$$x = \frac{1}{2}\ln\frac{1+\sqrt{T_e}}{1-\sqrt{T_e}}\frac{1-\sqrt{T_{e0}}}{1+\sqrt{T_{e0}}} \tag{15}$$
$$-\sqrt{T_e}\left(1+\frac{T_e}{3}\right)+\sqrt{T_{e0}}\left(1+\frac{T_{e0}}{3}\right).$$

The functions $T(x)$, $T_e(x)$, $\Theta(x)$ for $z = 1$, $M_1 = \infty$ are shown in Fig. (3). These curves are not assumed to be highly accurate, since as a result of the low electron temperature in the discontinuity, the exchange of energy in the discontinuity becomes substantial. The behavior of all the quantities in the discontinuity may be obtained for weak waves by including the terms in the viscosity $(\eta/3n)\,dv/dx$ and $-(\eta/3)\,dv/dx$ in the left hand sides of the first and second equations respectively of system (14).

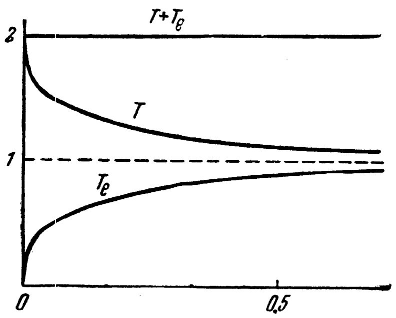

FIG. 3.

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286

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On the Angular Distribution of Deuterons from the $Be_4^9(pd)Be_4^8$ Reaction

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It is shown that even at proton energies ≥ 8 Mev the main contribution is from the region within the Be_4^9 nucleus. This significantly modifies the deuteron angular distribution, good agreement with experiment being obtained for proton energies of ≈ 22 Mev with a Be_4^8 radius $r_0 = 5 \times 10^{-13}$ cm.

WHEN ANALYZING the angular distribution of deuterons from the $Be_4^9(pd)Be_4^8$ reaction on the basis of the theoretical angular distribution from the $Be_4^8(pd)Be_4^9$ stripping reaction and the principle of detailed balance, difficulties arise related to the choice of the nuclear radius r_0. This is determined by agreement between the theoretical and experimental distribution curves at some single point. For nuclei that are not too light, the radius r_0 for the stripping reaction (using Butler's formula for the angular distribution) is given by $r_0 = (1.2A^{1/3} + 1.7) \times 10^{-13}$ cm, where A is the atomic weight of the target nucleus. This value of r_0 is in good agreement with that obtained by scattering of neutrons with energy $E \geq 1$ Mev by nuclei. For light nuclei the value of r_0 is found to be larger than that given by the above formula. Thus, for instance, for the direct and inverse reactions on Li_3^7, B_5^{10}, B_5^{11}, the nuclear radii lie in the interval between 4.5×10^{-13} and 5×10^{-13} cm,[1,2] and depend extremely weakly on the incident particle energies. For the $Be_4^9(dp)Be_4^{10}$ reaction at a deuteron energy of 3.6 Mev, the radius r_0 is found to be 6.1×10^{-13} cm.

At a proton energy $E_p \simeq 16-22$ Mev, however, in order to obtain agreement between the theoretical and experimental deuteron distributions from the

(pd) reaction on Be_4^9, it is necessary to choose r_0 for Be_4^8 in the interval between 3×10^{-13} and 2×10^{-13} cm, which is in sharp disagreement both with r_0 values for neighboring nuclei and for Be_4^9 for low deuteron energies.[1] Reynolds and Standing[1] conclude from this that r_0 must depend on the proton energy.

Previously Gordon[3] had suggested that this sharp decrease of r_0 is related to the increase in the contribution from the region within the nucleus as E_p increases. He did not, however, present any numerical calculations. We present below a calculation of the angular distribution of deuterons from the $Be_4^9(pd)Be_4^8$ reaction with the same assumptions as those made by Butler.[4] It is shown that the region within the Be_4^9 nucleus contributes fundamentally to the process when $E_p \geq 8$ Mev for all angles θ. This situation gives rise to many additional difficulties, since it is necessary to account for proton scattering by Be_4^8 and deformation of the deuteron wave functions due to interaction with this nucleus. For proton energies $E_p > V$ (where V is the potential well depth of the interaction of the proton or deuteron with Be_4^8), plane waves and an expansion in terms of the parameter V/E_p may be used for the zeroth approximation. Then good agreement between the theoretical and experimental distributions is obtained for $E_p = 22$ Mev when we choose $r_0 = 5 \times 10^{-13}$ cm and the radius of action of nuclear forces equal to 1.05×10^{-13} cm.

