$$\varphi_{\mathbf{k}}^{\text{incid}} \sim \frac{i}{k}\,\delta\left(1+\frac{\mathbf{kr}}{kr}\right)e^{-ikr}/r - \frac{i}{k}\,\delta\left(1-\frac{\mathbf{kr}}{kr}\right)e^{ikr}/r$$

and the completeness of system of spin functions, one can readily obtain from (3) the sought integral relation for the scattering matrix

$$2\pi\,[M(\mathbf{k},\mathbf{k}') - M^{+}(\mathbf{k}',\mathbf{k})] = ik\int M^{+}(\mathbf{k}',\mathbf{k}'')\,M(\mathbf{k},\mathbf{k}'')\,d\omega(\mathbf{k}''), \quad (4)$$

From (4) there immediately follow the integral relations for the coefficients of expansion of $M(\mathbf{k},\mathbf{k}')$ in terms of the invariant spin matrices given in Ref. 2 for nucleon-nucleon scattering and applied therein to the analysis of the full set of trials aimed at reestablishment of the scattering matrix.

If with $\mathbf{k}' = \mathbf{k}$ we utilize the expansion of $M(\mathbf{k},\mathbf{k}) = \sum_{\mu} \alpha_{\mu} S_{\mu}$ in the orthogonal and normalized Hermitian operators $S_{\mu}$ [the operators are orthogonal and normalized if $\operatorname{Sp} S_{\mu} S_{\nu} = (2s_1+1)(2s_2+1)\delta_{\mu\nu}$], then from (4) we can obtain the following relation

$$4\pi \operatorname{Im} \alpha_{\nu}(2s_1+1)(2s_2+1) = 4\pi \operatorname{Im} \operatorname{Sp} S_{\nu} M(\mathbf{k},\mathbf{k}) = k\int \operatorname{Sp} S_{\nu} M^{+}(\mathbf{k},\mathbf{k}'')\,M(\mathbf{k},\mathbf{k}'')\,d\omega(\mathbf{k}''). \quad (5)$$

In particular, assuming $S_{\nu}$ to be unity, we obtain the extension of the optical theory to the case of particles with spins

$$4\pi \operatorname{Im} \operatorname{Sp} M(\mathbf{k},\mathbf{k}) = k(2s_1+1)(2s_2+1)\,\sigma.$$

From the last relation there follows the inequality[3] $\sigma(0) \geq (k/2\pi)^2\sigma^2$, limiting the value of the cross section for elastic scattering to 0°. With $S_{\nu} \neq I$ the relations (5) connect $\operatorname{Im}\alpha_{\nu}$ with the integral with respect to $\operatorname{Sp} S_{\nu} M^{+} M$, determining the addition

$$(2s_1+1)^{-1}(2s_2+1)^{-1}\langle S_{\nu}\rangle_{\text{incid}} \operatorname{Sp} S_{\nu} M^{+} M$$

to the cross section for scattering of a nonpolarized beam from a nonpolarized target, due to the initial polarization of the colliding particles (in the initial state the mean value of $\langle S_{\nu}\rangle_{\text{incid}}$ of the quantity $S_{\nu}$ differs from zero).

The number of relations (5) is equal to the number of coefficients $\alpha_{\nu}$ that are not zero for $\mathbf{k}' = \mathbf{k}$. Thus in the case of scattering of mesons from nucleons we obtain only the optical theorem [the coefficient at $(\sigma\mathbf{n})$ in the expansion of the scattering amplitude vanishes when $\mathbf{k}' \to \mathbf{k}$]. In the case of nucleon-nucleon scattering we obtain three relations. In this case for $S_{\nu}$ one should select operators

$$I,\; 2^{-1/2}[(\sigma_1,\sigma_2) - (\sigma_1\mathbf{l})(\sigma_2\mathbf{l})],\; (\sigma_1\mathbf{l})(\sigma_2\mathbf{l})$$

(it is impossible to construct a greater number of scalar expressions from the vectors $\sigma_1$, $\sigma_2$ and $\mathbf{l} = \mathbf{k}/k$). In view of the invariance of the scattering matrix $M(\mathbf{k},\mathbf{k}')$ relative to time reversals, the traces containing the operators

$$2^{-1/2}[(\sigma_1\sigma_2) - (\sigma_1\mathbf{l})(\sigma_2\mathbf{l})],\; (\sigma_1\mathbf{l})(\sigma_2\mathbf{l})$$

and determining the additions to the cross section, can be expressed[2] in terms of the components of the tensor of correlation of the polarization arising incident to collisions of nonpolarized nucleons.

