of the order of $10^3 - 10^4$ v/cm). In the vicinity of the critical point the scattering should be strongly dependent on the pressure.

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ON THE EFFECTIVE FIELD IN A PLASMA

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The Bogoliubov equations for the "partial distribution functions" are used to compute the effective field acting on charged particles in a plasma. It is shown that the effective field differs from the mean field by a small quantity of the order $1/N$ where N is the number of particles within a sphere whose radius is equal to the Debye radius. This result also holds in the presence of a magnetic field.

As is well known, the electric field acting on an individual particle of a medium is not equal to the average field in the medium. For example, in a gas of free dipoles the effective field $\mathbf{E}_{\text{eff}}$ is given by the Lorentz formula $\mathbf{E}_{\text{eff}} = \mathbf{E} + 4\pi\mathbf{P}/3$ where $\mathbf{E}$ is the average field and $\mathbf{P}$ is the polarization of the medium. This formula is obtained on the assumption that the molecular dipoles are mutually impenetrable so that each dipole behaves as if it were placed inside a cavity in a polarized medium.

In the case of ionized plasma there is, of course, no basis for such an assumption. However, the effective field in a plasma should also, generally speaking, differ from the average field because there exists a certain correlation between the motions of individual particles. The question of the relation between E_{eff} and E in a plasma has been repeatedly discussed by different authors (see Ginzburg[1] and the literature cited there), and it has been shown that the effective field and the average field are the same. The most detailed proof of this assertion has been given by Ginzburg.[1] However, he used a method which did not allow him to obtain explicit expressions for the effective fields. As a result of this, in particular, there has remained the unsolved problem as to whether the effective and the mean fields remain the same in the presence of an external magnetic field.

In this paper Bogoliubov's method[2] is used to compute the effective field in a plasma. In the course of this computation the triple function is expressed approximately in terms of binary ones which corresponds to making an expansion with respect to the small parameter $1/nD^3$ where n is the density of particles and D is the Debye radius. By this method an analogous expansion is introduced in establishing the connection between the effective and the average fields and we restrict ourselves to the calculation of only the first order correction.

For the sake of simplicity we assume that the plasma in addition to electrons also contains only singly ionized ions of a single kind. We introduce the "microscopic density" of particles in phase space

$$F_e = \sum_k \delta(\mathbf{r} - \mathbf{r}_k(t))\,\delta(\mathbf{v} - \mathbf{v}_k(t))$$

for the electrons and

$$F_i = \sum_k \delta(\mathbf{r} - \mathbf{r}_k(t))\,\delta(\mathbf{v} - \mathbf{v}_k(t))$$

for the ions, where the summation is taken over all the electrons and ions respectively. It can be easily shown that in the absence of a magnetic field these functions satisfy the equations

$$\frac{\partial F_e}{\partial t} + (\mathbf{v}\nabla) F_e - \frac{e}{m}\mathbf{E}_m \frac{\partial F_e}{\partial \mathbf{v}} = 0, \tag{1}$$

$$\frac{\partial F_i}{\partial t} + (\mathbf{v}\nabla) F_i + \frac{e}{M}\mathbf{E}_m \frac{\partial F_e}{\partial \mathbf{v}} = 0, \tag{2}$$

which correspond to the system of Newton's equations of motion for all the electrons and ions. In (1) and (2) m is the mass of the electron, M the mass of the ion, and $\mathbf{E}_m$ the microscopic field, which can be obtained from the equation

$$\operatorname{div} \mathbf{E}_m = 4\pi e \int (F_i - F_e)\, d\mathbf{v}, \tag{3}$$

but in doing so one must omit in (1) and (2) the self-field of the electron or of the ion which is situated at the particular point in space.