For lower proton energies the plane wave approximation is too rough, and the distortion of the waves must be accounted for. Such calculations are at present being performed and will be published elsewhere.

The decay of Be_4^8 into two α-particles leads to no difficulties, since the half-life $\tau_1 \approx 10^{-14}$ sec of Be_4^8 is much greater than the time $\tau_2 \approx r_0/v_p \approx 10^{-20}$ sec of the process.

ANGULAR DISTRIBUTION OF DEUTERONS IN THE PLANE WAVE APPROXIMATION

As is well known, Be_4^9 is a loosely bound system analogous to the deuteron (the binding energy of the neutron in Be_4^9 is $\varepsilon = 1.63$ Mev). This makes it possible to develop the theory of stripping of Be_4^9 in the same way as was done by Butler for deuteron stripping.[4] The approximate nature of Butler's assumptions is well known (they have been discussed more than once in the literature), but experiment continues to give good agreement with Butler's theory.

In order to investigate the question of which region is important in the reaction under consideration, it is sufficient to perform the calculation in the plane wave approximation, neglecting the interaction of the proton or deuteron with the Be_4^8 nucleus.

We shall describe the state of the $Be_4^8 + n + p$ system in terms of the coordinate system $\mathbf{R}$, $\mathbf{r}_1$, ρ_1, or $\mathbf{R}$, $\mathbf{r}_2$, ρ_2, where

$$\mathbf{R} = \frac{M_a \mathbf{R}_a + M_n \mathbf{R}_n + M_p \mathbf{R}_p}{M}, \quad \mathbf{r}_1 = \mathbf{R}_n - \mathbf{R}_a; \ \rho_1 = \frac{\mathbf{R}_n M_n + \mathbf{R}_a M_a}{M_n + M_a} - \mathbf{R};$$

$$\mathbf{r}_2 = \mathbf{R}_n - \mathbf{R}_p; \ \rho_2 = \frac{\mathbf{R}_p M_p + \mathbf{R}_n M_n}{M_p + M_n} - \mathbf{R}; \ M = M_a + M_n + M_p.$$

Here $\mathbf{R}_a$, $\mathbf{R}_p$, and $\mathbf{R}_n$ are the radius vectors of the Be_4^8 nucleus (index a), the proton (index p), and the neutron (index n), respectively. We describe the wave function of the system before the reaction in terms of the coordinates $\mathbf{R}$, $\mathbf{r}_1$, ρ_1, and after the reaction in terms of the coordinates $\mathbf{R}$, $\mathbf{r}_2$, ρ_2. We then join the functions at the surface $|\mathbf{r}_2| = r_{20}$ of the second bound system, where in our case r_{20} is simply the radius of action of nuclear forces and varies between 1.05×10^{-23} and 1.7×10^{-23} cm.

The method of calculation is entirely analogous to Butler's,[4] so that we shall not go into detail. Butler has shown that the angular distribution $S(\theta)$ for stripping is given by the integral

$$\frac{1}{r^2}\Phi_{l_2 m_2}(r_2) = \int \psi(\mathbf{r}_1) Y^*_{l_2 m_2}(\theta_{\mathbf{r}_2}\varphi_{\mathbf{r}_2}) \psi_{\mathbf{k}_1}(\rho_1) \psi^*_{k_2}(\rho_2) \, d\Omega_{\mathbf{r}_2} d\rho_2$$

(we have omitted the spin functions and indices). Here $\psi(\mathbf{r}_1)$ is the wave function of the relative motion of the neutron in Be_4^9, $\psi_{\mathbf{k}_1}(\rho_1)$ is the wave function of the relative motion of the proton and Be_4^9 in the initial state, $\psi_{\mathbf{k}_2}(\rho_2)$ is the wave function of the relative motion of the deuteron and Be_4^8 (the system in the final state), l_2 is the orbital angular momentum of the neutron in the final state (in the present case the final nucleus is a deuteron, so that $l_2 = 0$), and we choose $\psi_{\mathbf{k}_1}$ and $\psi_{\mathbf{k}_2}$ in the form of plane waves.