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## Gadolinium Isotope with Mass 146

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WHEN TANTALUM is bombarded with 660 Mev protons there are formed new isotopes of gadolinium[1] hitherto unreported in the literature. Upon decay, these isotopes in a number of cases form known isotopes of europium, from which it is possible to determine the mass number of the parent substances — the new gadolinium isotopes. In fractions of europium separated from pure fractions of gadolinium (obtained 32 hours after cessation of bombardment) we observed a radioactive isotope that decays with a period of 1.6 days in accord with the tabular data for $Eu^{146}$. On the basis of measurements of this isotope from the time of its separation from the gadolinium fraction we evaluated the period of the parent substance $Gd^{146}$ to be $12 \pm 4$ hours. It should be noted that the mass number of $Gd^{146}$ was determined with the same degree of reliability as that of the daughter europium isotope, which belongs, according to Seaborg's[2] tables, in class C (mass number "reliable or probable").

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# Possibility of Constructing a Chain of Equations for Model Operators

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THE THEORY OF MODEL TRANSFORMATIONS[1] is characterized by the fact that the model operator $M_n$ transforming the model state

$$|\varphi_1 \ldots \varphi_n\rangle = \prod_{\gamma=1}^{n} \varphi(\gamma)$$

into the real state of the system $|\Psi\rangle = \Psi(1 \ldots n)$, is an operator function of all the dynamic variables of the system. To reduce the many-particle problem to a single-particle problem, let us introduce the sequence of generalized transition amplitudes

$$\langle\varphi_1 \ldots \varphi_n|\Psi\rangle;\ \langle\varphi_1 \ldots \varphi_{\alpha-1}, \varphi_{\alpha+1} \ldots \varphi_n|\Psi\rangle \equiv \langle\ldots(\varphi_\alpha)\ldots|\Psi\rangle, \ldots, \langle\varphi_\alpha \varphi_\beta|\Psi\rangle;\ \langle\varphi_\alpha|\Psi\rangle;\ |\Psi\rangle,$$

where, for example,

$$\langle\ldots(\varphi_\alpha)\ldots|\Psi\rangle = \int \frac{d\tau}{d\tau_\alpha} \prod_{\gamma\neq\alpha} \overset{+}{\varphi}(\gamma)\Psi(1\ldots n).$$

Assuming that the real and model states of the system are described by the wave equations

$$i\partial_t|\Psi\rangle = \Big\{\sum_\alpha T(\alpha) + \sum_{\alpha\beta} H(\alpha\beta)\Big\}|\Psi\rangle,\quad i\partial_t\varphi_\alpha = \{T(\alpha) + U(\alpha)\}\varphi_\alpha,$$

we obtain a system of equations

$$i\partial_t\langle\varphi_1\ldots\varphi_n|\Psi\rangle = \sum_{\alpha\beta}\langle\varphi_\alpha\varphi_\beta|H(\alpha\beta)|\langle\ldots(\varphi_\alpha\varphi_\beta)\ldots|\Psi\rangle\rangle - \sum_\alpha\langle\varphi_\alpha|U(\alpha)|\langle\ldots(\varphi_\alpha)\ldots|\Psi\rangle\rangle,$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots,$$

$$\Big\{i\partial_t - \sum_{\alpha\neq\gamma} T(\alpha) - \sum_{\alpha\beta\neq\gamma} H(\alpha\beta)\Big\}\langle\varphi_\gamma|\Psi\rangle = \langle\varphi_\gamma\Big|\sum_{\alpha\neq\gamma} H(\alpha\gamma) - U(\gamma)|\Psi\rangle,$$

similar to the system of equations for a density matrix.[2] In the stationary case the system assumes the form