Equations (1) and (2) can also be written

$$\frac{\partial F_e}{\partial t} + (\mathbf{v}\nabla) F_e - \frac{e}{m}\mathbf{E}_0 \frac{\partial F_e}{\partial \mathbf{v}} - \frac{e^2}{m}\frac{\partial}{\partial \mathbf{v}} \int \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} F_e(\mathbf{r}, \mathbf{v}, t) \{F_i(\mathbf{r}', \mathbf{v}', t) - F_e(\mathbf{r}', \mathbf{v}', t)\}\, d\mathbf{r}' d\mathbf{v}' = 0, \tag{4}$$

$$\frac{\partial F_i}{\partial t} + (\mathbf{v}\nabla) F_i + \frac{e}{M}\mathbf{E}_0 \frac{\partial F_i}{\partial \mathbf{v}} + \frac{e^2}{M}\frac{\partial}{\partial \mathbf{v}} \int \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} F_i(\mathbf{r}, \mathbf{v}, t) \{F_i(\mathbf{r}', \mathbf{v}', t) - F_e(\mathbf{r}', \mathbf{v}', t)\}\, d\mathbf{r}' d\mathbf{v}' = 0, \tag{5}$$

where in accordance with the stipulation made above the point $\mathbf{r}' = \mathbf{r}$ must be excluded in carrying out the integration over $\mathbf{r}'$. In Eqs. (4) and (5) $\mathbf{E}_0$ represents the external field; it satisfies the equation $\operatorname{div} \mathbf{E}_0 = 0$.

We shall go over from the exact microscopic equations (1) — (5) to a statistical treatment, and in order to do this we shall average all the quantities over a certain set of initial particle distributions. This means that we regard the exact densities F_e and F_i as random quantities. We shall denote the average values of F_e and F_i by f_e and f_i. The averaging of Eq. (3) yields

$$\operatorname{div} \mathbf{E} = 4\pi e \int \{f_i - f_e\}\, d\mathbf{v}, \tag{6}$$

where $\mathbf{E}$ denotes the average, i.e., the macroscopic field. On averaging (4) we obtain an equation which contains the binary distribution function:

$$\frac{\partial f_e}{\partial t} + (\mathbf{v}\nabla) f_e - \frac{e}{m}\mathbf{E}_0 \frac{\partial f_e}{\partial \mathbf{v}} - \frac{e^2}{m}\frac{\partial}{\partial \mathbf{v}} \int \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} \{f_{ei}(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; t) - f_{ee}(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; t)\}\, d\mathbf{r}' d\mathbf{v}' = 0. \tag{7}$$

Here

$$f_{ei}(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; t) = \langle F_e(\mathbf{r}, \mathbf{v}, t)\, F_i(\mathbf{r}', \mathbf{v}', t)\rangle,$$

(the brackets denote averaging) is the binary distribution function for electrons and ions. With respect to the average value of $F_e F_e'$ we can see that, strictly speaking, it is equal to

$$\langle F_e(\mathbf{r}, \mathbf{v}, t)\, F_e(\mathbf{r}', \mathbf{v}', t)\rangle = \langle \sum_j \delta(\mathbf{r}-\mathbf{r}_j)\,\delta(\mathbf{v}-\mathbf{v}_j)\,\delta(\mathbf{r}'-\mathbf{r}_j)\,\delta(\mathbf{v}'-\mathbf{v}_j)\rangle$$

$$+ \langle \sum_{j\neq k} \delta(\mathbf{r}-\mathbf{r}_j)\,\delta(\mathbf{v}-\mathbf{v}_j)\,\delta(\mathbf{r}'-\mathbf{r}_k)\,\delta(\mathbf{v}'-\mathbf{v}_k)\rangle = \delta(\mathbf{r}-\mathbf{r}')\,\delta(\mathbf{v}-\mathbf{v}')\, f_e(\mathbf{r}, \mathbf{v}, t) + f_{ee}(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; t),$$

where the binary function f_{ee} is the "non-singular" part of the average value of $F_e F_e'$. However, since in the integration over $\mathbf{r}'$ we have excluded the point $\mathbf{r}' = \mathbf{r}$ the term with the δ-functions turns out to be unimportant in the present case.

We assume that the electrostatic interaction energy of the particles is much smaller than their kinetic energy, i.e., $e^2 n^{1/3} \ll kT$, which is equivalent to the condition $nD^3 \gg 1$. This condition is almost always fulfilled. It is evident that under this condition the correlation between the particles is very small, so that we may write $f(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}', t) = f(\mathbf{r}, \mathbf{v}, t) f(\mathbf{r}', \mathbf{v}', t) + \varphi(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}', t)$ where φ is a small quantity. In accordance with this, Eq. (7) and the analogous equation for f_i can be brought into the form

