$$m' = m + e^2 M / 2, \tag{12}$$

where m' is the observed mass of the electron. The other terms which contain M go into the renormalized constant in the Green function.

Using the present method it is also possible to determine Green functions for three and more electrons in the Bloch–Nordsieck approximation; in these cases, however, the expressions are much more complicated.

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Green Function in Scalar Electrodynamics In The Boch-Nordsieck Approximation

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A METHOD WHICH HAS been applied earlier[1] is used to consider the Green function in scalar electrodynamics in the Boch-Nordsieck approximation,[2] *i.e.*, when the particle recoil can be neglected. As is well known, the infra-red catastrophe does not arise in this case.

The Green function is scalar electrodynamics satisfies the equation

$$\left\{m^2 - \left[i\frac{\partial}{\partial x^\mu} + \sqrt{4\pi}\, e A_\mu(x) + i\sqrt{4\pi}\, e \int D_{\mu\nu}(x,\xi)\frac{\delta}{\delta A_\nu(\xi)}\, d\xi\right]^2\right\} G(x, y \mid A) = \delta(x-y). \tag{1}$$

Using the invariance of the Green function against translation, in the momentum representation Eq. (1) can be written in the form

$$\left\{m^2 - \left[p_\mu - \int k_\mu A_\nu(k)\frac{\delta}{\delta A_\nu(k)}\, dk + \sqrt{4\pi}\frac{e}{(2\pi)^2}\int A_\mu(k)\, dk + i\sqrt{4\pi}\frac{e}{(2\pi)^2}\int D_{\mu\nu}(k)\frac{\delta}{\delta A_\nu(k)}\, dk\right]^2\right\} G(k \mid A) = 1. \tag{2}$$

In the Boch-Nordsieck approximation Eq. (2) assumes the form

$$\left\{m^2 - \left[u^\mu\left(p_\mu - \int k_\mu A_\nu(k)\frac{\delta}{\delta A_\nu(k)}\, dk + \sqrt{4\pi}\frac{e}{(2\pi)^2}\int A_\mu(k)\, dk + i\sqrt{4\pi}\frac{e}{(2\pi)^2}\int D_{\mu\nu}(k)\frac{\delta}{\delta A_\nu(k)}\, dk\right)\right]^2\right\} G(p \mid A) = 1;$$

$$u^\mu = p^\mu / |p|, \quad m \to m - i\varepsilon \quad (\varepsilon > 0,\ \varepsilon \to 0). \tag{3}$$

In this same approximation the photon Green function is

$$D_{\mu\nu}(k) = -g^{\mu\nu} / (k^2 + i\varepsilon) - g^{\mu\nu} / (M^2 - k^2 - i\varepsilon) \quad (M \to \infty).$$
$$g^{\mu\nu} = 0 \ (\mu \neq \nu), \ g^{00} = -g^{11} = -g^{22} = -g^{33} = 1, \tag{4}$$

where M is the Pauli-Villars auxiliary mass.

Using the proper-time method of Fock,[3] Eq. (3) can be solved exactly. Terms containing the auxiliary mass M are removed by renormalization and the following expression is found for the renormalized Green function $G' = (M/m')^{e^2/\pi} G$:

$$G'(p \mid A) = -\int_0^\infty d\nu \int_0^\infty d\mu e^{-\varepsilon(\nu+\mu)} \exp\{-i(m'-up)\nu - i(m'+up)\mu\} (m' \mid \nu+\mu \mid)^{e^2/\pi} \times \exp\left\{-\sqrt{4\pi}\frac{eu^\mu}{(2\pi)^2}\int \frac{\exp\{-i(uk)(\nu-\mu)\}-1}{(uk)} A_\mu(k)\,dk\right\}, \tag{5}$$

where $m' = m + e^2M/2$ is the observed mass. In particular, for $A = 0$, we have:[4]

$$G'(p/0) = \frac{1}{m'^2 - p^2}\left|\frac{m'^2}{m'^2-p^2}\right|^{e^2/\pi}. \tag{6}$$

Applying the Green function,[5] we calculate the probability of the following process. A particle is scattered in the external field and radiates an arbitrary number of long-wave photons with energies less than ε' and n additional photons with energies from E_1 to E_2. It is assumed that the energy and momentum of the radiated photons are small compared with the energy and the change of momentum of the particles in scattering, *i.e.*, $\varepsilon' \ll m'$ and $E_1 > \varepsilon'$. The probability of this process is given by the following expression:

$$dw = dw_0\left(\frac{\varepsilon'}{m'}\right)^{2e^2/\pi}\frac{1}{n!}\left\{\frac{2e^2}{\pi}\ln\frac{E_2}{E_1}\right\}^n, \tag{7}$$

where dw_0 is the scattering probability in the zeroth perturbation theory approximation.

If the condition $(2e^2/\pi)\ln(m'/\varepsilon') \ll 1$ is satisfied, Eq. (7) assumes the form

$$dw = dw_0\frac{1}{n!}\left\{\frac{2e^2}{\pi}\ln\frac{E_2}{E_1}\right\}^n.$$

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Coherent Scattering and Radiation of Electromagnetic Waves by a Plasma in an Inhomogeneous Magnetic Field

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IN VIEW OF THE INTEREST in the potentialities of the intense radiative acceleration interaction in a plasma[1] and the difficulty involved in the formation of stable plasma bunches which provide effective coherent radiation scattering, we propose another possible method for enhancing radiation scattering in a plasma, *viz.* the application to the plasma of local magnetic fields.

It is well known that the presence of a magnetic field in a plasma causes a change in its electromagnetic properties, in particular, in the dielectric constant ε; this change depends not only on the magnitude of the field but also on its direction. Furthermore, the use of an inhomogeneous magnetic field can bring about an inhomogeneous plasma density distribution because of the dependence of the ionization efficiency of a given ionizing agency, for example a high-frequency wave, on magnetic field. Hence, regions of the plasma in which the magnetic field differs noticeably in magnitude or direction are analagous to "optical inhomogeneities."

If the dimensions of the localized region of a given magnetic field are considerably smaller than the wavelength of the incident radiation, the radiation associated with the particles of the plasma is coherent; if there is no strong shielding of the field in the external region (small difference in dielectric constants in the internal and external regions) the radiation is proportional to the square of the number of particles.

For purposes of illustration, we consider coherent scattering of a quasi-plane wave of frequency ω propagating in a wave guide filled with plasma to which there is applied a longitudinal magnetic field to bring about colinear motion of the particles. If the plasma density falls off sharply between the axis of the wave guide and the walls, an effective scattering region may be formed by the application of a supplementary local field having, for example, a localized longitudinal component. The quantities which characterize the average values of the plasma parameters inside and outside the region of effec-