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ON INTERPRETATION OF THE LATERAL DISTRIBUTION OF NUCLEAR-ACTIVE PARTICLES IN EXTENSIVE AIR SHOWERS

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Experimental data referring to the shape of the lateral distribution curve of nuclear-active particles near the axis of extensive showers are considered. The data indicate a small value of the averaged minimum angles of emission of nuclear-active particles responsible for generation of the nuclear-active particles directly recorded in the experiments ($< 4 \times 10^{-3}$ radians). It is shown that Landau's theory of multiple particle production can yield such a small value for the emission angle only if it simultaneously predicts a high degree of concentration of energy of the incident particle on a single secondary particle in an elementary act of the nuclear cascade process at energies of $10^{12} - 10^{15}$ ev.

DETAILED measurements of the lateral distribution of nuclear-active particles (NAP) in extensive air showers were recently carried out both at high altitudes and near sea level.[1,2] The characteristic feature of the lateral distribution of NAP obtained from these measurements consists in the sharp increase of the NAP flux density with decreasing distance from the shower axis up to distances $\lesssim 2$ m, the increase being as fast as $1/r$ at least even for the case of showers with a small total number of all charged particles, $N = 6 \times 10^4$.

It is the aim of the present note to show that it follows from the experimental fact of the sharp increase of the NAP flux up to $r \lesssim 2$ m that the emission angles in nuclear interactions of NAP which generated the particles directly observed in the experiments[1,2] are very small. Furthermore, it will be shown that in the comparison of the obtained estimate of the angles of emission with the value for the angle of emission of the secondary particle possessing the highest energy, derived from the Landau theory, an agreement between the experimental estimate and the theoretical value can be reached only on condition of high average concentration of energy on a single secondary particle in each generation of the nuclear cascade process.

It was shown by Greisen et al.[3] that the steep rise of the lateral distribution function of NAP near the axis, going as $1/r$, follows from purely geometrical considerations, on condition that the detected NAP are produced by generating high-energy NAP traveling in a very narrow cone enveloping the shower core.*

The experimental data of Refs. 1 and 2 make it possible to estimate the emission angles of generating NAP. In fact, since the mean free path for nuclear interaction, corresponding to 70 g/cm^2 [4–7] in air† amounts to 800 m for the altitude of observation of Ref. 1 (Pamir station, 3860 m) and to 500 m near sea level, we can estimate the angle of emission of generating particles as follows:

$$\chi \leqslant 2/800 \approx 3 \cdot 10^{-3} \text{ radians} \quad (1)$$

for generating NAP in the core of a shower at the altitude of the Pamir station, and

$$\chi \leqslant 2/500 = 4 \cdot 10^{-3} \text{ radians} \quad (2)$$

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*In the limit, if the emission angle of generating particles in the core → 0, the $1/r$ law would hold for arbitrarily small distances from the axis.

†Such is the mean value of the nuclear interaction mean free path according to Refs. 4 — 7 in which the nuclear range was determined in air and graphite for NAP of $\sim 10^9 - 10^{10}$ ev. Since there is no basis for a decrease in the nuclear interaction cross-section for ultra-high energies than the mean free path for nuclear interaction for energies $> 10^{11}$ ev has to be regarded as essentially constant and equal to 70 g/cm^2.

for generating particles in the core of a shower near the sea level.* The minimum angles of emission are expected to be substantially smaller than these estimates.

We shall compare now the estimates for emission angles of generating NAP with the value of the average minimum angle of emission of particles which follows from the theory of Landau[8] for the case of nucleon—nucleus collision. For the estimate of the averaged minimum angle of emission of secondary particles in nucleon—nucleus collisions according to Landau's theory we made use of the model representing the collision as occurring between the nucleon and a column of nucleons of the nucleus forming a cylindrical volume of nuclear matter.[9]

We shall use the expression obtained by Landau for the angle of emission of a secondary particle in the ultra-relativistic case:

$$\chi = e^{-\lambda} / \gamma_c, \quad \gamma_c = 1/\sqrt{1 - v_c^2}. \tag{3}$$

where χ is the emission angle of particle in the laboratory system of coordinates, v_c is the velocity of the center of mass of colliding particles (c-system) with respect to the laboratory system and λ is a parameter of the Landau theory.