$$\frac{\partial f_e}{\partial t} + (\mathbf{v}\nabla) f_e - \frac{e}{m}\mathbf{E}\frac{\partial f_e}{\partial \mathbf{v}} - \frac{e^2}{m}\frac{\partial}{\partial \mathbf{v}}\int \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}'-\mathbf{r}'|^3}\{\varphi_{ei}(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; t) - \varphi_{ee}(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; t)\}\, d\mathbf{r}' d\mathbf{v}' = 0, \quad (8)$$

$$\frac{\partial f_i}{\partial t} + (\mathbf{v}\nabla) f_i + \frac{e}{M}\mathbf{E}\frac{\partial f_i}{\partial \mathbf{v}} + \frac{e^2}{M}\frac{\partial}{\partial \mathbf{v}}\int \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\{\varphi_{ii}(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; t) - \varphi_{ie}(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; t)\}\, d\mathbf{r}' d\mathbf{v}' = 0. \quad (9)$$

Our problem consists of finding the functions $\varphi_{\alpha\beta}$. In order to do this we shall set up equations for the binary distribution functions. We shall here discuss in detail only the equation for f_{ei} — the discussion of the other equations can be carried out in a completely analogous fashion.

We multiply Eq. (4) by $F_i(\mathbf{r}', \mathbf{v}', t)$, replace in (5) $\mathbf{r}, \mathbf{v}$ by $\mathbf{r}', \mathbf{v}'$ and after multiplying it by $F_e(\mathbf{r}, \mathbf{v}, t)$ we then add the results and take the average. Taking into account the fact that

$$\langle F_e(\mathbf{r}, \mathbf{v}, t)\, F_e(\mathbf{r}', \mathbf{v}', t)\, F_i(\mathbf{r}'', \mathbf{v}'', t)\rangle = f_{eei}(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; \mathbf{r}'', \mathbf{v}''; t) + \delta(\mathbf{r}-\mathbf{r}')\,\delta(\mathbf{v}-\mathbf{v}')\, f_{ei}(\mathbf{r}, \mathbf{v}; \mathbf{r}'', \mathbf{v}''; t)$$

etc., we obtain

$$\frac{\partial f_{ei}(\mathbf{r}, \mathbf{v}; \mathbf{r}'\, \mathbf{v}'; t)}{\partial t} + (\mathbf{v}\nabla) f_{ei} + (\mathbf{v}'\nabla') f_{ei} - \frac{e}{m}\mathbf{E}_0\frac{\partial f_{ei}}{\partial \mathbf{v}} + \frac{e}{M}\mathbf{E}_0'\frac{\partial f_{ei}}{\partial \mathbf{v}'} - \frac{e^2(\mathbf{r}-\mathbf{r}')}{m|\mathbf{r}-\mathbf{r}'|^3}\frac{\partial f_{ei}}{\partial \mathbf{v}} + \frac{e^2(\mathbf{r}-\mathbf{r}')}{M|\mathbf{r}-\mathbf{r}'|^3}\frac{\partial f_{ei}}{\partial \mathbf{v}'}$$

$$- \frac{e^2}{m}\frac{\partial}{\partial \mathbf{v}}\int \frac{\mathbf{r}-\mathbf{r}''}{|\mathbf{r}-\mathbf{r}''|^3}\{f_{eii}(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; \mathbf{r}'', \mathbf{v}''; t) - f_{eie}(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; \mathbf{r}'', \mathbf{v}''; t)\}\, d\mathbf{r}'' d\mathbf{v}''$$

$$+ \frac{e^2}{M}\frac{\partial}{\partial \mathbf{v}'}\int \frac{\mathbf{r}'-\mathbf{r}''}{|\mathbf{r}'-\mathbf{r}''|^3}\{f_{eii}(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; \mathbf{r}'', \mathbf{v}''; t) - f_{eie}(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; \mathbf{r}'', \mathbf{v}''; t)\}\, d\mathbf{r}'', d\mathbf{v}'' = 0. \quad (10)$$

