DIFFERENTIAL CROSS SECTION FOR PHOTOPRODUCTION OF π-MESONS ON NUCLEONS TAKING ACCOUNT OF d-STATES

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1. A wealth of experimental data has been obtained recently on the angular distribution of mesons* produced by photons on nucleons (cf. for example Refs. 1 and 2). To interpret the experimental data it is necessary to have a theoretical expression for the differential cross section $\sigma_p(\vartheta)$ for photoproduction of mesons on nucleons. Since there is, at the present time, no consistent analysis by which this problem can be examined, special interest attaches to those methods which are based on general considerations. Using two such relations — conservation of momentum and conservation of parity — expressions for $\sigma_p(\vartheta)$ have been obtained by various methods in several papers.[3-5]

In the present note we wish to call attention to a more refined expression for the meson-photoproduction differential cross section in nucleons[6] (details are given in Ref. 2). Using this expression we have calculated $\sigma_p(\vartheta)$ for cases in which mesons are produced in s-, p- and d-states. This formula contains as particular cases all the expressions for $\sigma_p(\vartheta)$ (without d-states) which have been obtained earlier.[4,5] Using this expression for $\sigma_p(\vartheta)$ it is possible to compute the relative yield due to the d-state in meson production at various photon energies and to establish the γ-ray energies for which this process must be taken into account. The interest in this analysis is explained by the fact that the experimental data[1,2] indicate the necessity for taking account of meson production in the d-state even at energies $E_\gamma \leq 300$ Mev.

Transition	L	j'	l'
E_{11}	1	1/2−	0
E_{13}	1	3/2−	2
M_{11}	1	1/2+	1
M_{13}	1	3/2+	1
E_{23}	2	3/2+	1
M_{23}	2	3/2−	2
M_{25}	2	5/2−	2
E_{35}	3	5/2−	2

2. We now consider the photoproduction of mesons in s-, p- and d-states. All the possible transitions associated with this process are given in the table. The symbols in the first column denote the matrix elements of the corresponding transitions; E denotes an electric transition while M denotes a magnetic transition.

If one takes account of all the transitions which have been enumerated and the interference between them, the photomeson angular distribution in the center-of-mass system is written as follows:†

$$\begin{aligned}
\frac{1}{2N}\sigma_p(\vartheta) &= P_0(\cos\vartheta)\,[\,|E_{11}|^2 + 2|E_{13}|^2 + |M_{11}|^2 + 2|M_{13}|^2 + 2|E_{23}|^2 + 2|M_{23}|^2 + 3|M_{25}|^2 + 3|E_{35}|^2] \\
&+ P_1(\cos\vartheta)\,\mathrm{Re}\Big[-2E_{11}^*M_{11} + 2E_{11}^*M_{13} + 2\sqrt{3}\,E_{11}^*E_{23} - 2E_{13}^*M_{11} + 2E_{13}^*M_{13} - 0.692\,E_{13}^*E_{23} + 2\sqrt{3}\,M_{11}^*M_{23} \\
&+ 0.692\,M_{13}^*M_{23} + \frac{18\sqrt{3}}{5}M_{13}^*M_{25} - 1.2\,E_{23}^*M_{23} + 6.77\,E_{23}^*E_{35} + 1.2\,E_{23}^*M_{25}\Big] + P_2(\cos\vartheta)\Big\{-|E_{13}|^2 - |M_{13}|^2 + |E_{23}|^2 \\
&+ |M_{23}|^2 + 1.711\,|M_{25}|^2 + \frac{18}{7}|E_{35}|^2 + \mathrm{Re}\Big[2E_{11}^*E_{13} - 2\sqrt{3}\,E_{11}^*M_{23} + 3.462\,E_{11}^*M_{25} + 4.884\,E_{11}^*E_{35} - 2\sqrt{3}\,E_{13}^*M_{23} \\
&+ 3.46\,E_{13}^*M_{25} - 1.399\,E_{13}^*E_{35} - 2M_{11}^*M_{13} - 2\sqrt{3}\,M_{11}^*E_{23} - 2M_{11}^*M_{13} + 2\sqrt{3}\,E_{23}M_{13} + \frac{6}{7}M_{23}^*M_{25} - 2.423\,M_{23}^*E_{35} \\
&+ 2.424\,M_{25}^*E_{35}\Big]\Big\} + P_3(\cos\vartheta)\,\mathrm{Re}\Big[4.148\,E_{13}^*E_{23} - 3.453\,M_{11}^*M_{25} - 4.894\,M_{11}^*E_{35} - \frac{12\sqrt{3}}{5}M_{13}^*M_{23} - 2.772\,M_{13}^*M_{25} \\
&+ 4.87\,M_{13}^*E_{35} - \frac{24}{5}E_{23}^*M_{23} + \frac{24}{5}E_{23}^*M_{25} + 1.692\,E_{23}^*E_{35}\Big] + P_4(\cos\vartheta)\{-1.707\,|M_{25}|^2 + 0.426\,|E_{35}|^2 \\
&+ \mathrm{Re}\,[6.301\,E_{13}^*E_{35} - 6.835\,M_{23}^*M_{25} - 6.056\,M_{23}^*E_{35} + 6.013\,M_{25}^*E_{35}]\},
\end{aligned} \tag{1}$$