We shall generalize relation (3) for the case of a collision between a nucleon and a cylinder of nuclear matter. The averaged (because of the statistical character of the theory) value of the minimum emission angle can be estimated taking for λ its maximum value λ_m. The latter can be found from the condition (cf. Ref. 8)

$$\int_{\lambda_m}^{L} C \exp(\sqrt{L^2 - \lambda^2})\, d\lambda = 1. \tag{4}$$

where $L = -\ln(\Delta/a)$, Δ is the longitudinal dimension of the system nucleon-cylinder of nuclear matter in the c-system at the end of the contraction stage,† a is its lateral dimension, and C is a constant determined from the normalization to the total number of produced particles.

Substituting λ_m found from (4) into (3) we obtain the value of the averaged minimum emission angle in the laboratory system:

$$\bar{\chi}_{\min} \approx {}^1/_2 A^{1/6} ({}^2/_3 A^{1/3} + 1) Mc^2 / E. \tag{5}$$

where E is the energy of the incident nucleon, A is the atomic weight of the target nucleus and Mc^2 is the rest energy of the nucleon ($\sim 10^9$ ev). For the case of collision between e nucleon and an air atom ($A \approx 14$) we have

$$\bar{\chi}_{\min} \approx 2Mc^2 / E. \tag{6}$$

Expressions (3) and (4) by means of which formula (6) has been obtained, are applicable in the region of the so-called non-trivial solution.[10] Since the non-trivial solution should merge with the solution representable in the form of a traveling wave corresponding to the secondary particle of maximum energy, one should expect that the value of the minimum emission angle (6) will be close to the value of the angle corresponding to the particle of maximum energy described by a traveling wave.

Comparing (6) with the estimates (1) and (2) of the emission angle of generating particles we conclude that the energies of NAP at the altitude of Pamir station and at sea level should equal, respectively,

$$E \gtrsim 10^{12} \text{ ev and } E \gtrsim 5 \cdot 10^{11} \text{ ev} \tag{7}$$

in order that theoretical values of the angles correspond to experimental estimates.

It can be now easily seen that generating NAP of such high energies can be present in the core of showers at the level of Pamir station and near sea level only on condition of an extremely high concentra-

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*More accurately, these levels are on the average two nuclear lengths above the shower detection point (Pamir station, Moscow) since the NAP observed directly are produced on the average at the distance of one nuclear mean free path above the detectors and the particles which generate them are in turn produced on the average one free path higher.

†The calculation was made for a cylinder of mean height $\bar{H}$, which was assumed to be equal to the mean chord of the sphere with radius $R = (\hbar/\mu c) A^{1/3}$, where μ is the π-meson mass, i.e., $\bar{H} = 4R/3$.

tion of the energy of incident particle on one secondary particle at all stages of the nuclear cascade process producing the recorded extensive showers.*

We shall denote by α the energy fraction of the incident particle, averaged for all stages of the nuclear cascade process, which is transferred to the secondary particle of maximum energy. The value of α necessary for safeguarding the presence of NAP of high energy at the elevation of Pamir station and at sea level can then be found from the relation

$$E = E_0\alpha^n. \tag{8}$$

Here E_0 is the energy of the shower-producing particle, n is the average number of nuclear lengths from the top of the atmosphere to levels two nuclear lengths above observation levels. In the determination of n it should be taken into account that for the case of showers observed in the lower half of the atmosphere by conventional detectors, the mean production level is at about ~100 g/cm^2 below the top of the atmosphere.[11]

According to Sarycheva[12] the energy of primary NAP initiating extensive air showers with total number of charged particles at sea level $N = 6 \times 10^4$ amounts to $E_0 \approx 10^{15}$ ev and the energy of particles initiating extensive air showers with the same number of particles at the elevation of Pamir station to $E_0 \approx 1.5 \times 10^{14}$ ev. It should be noted that the estimate of E_0 from the number of particles in a shower, given in Ref. 12, differs by less than 1.5 from the estimate using the coefficient of the relation between N and E_0 recently found by Greisen.[13]†

For the case of shower detection at Pamir elevation n = 7 and near sea level (Moscow) n = 12, under the assumption that the interaction mean free path in air for NAP is 70 g/cm^2.