In accordance with the assumption made earlier concerning the smallness of the correlation, we write

$$f(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; \mathbf{r}'', \mathbf{v}''; t) = f(\mathbf{r}, \mathbf{v}, t)\, f(\mathbf{r}', \mathbf{v}', t)\, f(\mathbf{r}'', \mathbf{v}'', t) + f(\mathbf{r}, \mathbf{v}, t)\,\varphi(\mathbf{r}', \mathbf{v}'; \mathbf{r}'', \mathbf{v}''; t)$$

$$+ f(\mathbf{r}', \mathbf{v}', t)\,\varphi(\mathbf{r}, \mathbf{v}; \mathbf{r}'', \mathbf{v}''; t) + f(\mathbf{r}'', \mathbf{v}'', t)\,\varphi(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; t) + \psi(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; \mathbf{r}'', \mathbf{v}''; t)$$

and neglect the last term on the right hand side. Then on taking (8) and (9) into account we obtain from (10)

$$\frac{\partial \varphi_{ei}(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; t)}{\partial t} + (\mathbf{v}\nabla)\varphi_{ei} + (\mathbf{v}'\nabla')\varphi_{ei} - \frac{e^2(\mathbf{r}-\mathbf{r}')}{m|\mathbf{r}-\mathbf{r}'|^3}\frac{\partial f_e}{\partial \mathbf{v}} f_i' - \frac{e^2(\mathbf{r}'-\mathbf{r})}{M|\mathbf{r}'-\mathbf{r}|^3} f_e \frac{\partial f_i'}{\partial \mathbf{v}'} - \frac{e}{m}\mathbf{E}\frac{\partial \varphi}{\partial \mathbf{v}} + \frac{e}{M}\mathbf{E}'\frac{\partial \varphi_{ei}}{\partial \mathbf{v}'}$$

$$= \frac{e^2(\mathbf{r}-\mathbf{r}')}{M|\mathbf{r}-\mathbf{r}'|^3}\frac{\partial \varphi_{ei}}{\partial \mathbf{v}} + \frac{e^2(\mathbf{r}'-\mathbf{r})}{M|\mathbf{r}'-\mathbf{r}|^3}\frac{\partial \varphi_{ei}}{\partial \mathbf{v}'} + \frac{e^2}{m}\frac{\partial f_e}{\partial \mathbf{v}}\int \frac{\mathbf{r}-\mathbf{r}''}{|\mathbf{r}-\mathbf{r}''|^3}\{\varphi_{ii}(\mathbf{r}', \mathbf{v}'; \mathbf{r}'', \mathbf{v}''; t) - \varphi_{ie}(\mathbf{r}', \mathbf{v}'; \mathbf{r}'', \mathbf{v}''; t)\}\, d\mathbf{r}'' d\mathbf{v}''$$

$$+ \frac{e^2}{M}\frac{\partial f_i'}{\partial \mathbf{v}'}\int \frac{\mathbf{r}'-\mathbf{r}''}{|\mathbf{r}'-\mathbf{r}''|^3}\{\varphi_{ee}(\mathbf{r}, \mathbf{v}; \mathbf{r}'', \mathbf{v}''; t) - \varphi_{ei}(\mathbf{r}, \mathbf{v}; \mathbf{r}'', \mathbf{v}''; t)\}\, d\mathbf{r}'' d\mathbf{v}''. \quad (11)$$

The quantities appearing on the right hand side of this equation are small, but they cannot be simply neglected since they turn out to be essential for large and for small values of $|\mathbf{r}-\mathbf{r}'|$. In contrast to the method adopted by Bogoliubov[2] of expanding in terms of a small parameter, we shall solve (11) in the following way. First we solve the stationary equation. We set

$$\mathbf{E}=0,\; f_e = n\left(\frac{m}{2\pi kT_e}\right)^{3/2}\exp\left(-\frac{mv^2}{2kT_e}\right),\; f_i = n\left(\frac{M}{2\pi kT_i}\right)^{3/2}\exp\left(-\frac{Mv^2}{2kT_i}\right),\quad \varphi_{ee}=f_e f_e'\chi(|\mathbf{r}-\mathbf{r}'|)$$

etc., where n is the density of electrons (and ions), T_e the electron temperature, and T_i the ion temperature. We substitute this into (11) and in accordance with the assumption that $T_e \neq T_i$ we neglect the velocity of the ions in comparison with the velocity of the electrons. Since $\mathbf{v}$ is arbitrary we obtain

$$\nabla\chi_{ei}(r)+\frac{e^2\mathbf{r}}{kT_e r^3} = -\frac{e^2\mathbf{r}}{kT_e r^3}\chi_{ei}(r)-\frac{ne^2}{kT_e}\int\frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\{\chi_{ii}(r')-\chi_{ei}(r')\}\,d\mathbf{r}'. \tag{12}$$

For $r > e^2/kt$ we may neglect the first term on the right hand side and from this, taking it into account that $\chi \rightarrow 0$ as $r \rightarrow \infty$, we obtain:

$$\chi_{ei}(r)-\frac{ne^2}{kT_e}\int\frac{1}{|\mathbf{r}-\mathbf{r}'|}\{\chi_{ii}(r')-\chi_{ei}(r')\}\,d\mathbf{r}'=\frac{e^2}{rkT_e}. \tag{13}$$

In a similar fashion we can obtain equations for χ_{ee} and χ_{ii}

$$\chi_{ee}(r)-\frac{ne^2}{kT_e}\int\frac{1}{|\mathbf{r}-\mathbf{r}'|}\{\chi_{ei}(r')-\chi_{ee}(r')\}\,d\mathbf{r}'=-\frac{e^2}{rkT_e}, \tag{14}$$

$$\chi_{ii}(r)-\frac{ne^2}{kT_i}\int\frac{1}{|\mathbf{r}-\mathbf{r}'|}\{\chi_{ei}(r')-\chi_{ii}(r')\}\,d\mathbf{r}'=-\frac{1}{r}\frac{e^2}{kT_i}. \tag{15}$$

The solution of the system of equations (13) – (15) is

$$\chi_{ei}(r)=-\chi_{ee}(r)=(e^2/kT_e)\,e^{-r/D}/r,\quad \chi_{ii}=-(e^2/kT_i)\,e^{-r/D}/r, \tag{16}$$

where D is the Debye radius:

$$D=\{kT_eT_i/4\pi e^2 n(T_e+T_i)\}^{1/2}.$$

At distances smaller than e^2/kT Eq. (12) does not have a sensible solution. This is explained by the fact that the coincidence of particles of opposite sign, i.e., the formation of bound states, turns out to be statistically more favorable. Therefore, strictly speaking, a quantum mechanical treatment is required for the investigation of this region. However, one can make classical mechanics serve the purpose if one entirely excludes all elliptic orbits whose formation is not very probable as it requires triple collisions. In doing this it would be necessary to solve Eq. (11) without assuming a Maxwellian velocity distribution. Such a solution presents no difficulties but requires awkward calculations. Since the region of small $(\mathbf{r}-\mathbf{r}')$ occupies a small portion of the correlation region and makes only a small contribution to the final result, we shall not undertake this task. We shall simply neglect the first two terms in the right hand side of Eq. (11), which is equivalent to neglecting the bending of the particle trajectory as a result of collisions. In accordance with this we shall in future exclude collisions with impact parameters smaller than $\rho_0 = e^2/kT$.

We shall assume that the functions f_e and f_i do not differ appreciably from the Maxwellian ones. Then one can substitute the solutions $\varphi_{\alpha\beta} = f_\alpha f_\beta' \chi_{\alpha\beta}(|\mathbf{r}-\mathbf{r}'|)$ found above into the right hand side of Eq. (11), which is small in terms of $1/nD^3$. By limiting ourselves to the linear approximation for the field we substitute the stationary solution also into the last two terms in the left hand side of Eq. (11). From this, after setting for convenience $\varphi_{\alpha\beta}=\varphi'_{\alpha\beta}+\varphi''_{\alpha\beta}$, we obtain:

$$\frac{\partial\varphi'_{ei}}{\partial t}+(\mathbf{v}\nabla)\varphi'_{ei}+(\mathbf{v}'\nabla')\varphi'_{ei}=-\frac{e\mathbf{v}}{kT_e}\mathbf{E}(r,t)\chi_{ei}(|\mathbf{r}-\mathbf{r}'|)f_e(\mathbf{r},\mathbf{v},t)f_i(\mathbf{r}',\mathbf{v}',t)$$
$$+\frac{e\mathbf{v}'}{kT_i}\mathbf{E}(\mathbf{r}',t)\chi_{ei}(|\mathbf{r}-\mathbf{r}'|)f_e(\mathbf{r},\mathbf{v},t)f_i(\mathbf{r}',\mathbf{v}',t); \tag{17}$$

$$\frac{\partial\varphi''_{ei}}{\partial t}+(\mathbf{v}\nabla)\varphi''_{ei}+(\mathbf{v}'\nabla')\varphi''_{ei}=\left\{\frac{1}{m}\frac{\partial f_e(\mathbf{r},\mathbf{v},t)}{\partial\mathbf{v}}f_i(\mathbf{r}',\mathbf{v}',t)-\frac{1}{M}f_e(\mathbf{r},\mathbf{v},t)\frac{\partial f_i(\mathbf{r}',\mathbf{v}',t)}{\partial\mathbf{v}'}\right\}\nabla U(|\mathbf{r}-\mathbf{r}'|), \tag{18}$$

where $U(r) = -e^2e^{-r/D}/r$. Analogous equations may also be obtained for φ_{ee} and φ_{ii}.

If one substitutes the solutions of these equations into (8), (9) then terms of the form $\varphi''_{\alpha\beta}$ will give rise to the collision term (see Appendix), while $\varphi'_{\alpha\beta}$ will give the correction to the average field. We define the effective fields E^e_{eff} for the electrons and E^i_{eff} for the ions by means of the relation:

$$\mathbf{E}^{\alpha}_{ef} = \mathbf{E} + [f_\alpha(\mathbf{r}, \mathbf{v}, t)]^{-1} \int \frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} \{\varphi'_{\alpha i}(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; t) - \varphi_{\alpha e}(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; t)\}\, d\mathbf{r}' d\mathbf{v}'.$$

Then the kinetic equations can be written in the following form:

$$\frac{\partial f_e}{\partial t} + (\mathbf{v}\nabla) f_e - \frac{e}{m}\frac{\partial}{\partial \mathbf{v}} \mathbf{E}^e_{ef} f_e = \mathrm{St}(f_e), \qquad \frac{\partial f_i}{\partial t} + (\mathbf{v}\nabla) f_i + \frac{e}{M}\frac{\partial}{\partial \mathbf{v}} \mathbf{E}^i_{ef} f_i = \mathrm{St}(f_i),$$

where St denotes the collision term.

In order to find explicit expressions for the effective fields one must solve Eq. (17) and the analogous equations for φ'_{ee} and φ'_{ii}. In Eq. (17) we may neglect terms proportional to the ion velocity, and its solution is then given in the form:

$$\varphi'_{ei}(\mathbf{r}, \mathbf{v}; \mathbf{r}', \mathbf{v}'; t) = -f_e(\mathbf{v}) f_i(\mathbf{v}') \int\limits_{-\infty}^{t} \frac{e\mathbf{v}}{kT_{e_i}} \mathbf{E}(\mathbf{r} - \mathbf{v}(t-t'), t')\, \chi_{ei}(|\mathbf{r} - \mathbf{v}(t-t') - \mathbf{r}'|)\, dt', \tag{19}$$

where we have taken the functions f_e, f_i outside the integral having assumed that they do not change over a distance $\sim D$ and during a time $\sim 1/\omega_0$ where $\omega_0 = \sqrt{4\pi e^2 n/m}$ is the plasma oscillation frequency.

Substituting (19) and the similar solutions for φ'_{ee} and φ'_{ii} into (8) and (9) we can easily obtain expressions for the effective fields. Since they depend on the velocity of the particle in a fairly complicated manner we shall average over this velocity. The terms with φ'_{ee} and φ'_{ii} evidently drop out as a result. We set $E \sim e^{i\omega t}$ and we neglect in (19) the dependence of E on **r** by assuming that the wavelength $\lambda \gg D$. Then up to terms in $1/nD^3$ we obtain

$$\langle \mathbf{E}^e_{eff} \rangle_v = \langle \mathbf{E}^i_{eff} \rangle_v = \begin{cases} \mathbf{E}[1 - T_i^2/12\pi n D^3 (T_i + T_e)^2] & \text{for } \omega \ll \omega_0, \\ \mathbf{E}[1 - (\omega_0/i\omega)\sqrt{2}\cdot T_i^{3/2}/12\pi^{3/2} n D^3 (T_i + T_e)^2] \approx \mathbf{E} \text{ for } \omega \gg \omega_0. \end{cases} \tag{20}$$

We see that in a plasma the effective field coincides with the average field under the condition $nD^3 \gg 1$ which is usually fulfilled. For example, in the ionosphere, where according to Ginzburg[1] $n \sim 10^{16}$, $T \sim 300°$, we obtain $E_{eff} - E \sim 10^{-5} E$.