After some operations we obtain the angular distribution function in the form

$$S(\theta_{\mathbf{k}_1\mathbf{k}_2}) = |Q_{l_1}(x_1)|^2 \, |I_{l_2} j_{l_2}(x_2) - II_{l_2} x_2 j_{l_2+1}(x_2)|^2;$$

$$x_1 = \left| \left(\frac{M_p}{M}\right)^{1/2} \left(\frac{M_a}{M_a + M_n}\right) \frac{\sqrt{2(M_a + M_n)W_1}}{\hbar} \mathbf{n}_1 - \left(\frac{M_p + M_n}{M}\right)^{1/2} \frac{\sqrt{2M_a W_2}}{\hbar} \mathbf{n}_2 \right| r_0;$$

$$x_2 = \left| \left(\frac{M_p}{M}\right)^{1/2} \frac{\sqrt{2(M_a + M_n)W_1}}{\hbar} \mathbf{n}_1 - \left(\frac{M_p^2}{M(M_p + M_n)}\right)^{1/2} \frac{\sqrt{2M_a W_2}}{\hbar} \mathbf{n}_2 \right| r_{20};$$

Here $\mathbf{n}_1$ and $\mathbf{n}_2$ are unit vectors in the directions of $\mathbf{k}_1$ and $\mathbf{k}_2$, respectively, $W_1 = E_p(M_a + M_n)/M$, $W_2 = W_1 + Q$, Q is the energy released by the reaction, r_0 is the Be_4^8 radius in the $Be_4^8 + n$ system, r_{20} is the radius at which the functions are joined and lies between 1.05×10^{-13} and 1.7×10^{-13} cm, j is the spherical Bessel function of half-integer index,

$$I_{l_2} = \sum_{q=0}^{l_2} \frac{(l_2+q)!\,[l_2+q+k_n r_{20}]}{q!\,(l_2-q)!\,(2k_n r_{20})^q}, \quad II_{l_2} = \sum_{q=0}^{l_2} \frac{(l_2+q)!}{-q)!\,(2k_n r_{20})^q},$$

and k_n is the wave number of the relative motion of the neutron in the bound state in the final system.

In the case of the (pd) reaction,

$$I_{l_2} = (k_d r_{20}), \quad II_{l_2} = 1, \quad k_d = 0{,}23 \cdot 10^{13} \text{ cm}^{-1},$$

so that

$$S(\theta_{\mathbf{k}_1\mathbf{k}_2}) = |Q_{l_1}(x_1)|^2 \, |(k_d r_{20})\, j_0(x_2) - x_2 j_1(x_2)|^2;$$

$$Q_{l_1}(x_1) = \int_0^\infty R_{l_1}(r_1)\, j_{l_1}(x_1 r_1/r_0)\, r_1^2 dr_1,$$

where R_{l_1} is the radial part of the neutron wave function in the Be_4^9 system. This function can be obtained only for specific models. We shall make use of the wave function used by Guth and Mullin[5] in calculating the cross section for the (γn) reaction for gamma energies between 1.7 and 4 Mev, namely

$$R_{l_1}(r_1) = \begin{cases} A_1 j_1(\beta r_1) & \text{for } r_1 \leqslant r_0, \\ B_1(1 + \alpha r_1)(\alpha r_1)^{-2} e^{-\alpha(r_1 - r_0)} & \text{for } r_1 \geqslant r_0, \end{cases}$$

$$B_1 = -A_1 \sin\beta r_0; \ \beta = [2\mu\hbar^{-2}(V - \varepsilon)]^{1/2}; \ \alpha = \sqrt{2\mu\varepsilon}/\hbar; \ \mu = 8M_n/9,$$

where $V = 12.16$ Mev is the interaction potential well depth of $Be_4^8 + n$, $r_0 = 5 \times 10^{-13}$ cm, and $\varepsilon = 1.63$ Mev. Inserting this function into the integral, we obtain

$$Q_{l_1} = Q_1 + Q_2 = \frac{0.267 x_1 j_0(x_1) + 0.236 j_1(x_1)}{11.5 - x_1^2} + \frac{0.32 x_1 j_0(x_1) - 0.242 j_1(x_1)}{1.77 + x_1^2}$$

The dependence of these quantities on x_1 is given in Figs. 1 and 2. Here Q_1 is the contribution from the region $r_1 \leq r_0$, and Q_2 is the contribution from the region outside the nucleus $r_1 \geq r_0$. It is clearly seen that Q_1 is of the same order as Q_2 even for small values of x_1, so that

$$1.75 \leqslant x_1 \leqslant 6.3 : |Q_1| \geqslant |Q_2|.$$

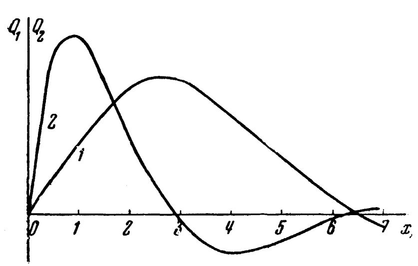

FIG. 1. 1 – plot of $Q_1(x_1)$ (in arbitrary units); 2 – plot of $Q_2(x_1)$.

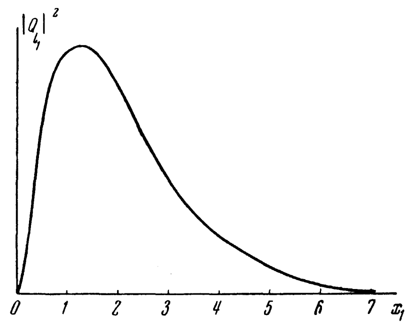

FIG. 2. Plot of $|Q_1(x_1)|^2$ (in arbitrary units).

In the $Be_4^9(pd)Be_4^8$ reaction, the interval over which x_1 varies depends on the proton energy:

$$\begin{aligned}
E_p &= 2 \text{ Mev} \quad 0.92 \leq x_1 \leq 3.57; \quad x_1 = 1.7 \text{ for } \theta = 50^\circ \\
E_p &= 3.6 \text{ Mev} \quad 1.05 \leq x_1 \leq 4.5\ ; \quad x_1 = 1.7 \text{ for } \theta = 33^\circ \\
E_p &= 8 \text{ Mev} \quad 1.37 \leq x_1 \leq 6.0\ ; \quad x_1 = 1.7 \text{ for } \theta = 14^\circ \\
E_p &= 22 \text{ Mev} \quad 2.16 \leq x_1 \leq 10.4\ ;
\end{aligned}$$

thus, even at proton energies $E_p \simeq 8$ Mev, the contribution of the internal region of the Be_4^9 nucleus is significant and comparable with the contribution of the external region. Therefore the angular distribution obtained in the plane wave approximation cannot be used for energies of the order of the potential well depth. The proton distribution as obtained by Butler's theory should lead to large differences between the experimental and theoretical deuteron distribution curves, and these are indeed observed in the form of the sharp difference in the values of r_0 for low and high energies. (Let us bear in mind that r_0 is usually determined from the position of the first maximum in the distribution, which means that for our reaction it is determined from the data for $\theta \sim 25^\circ$ and $E_d \approx 3.6$ Mev, where the contribution from the external region is still most significant.)

When $E_p \gg V$ one may neglect the deformation of the function in the zeroth approximation in V/E_p, and calculate the angular distribution with plane waves. The angular distribution for $E_p = 22$ Mev is shown in Fig. 3. Here the reaction parameters are taken as $r_0 = 5 \times 10^{-13}$ cm (rather than 2×10^{-13} cm, as elsewhere[1,3]), $r_{20} = 1.05 \times 10^{-13}$ cm, and $r_{20} = 1.7 \times 10^{-13}$ cm. The distribution obtained is in good agreement with experiment. It is important to note that the angular distribution is determined basically by the factor $|Q_{l_1}|^2$, and since the wave functions of the proton and deuteron enter primarily into the slowly varying second factor, this may explain the independence of the shape of the angular distribution on the proton energy.

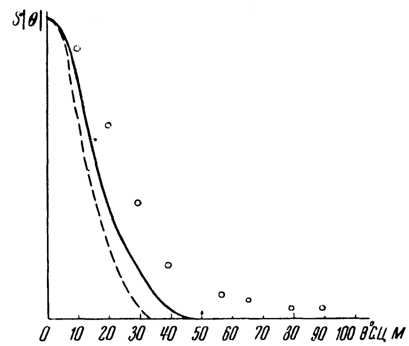

FIG. 3. Angular distribution of deuterons (arbitrary units) obtained in the plane wave approximation. The solid curve corresponds to $r_{20} = 1.05 \times 10^{-13}$ cm, and the dotted curve to $r_{20} = 1.7 \times 10^{-13}$ cm; the proton energy is 22 Mev in the laboratory coordinate system. The circles give the experimental angular distribution.[6]

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287