where $P_n(\cos\vartheta)$ is the Legendre polynomial; $N = 1/16k^2$ and k is the photon wave number. It can easily be shown that if all transitions associated with the production of mesons in the d-state are neglected (that is, if we set $E_{13} = M_{23} = M_{25} = E_{35} = 0$ the expression in (1) goes over to the expression for $\sigma_p(\vartheta)$

*In what follows the term meson will always mean π-meson.

†The calculations are simplified considerably if use is made of the tables of numerical values for the Racah coefficients[8] and the Z coefficients.[9] These coefficients are considered in greater detail in Ref. 7.

obtained earlier* [cf. Ref. 10, Eq. (27)]. Similarly, we can obtain the particular forms for $\sigma_p(\vartheta)$ found by Feld.[4]

3. An analysis of the experimental data on the angular distribution of mesons produced by photons on nucleons in the energy region $E_\gamma \leq 400$ Mev has been carried out in a paper by Watson et al. (this work is considered in greater detail in Ref. 2). This analysis has shown that in the indicated energy region it is impossible to obtain good agreement between the theory and the experimental data if meson production in only s- and p-states is taken into account. In this connection it would be of interest to analyze the indicated experimental data using the method used in Ref. 1 but taking account of meson production in d-states using the expression given in (1).

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*It is necessary to make the following substitutions for the unknown matrix elements $E_{11} \to E_{11}$, $M_{11} \to -M_{11}$, $M_{13} \to -M_{13}$, $E_{23} \to E_{23}/2\sqrt{3}$.

PHASE OF A SCATTERED WAVE

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We consider here the application of one form of the method of variation of constants for determining the phase of the scattered wave in a spherically-symmetric one-particle problem in quantum mechanics. We have the equation

$$d^2G / d\rho^2 + [1 - l(l+1)/\rho^2 - U(\rho)]\, G = 0, \tag{1}$$

where $l = 0, 1, 2\ldots$, while $U(\rho)$ satisfies the condition $\int^{\infty} U(\rho)\, d\rho < C$ and may be given as $U(\rho) = \gamma(\rho)/\rho$, where $\gamma(\rho) = \gamma_0 + \gamma_1\rho + \ldots$.

We seek a solution of Eq. (1) which may be represented in the following form when $\rho \to 0$

$$G = A_0 \rho^{l+1} \tag{2}$$

and which takes on the following asymptotic form at large values of ρ

$$G = \mathrm{const}\cdot\sin\left(\rho - \frac{\pi l}{2} + \delta_l\right); \quad \delta_l = \mathrm{const}. \tag{3}$$