The difference between E_{eff} and E increases as the density increases and as the temperature decreases. However, even in an electric spark,[4] where $n \sim 10^{17}$, $T \sim (4 \times 10^4)°$, the difference is $E_{eff} - E \sim 10^{-3} E$, i.e., still negligibly small.

The calculation of the effective field can be easily generalized to the case when a constant magnetic field H is present. For this it is sufficient merely to add to (17) the Lorentz force term $-(e/c)\mathbf{v} \times \mathbf{H}$. For a field sufficiently weak so that $\Omega = eH/mc \ll \omega_0$ this leads to the same result as before, while in the opposite case when $\Omega \gg \omega_0$.

$$\langle \mathbf{E}^e_{eff} \rangle_v = \langle \mathbf{E}^i_{eff} \rangle_v = \begin{cases} \mathbf{E} - \dfrac{T_i^2}{4\pi n D^3 (T_i + T_e)^2} \dfrac{\mathbf{H}(\mathbf{HE})}{H^2} & \text{for } \omega \ll \omega_0, \\ \approx \mathbf{E} & \text{for } \omega \gg \omega_0 \end{cases} \tag{21}$$

We see that the frequency $\omega = \Omega$ is not distinguished in any way. This can be easily understood since during the transit time of the electron through the correlation region which is of order $\sim 1/\omega_0$ no resonance effects have time to make themselves felt.

It is easy to see the physical reason for the difference between the effective field from the average field. In the equilibrium state a correlated electron "cloud" is formed near each ion. When an electric field is applied this "cloud" is displaced with respect to the ion, producing an additional field directed oppositely to the average field. In a strong magnetic field such a "polarization" occurs only along the magnetic field, which accounts for the form of formula (21).

APPENDIX

For the sake of completeness we shall briefly examine here the manner in which the collision term originates. We shall consider that the functions f_e and f_i vary sufficiently smoothly that they remain essentially constant over a distance D. Then it is possible in Eq. (18) first to neglect the time derivative and second to assume that $\varphi''_{ei} = \varphi_{ei}(\mathbf{r} - \mathbf{r}', \mathbf{v}, \mathbf{v}')$. On introducing a cylindrical coordinate system with the z axis directed along $\mathbf{v}-\mathbf{v}'$, and on setting $\varphi''_{ei} = 0$ for $z = -\infty$ we obtain from (18):

$$\varphi''_{ei}(\rho, \varphi, z; \mathbf{v}, \mathbf{v}') = \left\{\frac{1}{m}\frac{\partial f_e(\mathbf{v})}{\partial \mathbf{v}} f_i(\mathbf{v}') - \frac{1}{M} f_e(\mathbf{v}) \frac{\partial f_i(\mathbf{v}')}{\partial \mathbf{v}'}\right\} \nabla \int\limits_{-\infty}^{z} \frac{U(\sqrt{\rho^2+z^2})}{|\mathbf{v}-\mathbf{v}'|} dz'$$

and similarly for φ''_{ii} and φ''_{ee}.

Substituting this into (7) and carrying out some straightforward calculations we obtain the collision term in Landau's form:[3]

$$\mathrm{St}(f_\alpha) = \left(\frac{2\pi e^2}{m_\alpha}\right) \ln\left(\frac{D}{\rho_0}\right) \sum_\beta \sum_{k,j=1}^{3} \frac{\partial}{\partial v_k} \int \frac{u^2\delta_{jk} - u_j u_k}{u^3} \left\{\frac{1}{m_\alpha}\frac{\partial f_\alpha(\mathbf{v})}{\partial v_j} f_\beta(\mathbf{v}') - \frac{1}{m_\beta} f_\alpha(\mathbf{v}) \frac{\partial f_\beta(\mathbf{v}')}{\partial v'_j}\right\} d\mathbf{v}'.$$

Here $\alpha, \beta = e, i$, $u_k = v_k - v'_k$ is the k-th component of the relative velocity, and ρ_0 is the minimum impact parameter.

We note that in such a derivation the cut-off at the maximum impact parameter is taken into account automatically, while if Eq. (11) had been solved more accurately, the cut-off at the lower limit would also have come out automatically.

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