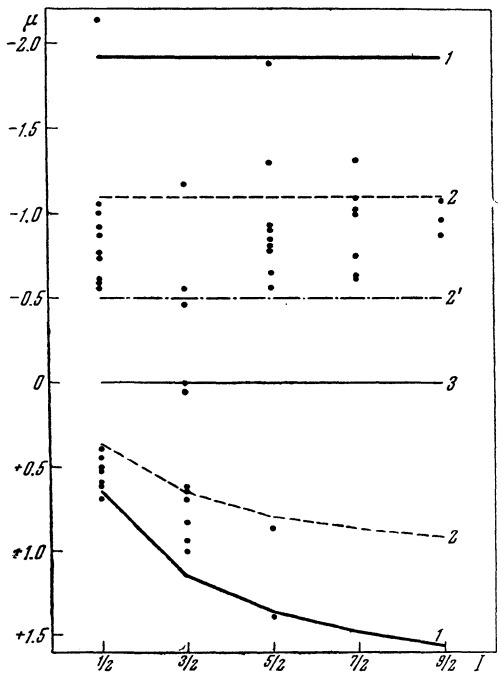

FIG. 2. Classification of magnetic moments of nuclei with odd N and A. (1) Schmidt line, (2) line a ($\Delta g_s/2 =$ 0.8 n.m.), (2') line b ($\Delta g_s/2 = 1.4$ n.m.) (3) Dirac line.

1. For $I = \ell + \frac{1}{2}$ (for both odd Z and for odd N) the overwhelming majority of values of μ, as is evident from the table and Figs. 1 and 2, are concentrated in a comparatively narrow strip, running parallel to the Schmidt line but remaining a considerable distance from it. Only a few values of μ lie between this strip and the Schmidt line. This is one of the reasons to separate, for $I = \ell + \frac{1}{2}$, a group of values constituting this strip, on the one hand, and a group of values lying between this strip and the upper Schmidt line, on the other. We designate the line Aa as the boundary between these groups. The Dirac line is the other limit of the strip of maximum concentration of values of μ for $I = \ell + \frac{1}{2}$ for nuclei with odd Z. For nuclei with odd N and with $I = \ell + \frac{1}{2}$, one should draw a special line, which, as is shown by values of μ for spins $\frac{1}{2}$, $\frac{3}{2}$ and $\frac{5}{2}$, should pass at a height of approximately -0.5 nuclear magnetons (n.m.), parallel to the Schmidt line, at a distance of approximately 1.4 n.m. from it. We designate it as the line Ab. In connection with this it is possible for $I = \ell + \frac{1}{2}$ to distinguish in the area between the Schmidt line and Dirac line (for odd Z—the lower Dirac

TABLE. Deviations of the value of the nuclear magnetic moment from the Schmidt line, calculated as changes in the $g_S/2$-factor (in nuclear magnetons)

Odd Z

	Group	Line	1/2 Z	1/2 $\Delta g_s/2$	3/2 Z	3/2 $\Delta g_s/2$	5/2 Z	5/2 $\Delta g_s/2$	7/2 Z	7/2 $\Delta g_s/2$	9/2 Z	9/2 $\Delta g_s/2$
$I = l + {}^1/_2$	A-1-Z	Line *Aa* ($\Delta g_s/2$=1.04)	1	−0.19								
			9	0.16								
					3	0.53						
									23	0.64	41	0.63
	A-2-Z		81	1.16	37	1.04	59	0.99	21	1.03	43	1.11
			81	1.18	5	1.10	13	1.15	27	1.14		
					31	1.23					49	1.29
							55	1.31			49	1.30
							25	1.32				
							63	1.39				
					29	1.41	51	1.43				
					35	1.52						
					29	1.56						
					11	1.57	75	1.61				
			15	1.66	35	1.68	75	1.65				
		Upper Dirac Line ($\Delta g_s/2$ =1.79)			31	1.77						
	A-3-Z						53	1.98				
		Lower Dirac Line ($\Delta g_s/2$=1.79)			33	2.35						
$I = l - {}^1/_2$	B-2-Z	Line *Ba* ($\Delta g_s/2$=1.04)									83	1.78
									55	1.44		
									57	1.35		
									55	1.30		
									53	1.16		
					17	1.16			71	1.13		
									55	1.11		
									51	1.05		
	B-1-Z				17	0.92						
							63	0.89				
							37	0.69				
			45	0.46	19	0.44			72	0.49		
			47	0.46								
			47	0.40								
			39	0.37	19	0.14						
					77	0.12						
			7	−0.05	79	0.02						

Odd N

	Group	Line	1/2 N	1/2 $\Delta g_s/2$	3/2 N	3/2 $\Delta g_s/2$	5/2 N	5/2 $\Delta g_s/2$	7/2 N	7/2 $\Delta g_s/2$	9/2 N	9/2 $\Delta g_s/2$
$I = l + {}^1/_2$	A-1-N	Line *Aa* ($\Delta g_s/2$= 0.8)	1	−0.22								
							9	0.02				
							51	0.61	23	0.60		
					5	0.73						
	A-2-N		69	0.86					27	0.81	49	0.82
			67	0.91			55	0.98	45	0.90		
			65	0.99			53	1.00	83	0.91	47	0.94
			73	1.03			13	1.06			41	1.03
			75	1.14			143	1.11				
			71	1.18			25	1.12	85	1.15		
			65	1.29					87	1.27		
			63	1.32			103	1.26	85	1.29		
		Line *Ab* , ($\Delta g_s/2$=1.4)	15	1.36	121	1.35	59	1.34				
	A-3-N				29	1.44						
					33	1.91						
		Dirac Line ($\Delta g_s/2$=1.91)			31	1.96						
$I = l - {}^1/_2$	B-2-N											
	B-1-N	Line *Ba* ($\Delta g_s/2$=0.8)			17	0.84						
					113	0.83						
					77	0.76						
			145	0.7			37	0.69				
			101	0.56	79	0.53						
			119	0.41								
			43	0.32	81	0.34						
					19	0.24						
			125	0.14								
			117	0.08								
			7	−0.19			147	−0.07				

line) three groups of values of μ, which we call A-1-Z, A-2-Z, A-3-Z, A-1-N, A-2-N, A-3-N, where A serves as a designation of parallel orientation of the spin and orbital moments. Here the group A-3-Z coincides with the "forbidden zone" between upper and lower Dirac lines.

The moments for the nuclei with Z = 33 and 53 lie in the "forbidden zone." The magnitude of the magnetic moment of the nucleus As^{75} with Z = 33 is nearer to the lower Schmidt line than to the upper. However, with respect to its spin ($^3/_2$) this nucleus does not differ from the series of nuclei preceding and following it, whose spins and orbital moments are parallel. Also, from the point of view of the shell model, there is no basis to propose the state $d_{3/2}$ in this case. The nucleus I^{127} with Z = 53 has the same spin $^5/_2$ as the nucleus Sb^{121} with Z = 51 and $I = \ell + ^1/_2$. Therefore the magnetic moments of the "forbidden zone" can be attributed to group A.

For $I = \ell - ^1/_2$ the deviations of values of μ from the Schmidt line are, on the average, considerably smaller than for $I = \ell + ^1/_2$, and the majority of these values are concentrated in a strip bounded, on one side, by the Schmidt line, and on the other by the line Ba, which is analogous in nature to the line Aa (B designates an antiparallel orientation of spin and orbital moments). For $I = \ell - ^1/_2$ the groups B-1-Z and B-1-N can be distinguished between the Schmidt line and the line Ba, and the groups B-2-Z and B-2-N within the line Ba and the Dirac line (for odd Z — the lower Dirac line).

2. From Figs. 1 and 2 and the table it is clear that for $I = \ell + ^1/_2$ the values of μ which form the upper limit of each of the groups A-2, almost all deviate from the Schmidt line. For odd Z and $I = \ell + ^1/_2$ they show precisely the same dependence on nuclear spin as the upper Schmidt line, forming the series 1.6, 2.8, 3.8, 4.8, 5.7 n.m. For odd N the difference between the Schmidt numbers and the values of μ which form the upper boundary of the group A-2 remains almost constant with change in I. Since this difference for both odd Z and odd N does not exhibit any appreciable dependence on the magnitude of ℓ and I, it is possible that it comes from the change of only the spin part of the magnetic moment $g_s/2$, on account of its deviation from the magnitude of the magnetic moment of a free nucleon.

Thus, for the value of μ forming the upper limit of each of the groups A-2, we can take, to a good approximation, the formula

$$\mu = g_l l + g_{s1} s, \tag{1}$$

representing a Schmidt formula, with the part relating to the spin moment changed. Here it is possible to take for protons $g_\ell = 1$, $g_{s1}/2 = g_s/2 - 1.04$ n.m. and for neutrons $g_\ell = 0$, $g_{s1}/2 = g_s/2 + 0.8$ n.m. There is an analogous line for $I = \ell - ^1/_2$. Its formula is

$$\mu = g_l (l+1)(2l-1)/(2l+1) - g_{s1}(2l-1)/2(2l+1). \tag{2}$$

The line Ba, representing this formula, for odd N coincides sufficiently closely with the values of μ which deviate most from the Schmidt line. It is important that the value of $\Delta g_s/2$ for the lines Aa and Ba is the same. For odd N one thus observes the coincidence of maximum deviation of $g_s/2$ for $I = \ell - ^1/_2$ with the minimum of mass deviations for $I = \ell + ^1/_2$. For odd Z the line Ba for $I = ^1/_2, ^3/_2, ^5/_2$ is near to the maximum limit of the deviations, and for $I = ^7/_2$ it coincides accurately with their minimum limit. The lines Aa and Ba for odd Z are somewhat further from the Schmidt line than for odd N, as is to be expected.

3. The nuclei whose moments enter into groups A-1-Z and A-1-N are characterized by several particularities in their shell structure; almost all of them have only one nucleon above a closed orbital term* (or above a doubly-magic core), whereas all relate to the first half of the periodic system where, following the closing of the orbital term, a much greater change occurs in the energy level than in the second half [after closing a sub-orbital term the deviation from the Schmidt line is almost always very large, that is, after (a) 14, 28 or 82 protons or neutrons; (b) 50 protons; (c) 126 neutrons, especially in the Bi^{209} nucleus, after the simultaneous closing of two sub-orbital terms]. It is interesting that of nine values of μ relating to group A-1, the deviations of the four which differ most from the Schmidt lines are, for both odd Z and odd N, practically the same and near to 0.6 n.m.

4. Mayer and Jensen,[3] in considering the appearance of isomers, indicate that the magnitude of the matrix element for the M-4 transition can be brought into agreement with experiment if, in the relevant equations, the value 0.8 n.m. is substituted for μ_p rather than its value for a free nucleon, and the value

*By orbital term we understand the totality of adjacent nucleons of the same type having the same ℓ; by sub-orbital, having the same j.

0.6 n.m. for μ_N. The magnitudes of these are near, for protons, to one of the Dirac lines, and for neutrons, to one of the lines proposed by us (Ab).

We have justification to expect that in the future other phenomena will be found confirming the real significance of both the Dirac line and the line proposed by us.

It would appear that the simplest and most natural explanation of the characteristic properties of the lines proposed by us could be given by the hypothesis[2,4,5] about the quenching of the anomalous part of the magnetic moment of the unpaired nucleon in nuclear matter. (We note that acceptance of the theory of quenching means acceptance of the single-particle model only as applied to the spin, and not to the orbital part of the nuclear magnetic moment.) In several cases the influence of the quenching appears very clearly, for example, in relation to μ of Bi^{209}, the value of which (4.08 n.m.) almost exactly coincides with the Dirac line (4.09 n.m.). We add that the value of μ for Bi^{209} cannot be explained either from the theory of $j-j$ coupling between proton and neutrons,[6] according to which there should be no deviation from the Schmidt line in the given case, nor by the collective model,[7] in view of the small value of the quadrupole moment of this nucleus. The calculations of Blin-Stoyle[8] give only a very ill-defined value for the deviation, 0.81—1.68 n.m., and the calculations of Arima and Horie[9] a value of 0.81 n.m., with an overall deviation of 1.46 n.m.

Together with the coincidence noted above of the deviations for four values of μ which are part of the groups A-1-Z and A-1-N, it is impossible to believe such facts to be purely coincidental as, for example, the exact agreement of the deviations of the pair of nuclei $_{37}Rb_{48}$ and $_{30}Zn_{37}$ if a change in the g_S-factor is taken as the deviation. It is characteristic that just these two nuclei differ from all preceding odd nuclei also by other particularities: a spin $5/2$ in place of $3/2$ for $I = \ell - 1/2$ in place of $I = \ell + 1/2$. Together with this, in the majority of other cases it is necessary to combine the theories of quenching and deformation. Deformation has a particularly great influence on the values of μ in group A-3. However, the influence of quenching apparently is none the less more important than many authors have assumed (see Ref. 10 and others) and it is possible that their ideas concerning this question should be considered only as evidence of the incompleteness of contemporary concepts about nuclear properties. This would signify that the theory of quenching needs further development on a substantially broader basis.

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