Consequently, we find for α [from (8)] the value $\alpha \gtrsim 0.5$ by starting with either (1) or (2) and by assuming the correctness of expression (6) for the averaged minimum emission angle, obtained from Landau's theory.‡ It follows that this theory will yield the value of emission angles of generating NAP required by experimental evidence only in the case when it predicts simultaneously a very high degree of concentration of energy, not less than 50%, on a single secondary particle in elementary processes at the energies of $10^{12} - 10^{15}$ ev.

Gerasimova and Chernavskii calculated the fraction of energy of the incident nucleon carried away by the traveling wave.[10] They found that the average energy concentration on the secondary particle of maximum energy in the energy interval $10^{12} - 10^{15}$ ev amounts to about 35%. The problem of the fraction of energy of the incident particle, carried away by the secondary particle of maximum energy according to the hydrodynamical theory, is extremely difficult and has not been so far worked out in any detail.

It has been pointed out in the work of Vernov et al.[14] that in interactions of the nucleons with nucleons and with light nuclei at ultra-high energies, the total energy of the incident nucleon is in most cases not dissipated within a unique thermodynamical system. The greater part of the energy, on the order of 70%, remains in the nucleon undergoing the collision. The considerations of the present note do not contradict such a view.

In conclusion, I wish to express my deep gratitude to N. A. Dobrotin, G. T. Zatsepin, G. A. Milekhin, and D. S. Chernavskii for discussion of the problems raised in the present note.

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*The deduction that NAP of high energies ($>10^{11} - 10^{12}$ ev) should be found in the cores of extensive showers at lower atmospheric depths (sea level) was arrived at earlier by Zatsepin. It was not, however, fully substantiated since the relation $\chi_{\min} = Mc^2/E$ was assumed arbitrarily.

†The coefficient in the relation between N and E_0 was obtained in Ref. 13 by means of very general considerations computing the total energy spent on ionization by all charged particles in the total atmospheric depth using experimental data on the longitudinal development of showers.

‡It should be noted that in spite of the fact that formula (6) was derived from the Landau solution which is approximate at the boundary with the traveling wave, this formula nevertheless yields for a given value of the energy of generating particle a value of $\overline{\chi}_{\min}$ only twice as large as that obtained from a more accurate solution of the hydrodynamical equations (G. A. Milekhin — private communication). Since α is almost independent of the energy of the generating particle of maximum energy, estimated from the emission angle at a given level [cf. Eq. (8)], the increase in the estimated value of α due to the use of a slightly inaccurate formula (6) is not very considerable.

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SOME CHARACTERISTICS OF M1-CONVERSION NUCLEAR TRANSITIONS

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M1-conversion nuclear transitions are investigated within the framework of the single-particle model with LS coupling.

WITHIN the framework of the strictly single-particle model of the nucleus M1-conversion on nucleonic transitions of the types $n_1 s_{1/2} - n_2 s_{1/2}$ and $n_1 d_{1/2} - n_2 s_{3/2}$ are "forbidden". The contribution from the field of the external electronic current of the transition disappears in the case of a $n_1 s_{1/2} - n_2 s_{1/2}$ transition because of the orthogonality of the nucleonic radial functions and in the case of a $n_1 s_{1/2} - n_2 d_{3/2}$ transition because of the orthogonality of the nucleonic of the nucleonic angular functions. In these transitions the conversion probability is determined only by the contribution of the "internal" electronic transition current. However the spin-orbit interaction of the nucleons and the correlation of nucleonic motion which results from their interaction lead to the inclusion of an interaction with the external electronic current. The conversion probability in these cases is determined by entirely different matrix elements than for the usual "allowed" M1 nuclear transitions (such as $p_{1/2} - p_{3/2}$). The results of calculations of K-electron conversion probabilities for various M1 nuclear transitions are given below. It is assumed that the momentum of the final electronic state $p \gg Ze^2$; the K-electron transition to the state $j = 3/2, \lambda = +1/2, \ell = 2$ is neglected because the probability of this transition is smaller by the factor $\sim (Ze^2)^2/2$ than the transition probability to the state $j = 1/2, \lambda = -1/2, \ell = 0$. The relativistic units $\hbar = m_e = c = 1$ are used throughout.

In the model under consideration the nucleonic wave function is separated into radial and angular parts: